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Differential detection for measurements of Faraday rotation by means of ac magnetic fields

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Abstract

We demonstrate that by using a combination of a Wollaston prism and two photodiodes the accuracy in the measurements of Faraday rotation with ac magnetic fields can be greatly improved. Our experiments were performed on microscope cover glass plates with thicknesses between 0.13 and 0.16 mm. We show that our setup is capable of distinguishing between the Faraday rotation signals of glass plates having a difference in thickness of a few micrometers, corresponding to Faraday rotations of hundreds of microdegrees per Tesla only.

(Some figures in this article are in colour only in the electronic version)

1. Introduction: the Faraday effect

The Faraday effect corresponds to an interaction between light and a magnetic field in a dielectric medium. More specifically, the phenomenon occurs when the magnetic field is applied along the direction of propagation of the electromagnetic wave where it causes a magnetic field induced birefringence, which results in a rotation of the polarization of linearly polarized light. Empirically, it is found that the angle ϕ of this rotation is given by the formula $\phi = VBL$, where B is the component of the magnetic induction field parallel to the direction of propagation of light, L is the length of the sample and V is the Verdet constant. Materials with large Verdet constants are used in optical switches, modulators, nonreciprocal elements in laser gyroscopes, optical circulators, laser isolators [1] and sensors of magnetic fields and electric currents [2, 3]. The Verdet constant is generally dependent on the temperature, the incident wavelength and the refractive index of the material. For instance, in diamagnetic glasses at 632.8 nm and at room temperature, silicate glasses have a V of 230–1548° T^{-1} m^{-1}. The lowest values are obtained for standard optical glasses such as BK7, while the highest are

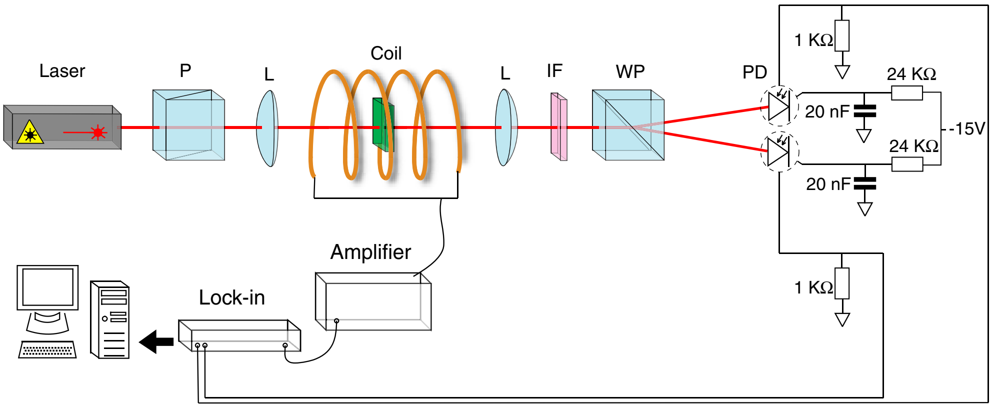

Figure 1. Experimental setup. After passing the polarizer P, the laser beam is focused on the sample by a lens L. It is then made parallel again with a second lens and then traverses an interference bandpass filter IF. Finally, it is directed through the Wollaston prism WP at the photodiodes (PD). The lock-in drives the magnetic coil through the amplifier and detects the signal from the PD at the same frequency.

observed in glasses with high refractive index and large dispersion. *V* is weakly temperature dependent in glasses and it decreases with increasing wavelength [4].

Combining elements of optics and electromagnetism, measurements of Faraday rotation constitute a good introduction to these areas of physics for undergraduate and graduate students [5–8]. In [7] it was proposed that ac rather than dc magnetic fields [9, 10] are more practical and less expensive and offer the additional advantage of introducing students to the phase sensitive lock-in technique [11, 12]. Here, we propose a modified detection scheme for the experiment described in [7]. Indeed, replacing the analyser and the photodiode with a Wollaston prism, which is essentially a polarizing beam splitter [13], and a combination of two identical photodiodes, respectively, allows us to identify the above-mentioned experiment as a special case of ours. Furthermore, not only is our experimental procedure more general but it is also significantly more accurate; we show that our setup is capable of distinguishing between the Faraday rotation signals of glass plates having a difference in thickness of a few micrometers, corresponding to Faraday rotations of hundreds of microdegrees per Tesla only. This accuracy can be achieved because of the main advantage of the differential detection technique, which is that intensity fluctuations in the light source are eliminated.

These improvements are very significant, having in mind that the price of a Wollaston prism is comparable to that of an optical analyser and they constitute a good introduction for students to the differential detection technique.

2. Experimental setup

The Faraday rotation measurements were performed using a Melles Griot 56DOL507 continuous wave laser at a wavelength of 830 nm with a maximum power of 35 mW (see figure 1). The light intensity was adjusted by a Melles Griot diode laser driver 06DLD03. If such a controller is not available, for instance when the light source is a lamp, an alternative is to set the intensity by using neutral density filters. The initial linear polarization of the beam is determined by a polarizer. A lens focuses the light on the sample, which is placed within a magnetic coil. The coil was made from 1 mm thick copper wire and consists of 80 turns in 20 layers. Its length and diameter are 8 cm. The beam is then collimated and, after passing through a Wollaston prism, which is set at 45° with respect to the polarizer to ensure maximum sensitivity, it is split in two parts with different polarization. Each of these beam paths ends at a Hamamatsu S1722 photodiode. At the entrance of the Wollaston prism, an interference bandpass filter ensures that only light at 830 nm is detected. The electrical signal's dc and ac components of both photodiodes are sent to a Stanford Research Systems SR830 lock-in amplifier at channels A and B. This amplifier, which is operated in the A–B mode, generates a reference wave at the desired frequency and amplitude, which after amplification drives the magnetic coil. The magnetic field was modulated by varying the wave amplitude at a frequency of 175 Hz. The lock-in is connected to a computer via RS232, and the entire experiment is operated by a LabView program.

The magnetic field was calibrated with a Magnet-Physik Dr Steingroever GmbH FH-51 gaussmeter, which provides root mean squared values for the ac magnetic field.

3. Theory

We can represent the incoming laser beam with the Jones vector expression for an initial electric field of linearly polarized light along the x axis and propagating along the z direction:

$$E(z,t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_0 \exp(\mathrm{i}kt - \mathrm{i}\omega t) \tag{1}$$

where A_0 is the amplitude, k is the wave number, ω is the angular frequency and t represents the time. Then for a Faraday rotation of ϕ, we obtain

$$E(z,t) = \begin{pmatrix} \cos\phi \\ \sin\phi \end{pmatrix} A_0 \exp(\mathrm{i}kt - \mathrm{i}\omega t). \tag{2}$$

For a Wollaston prism rotated at an angle α we then have

$$E(z,t) = \begin{pmatrix} \cos\alpha\cos\phi - \sin\alpha\sin\phi \\ \sin\alpha\cos\phi + \cos\alpha\sin\phi \end{pmatrix} A_0 \exp(\mathrm{i}kt - \mathrm{i}\omega t) \tag{3}$$

where we recognize that

$$E(z,t) = \begin{pmatrix} \cos(\alpha+\phi) \\ \sin(\alpha+\phi) \end{pmatrix} A_0 \exp(\mathrm{i}kt - \mathrm{i}\omega t). \tag{4}$$

If we position the Wollaston prism along 45°, (4) becomes

$$E(z,t) = \tfrac{1}{\sqrt{2}} \begin{pmatrix} \cos\phi - \sin\phi \\ \cos\phi + \sin\phi \end{pmatrix} A_0 \exp(\mathrm{i}kt - \mathrm{i}\omega t). \tag{5}$$

We can then express the intensity of the two different light waves I_A and I_B, outgoing from the Wollaston prism, as

$$\begin{pmatrix} I_A \\ I_B \end{pmatrix} = \frac{A_0^2}{2} \begin{pmatrix} \cos^2(\phi) + \sin^2(\phi) - 2\cos(\phi)\sin(\phi) \\ \cos^2(\phi) + \sin^2(\phi) + 2\cos(\phi)\sin(\phi) \end{pmatrix} = \frac{A_0^2}{2} \begin{pmatrix} 1 - 2\cos(\phi)\sin(\phi) \\ 1 + 2\cos(\phi)\sin(\phi) \end{pmatrix}, \tag{6}$$

and for small value of the angle ϕ, we see that

$$\begin{pmatrix} I_A \\ I_B \end{pmatrix} = \frac{A_0^2}{2} \begin{pmatrix} 1 - 2\phi \\ 1 + 2\phi \end{pmatrix}. \tag{7}$$

If we chose to measure the signal on only one of the photodiodes, for instance the one that is connected to channel A of the lock-in amplifier (I_A), we see that

$$I_A = \frac{A_0^2}{2}(1 + 2\phi), \tag{8}$$

and since we know that ϕ is proportional to the oscillating magnetic field, i.e. $\phi = \phi_0 \cos(\Omega t)$, where Ω is the frequency of the ac magnetic field, we can write

$$I_A = \frac{A_0^2}{2} + A_0^2 \phi_0 \cos(\Omega t) = V_{\text{dc},A} + V_{\text{ac},A} \cos(\Omega t) \tag{9}$$

where $V_{\text{dc},A}$ and $V_{\text{ac},A}$ are the voltages of the dc and ac components of the detected signal I_A, respectively. $V_{\text{ac},A}$ originates from the modulation of the light intensity at the frequency of the oscillating magnetic field, and $V_{\text{dc},A}$ corresponds to the average light intensity detected by the photodiode. We can identify the parts of (9) to obtain

$$V_{\text{dc},A} = \frac{A_0^2}{2} \qquad \text{and} \qquad V_{\text{ac},A} = A_0^2 \phi_0. \tag{10}$$

It follows immediately that

$$\phi_0 = \frac{V_{\text{ac},A}}{A_0^2} = \frac{V_{\text{ac},A}}{2V_{\text{dc},A}}, \tag{11}$$

which is also the expression obtained in [7]. Note, however, that since the Wollaston prism is positioned at 45° with respect to the polarizer, each of the photodiodes detects only half of the incoming intensity. Consequently, to obtain the actual Faraday rotation, the right-hand side of (11) has to be multiplied by a factor of 2.

A more accurate way to evaluate the Faraday rotation is to subtract the x and y signals from the Wollaston prism. Substituting with $\phi = \phi_0 \cos(\Omega t)$ in (7), we obtain then

$$I_A - I_B = 2A_0^2 \phi_0 \cos(\Omega t) = V_{\text{ac}} \cos(\Omega t), \tag{12}$$

where V_{ac} is the signal as measured by the lock-in in the A–B channel mode. Consequently, using the expression for A_0^2 in (10),

$$\phi_0 = \frac{V_{\text{ac}}}{2A_0^2} = \frac{V_{\text{ac}}}{4V_{\text{dc},A}} = \frac{V_{\text{ac}}}{4V_{\text{dc},B}}. \tag{13}$$

4. Experimental procedure

In all of our measurements, care was taken to avoid electronic interference and the recorded data were obtained from high light intensity signals to avoid the average light intensity issue discussed in [8]. The samples used for demonstrating the accuracy of the experiment were 22×22 mm^2 microscope borosilicate glass cover slides from VWR international. Similar microscope glass slides are widely available in undergraduate laboratories. According to the manufacturer, the thickness of the VWR slides is between 0.13 and 0.16 mm. We measured the thickness of the glass plates with a mechanical micrometer (model 806.D25 from Facom), which has an accuracy of 2.5 μm. In the three batches that we examined, most glass slides were found to have thickness comprised between 0.15 and 0.16 mm. We selected three glass plates having sufficiently distinct thicknesses: 0.155, 0.160 and 0.150 mm. Each of the samples was measured five times. In between measurements, the samples were taken several times in and out of the sample holder in order to ensure that the differences measured between samples were not attributed to an experimental artefact.

Figure 2 shows the lock-in response versus amplitude of the applied magnetic field for all the measurements. The Faraday rotation responses from all three samples are clearly distinguishable and the reproducibility in the data for each of the samples is very high. In the inset, the measured sample thickness is plotted versus the average slope of the curves per sample in figure 2. Illustrating that the Faraday rotation is directly proportional to the thickness

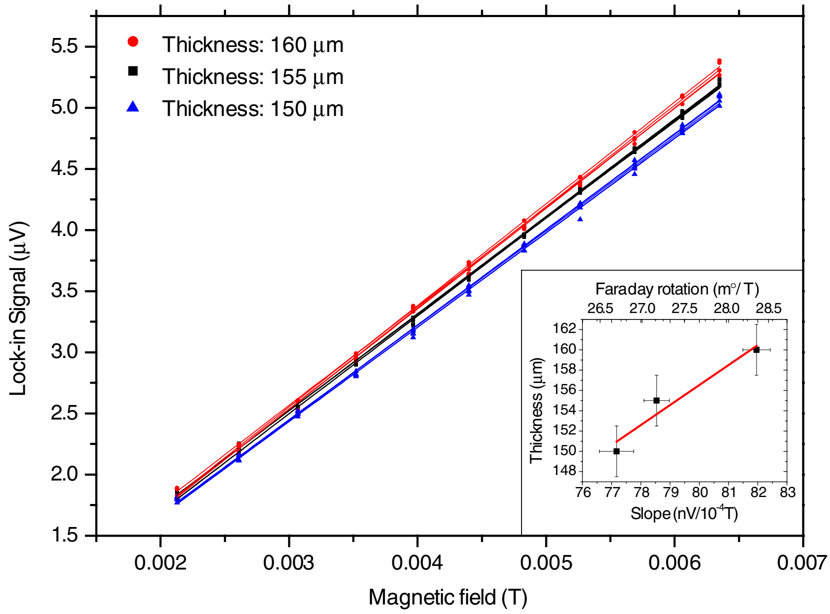

Figure 2. Lock-in signal versus applied magnetic field for three samples with different thicknesses during the measurement of Faraday rotation in borosilicate glass. In the inset, the average slopes per sample versus the sample thickness and the corresponding Faraday rotation in mdeg T^{-1}.

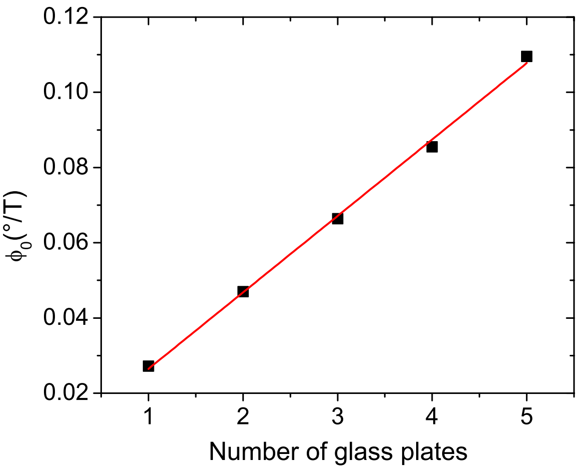

Figure 3. Faraday rotation in deg T^{-1} versus the borosilicate glass thickness, expressed in number of glass plates.

of the material, a linear dependence is observed. While the difference in slopes corresponds to a Faraday rotation of the order of 1 mdeg T^{-1}, the error bars on the slopes, representing the standard deviation, correspond to Faraday rotations smaller than 200 μdeg T^{-1}.

In order to obtain the Verdet constant of the borosilicate glass, the Faraday rotation was measured as a function of the thickness of the material (see figure 3). In this manner, the obtained slope is directly proportional to the Faraday rotation of the glass and the intercept, which is sensitive to the noise floor of the lock-in amplifier, becomes irrelevant. Taking the thickness values specified by the manufacturer, the Verdet constant is found to be between 127 and 156 deg T^{-1} m^{-1}, which is in very good agreement with literature values (see [7, 8] and references therein). Note that in this case, the precision is mainly limited by the measurement of the sample thickness and not by the corresponding Faraday rotation. It was, of course, possible to measure this thickness in a more accurate way using alternative methods; however, a mechanical micrometer offers the advantages of being cheaper and more likely available in an undergraduate or graduate level laboratory. It should also be noted that multiple reflections are likely to play a role while stacking up a large number of glass plates. If this occurs, the samples can be positioned at a small angle with respect to the incident light beam.

5. Conclusions

We have presented a more general way of measuring the Faraday rotation with ac magnetic fields. By making use of a Wollaston prism in combination with two photodiodes, the Faraday rotation setup becomes highly accurate, capable of measuring differences between the Faraday rotation signals of glass plates having a difference in thickness of a few micrometers, corresponding to Faraday rotations of tens of nanodegrees per Gauss only.

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