

Precise measurements of Faraday rotation using ac magnetic fields

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We discuss several important issues concerning the use of a lock-in amplifier and the experimental procedure that need to be addressed to obtain accurate measurements of the Faraday rotation using ac magnetic fields. Our study was conducted on BK7 glass. We show that if electronic interference is avoided and the dependence of the signal on the average light intensity is taken into account, an accurate value of the Verdet constant for thin samples can be determined. © 2008 American Association of Physics Teachers.

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I. INTRODUCTION

The Faraday effect is a magneto-optical phenomenon that occurs when a magnetic field is applied to a dielectric material along the direction of propagation of a light beam. In particular, Faraday rotation is the rotation in the plane of polarization of linearly polarized light due to magnetic field induced birefringence. The angle ϕ of this rotation is found empirically to be given by the relation $\phi = VBL$,¹ where B is the component of the magnetic induction field parallel to the direction of propagation of light, L is the length of the sample, and V is the Verdet constant. Materials with large Verdet constants are used in optical switches, modulators, nonreciprocal elements in laser gyroscopes, optical circulators, laser isolators,² and sensors of magnetic fields and electric currents.^{3,4}

Because the Faraday rotation combines elements of optics and electromagnetism, measurements of Faraday rotation are a worthwhile introduction to these areas of physics for senior undergraduate and graduate students.^{5,6} In Ref. 7 it was proposed that ac rather than dc magnetic fields^{8,9} are more practical and less expensive and offer the additional advantage of introducing students to the phase sensitive lock-in technique.^{10,11}

Although the ac field method is straightforward, a few simple precautions involving the use of a lock-in amplifier and the experimental procedure should be taken. Failure to do so might yield noisy or even incorrect results. The need for such precautions becomes especially apparent when small signals are measured, for instance in thin films, or when only a low intensity light source is available. We show that using a lock-in and a voltage amplifier can lead to unexpected electronic interference, and even render the results inaccurate. The experimental procedures must be extended to include measurements at different light intensities to take into account the average light intensity dependence of the measurements. After addressing these issues, it is demonstrated that it is possible to obtain an accurate value of the Verdet constant for thin samples, and/or samples illuminated by a light source of low intensity, producing signals of only a few μV .

II. APPARATUS

The Faraday rotation measurements were performed using a Melles Griot 56DOL507 continuous wave laser at a wavelength of 830 nm with a maximum power of 35 mW (see Fig. 1). The light intensity was adjusted by a Melles Griot

diode laser driver 06DLD03. If such a controller is not available, for instance when the light source is a lamp, an alternative is to set the intensity by using neutral density filters. The initial linear polarization of the beam is determined by a polarizer. A lens focuses the light on the sample, which is placed within a magnetic coil. The coil was made from 1-mm-thick copper wire and consists of 80 turns in 20 layers. Its length and diameter are 8 cm. The beam is then collimated and, after passing through an analyzer, which is set at 45° with respect to the polarizer to ensure maximum sensitivity,⁷ reaches the Hamamatsu S1722 photodiode. At the entrance of the photodiode, an interference filter ensures that only light at 830 nm is detected. Both the electrical signal's dc and ac components are sent to a Stanford Research Systems SR830 lock-in amplifier. This amplifier generates a reference wave at the desired frequency and amplitude, which after amplification drives the magnetic coil. The magnetic field can be modulated by either varying the frequency at a given wave amplitude, or vice versa. The lock-in is connected to a computer via RS232, and the entire experiment is operated by a LABVIEW program, which took one of our undergraduates, who had previously been unfamiliar with the programming language but was acquainted with other programming languages, two weeks to complete.

The magnetic field was calibrated with a Magnet-Physik Dr. Steingroever GmbH FH-51 gaussmeter, which provides root mean squared values for the ac magnetic field.

III. THEORY

We start with the Jones vector expression for an initial electric field of linearly polarized light along the x axis and propagating along the z direction,

$$E_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} A_0 \exp(ikz - i\omega t), \quad (1)$$

where A_0 is the amplitude, k is the wave number, ω is the angular frequency and t represents the time. It can be shown that the Faraday rotation angle ϕ from the direction x is given by⁷

$$\phi = \frac{V_{ac}}{2V_{dc}} = VLB, \quad (2)$$

where V_{ac} and V_{dc} are the voltages of the ac and dc components of the detected signal, respectively. The former originates from the modulation of the light intensity at the frequency of the oscillating magnetic field, and the latter

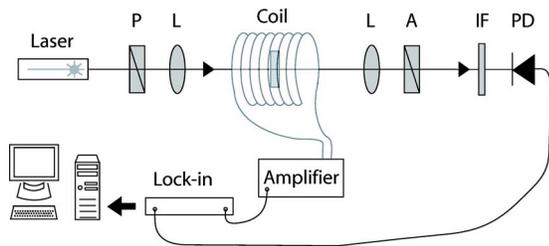


Fig. 1. Experimental setup. After passing the polarizer P, the laser beam is focused on the sample by a lens L. It is then made parallel again with a second lens and then traverses the analyzer A. Finally, it is directed through an interference filter IF at the photodiode (PD). The lock-in drives the magnetic coil through the amplifier and detects the signal from the PD at the same frequency.

corresponds to the average light intensity detected by the photodiode. Note that V_{ac} as measured by the lock-in amplifier has units of Volts root mean squared. As implied in Ref. 7 and explicitly stated in Ref. 8, the ratio $V_{ac}/2V_{dc}$ is independent of the average light intensity. We will address this point in Sec. IV.

IV. EXPERIMENTAL PROCEDURE

Like most lock-in amplifiers, the SR830 is conveniently equipped with auxiliary inputs which can be used for the measurement of V_{dc} . The signal cable from the photodiode can be connected to the signal input (to measure V_{ac}) and the auxiliary input of the lock-in, by means of an electrical T connection. This connection allows the user to simultaneously measure the two quantities necessary for determining the Faraday rotation.

Although this wiring appears to meet the requirements of the experiment, it introduces an undesired signal because there is an internal leak from the SR830 lock-in frequency generator to the auxiliary inputs. The existence of this leak was verified on three SR830 instruments and on one SR530 unit; we suspect that this leak is an issue in other models, probably originating in the internal wiring of the lock-in frequency generator. The reference signal of the lock-in frequency generator travels through the auxiliary input and into the signal input, where it mixes with the signal from the photodiode. The waves have the same frequency, but different phase and amplitude. Whenever small Faraday rotations are being measured, the unwanted signal can be greater by more than an order of magnitude.

Because users are not likely to expect a voltage input to behave as an electrical source, they might assume that they are observing noise. The usual practice then is to block the beam and determine the baseline, which is to be subtracted from the actual measurement. However, it should be noted that we are not discussing additive quantities, but waves.

When the beam is blocked, the lock-in will detect mainly the signal leakage through the auxiliary input. As the beam is released, both signals mix and, depending on their relative phase, the result could be anything between a minimum and a maximum. Therefore, the measurement of the Faraday rotation will be inaccurate.

This experimental artifact introduces a relation between the lock-in signal R and the lock-in phase θ , which are unrelated in a proper experiment. The correlation is visible in Fig. 2, where instead of being random, the data exhibit clear patterns. The patterns do not appear to be symmetrical or

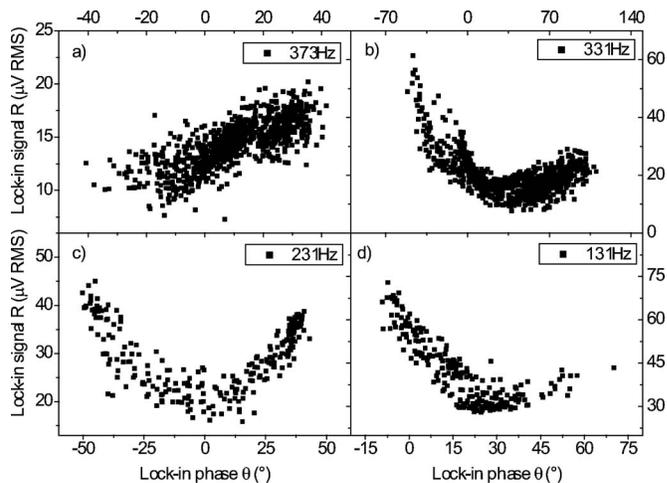


Fig. 2. Lock-in signal R versus the lock-in phase θ for different frequencies generated by the lock-in during the measurement of Faraday rotation in 1-mm-thick BK7 glass plate. Although the patterns should be random, the correlation between R and θ is visible in the data.

follow a 360° phase rotation pattern (shifting systematically as a function of angle to repeat itself as the phase angle passes through 360°), suggesting that the correlation is only partial.

As mentioned in Ref. 8, the use of a separate detector, such as a digital voltmeter, results in a similar problem. The best way to resolve the difficulty is to measure V_{dc} before and after the experiment. After disconnecting the cable from the auxiliary input, our data yield random patterns.

Electrical signal leakage can also be observed through the ground connection between the voltage amplifier and the photodiode. This fact can be revealed, for example, by examining the Fourier spectrum of the photodiode signal with an oscilloscope after blocking the beam. In addition to the correlation of R and θ , the noise source skews the distribution of the signal values, as illustrated in Fig. 3(a). Users who take only a few experimental points and do not perform sufficient averaging are likely to obtain very different Faraday rotations from different samples of the same material, because of the statistics.

This issue can be addressed by placing the photodiode on an independent power supply, ideally batteries. The result is a clear improvement of the statistics [see Fig. 3(b)].

A somewhat less important difficulty is the possibility of external interference with the measurements. This interference is likely to occur in a laboratory where many instruments are functioning simultaneously or where students are operating several Faraday setups at the same time. An example can be seen in Fig. 3(b) for 1025 and 975 Hz Faraday rotations corresponding to magnetic fields generated around these frequencies. We solved the problem by choosing a single, interference-free frequency at which the magnetic field is generated. Whenever multiple Faraday setups are in use, for instance in a classroom, they should all be set at a different frequency. Variation of the magnitude of the field is achieved by modulating the amplitude of the oscillations.

As mentioned in Sec. III, it is assumed in Eq. (2) that the ratio V_{ac}/V_{dc} is independent of the average light intensity. That is, V_{ac} is of the form $V_{ac}=bV_{dc}+a$ with $a=0$. Experimentally we find that $a \neq 0$ [see Fig. 4(a)]. This difference can be attributed to the experimental noise, and is impossible

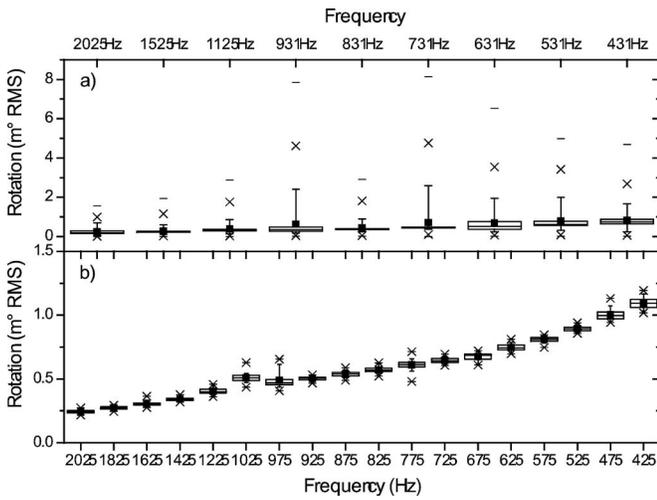


Fig. 3. (a) Skewed Faraday rotation distribution from 1-mm-thick BK7 glass (1000 data points per frequency) when signal leaks from the voltage amplifier to the photodiode. (b) Faraday rotation from the same material, after placing the photodiode on an independent power supply. Because of the significant signal-to-noise improvement, 50 points per frequency were sufficient. The mean is indicated by full squares, the boxes enclose 25-75% of the points, the median is indicated by a line in the middle of the boxes, 99% of the points are between cross signs and the dashes designate the maximum and minimum.

to eliminate entirely. It is also present in the linear dependence of V_{ac} on the applied magnetic field for a given average light intensity [see Fig. 4(b)].

It follows from Eq. (2) that the Faraday rotation is given by $\phi = b/2 + a/2V_{dc}$. The second term is responsible for the light intensity dependence of ϕ and hence affects the measurement of the Verdet constant. This dependence can be dramatic as illustrated in Fig. 5(a).

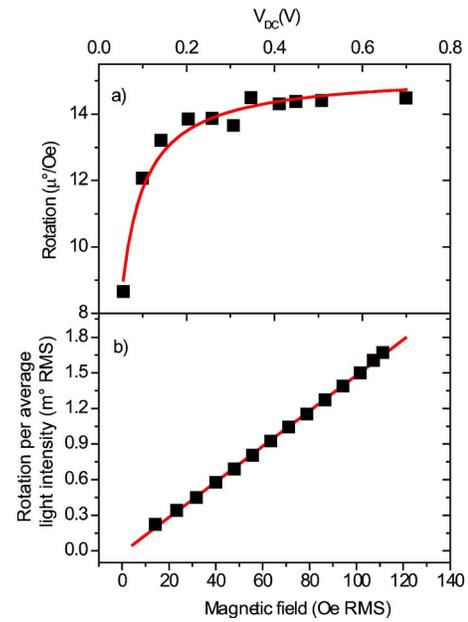


Fig. 5. (a) Faraday rotation per Oe, as a function of average light intensity in 1 mm thick BK7 glass. (b) Faraday rotation per average light intensity, as a function of the magnetic field in 1 mm thick BK7 glass.

To avoid the light intensity dependence of the measured Verdet constant, we can measure V_{ac} as function of V_{dc} for different magnetic fields [as in Fig. 4(a)]. The slopes of these curves, divided by two, give a ratio which is independent of the average light intensity. When plotted as a function of the magnetic field, this slope gives the amount of Faraday rotation [see Fig. 5(b)]. This method gives the Faraday rotation per Oe as $\phi = b/2 = 0.01498$ m°/Oe.

Similarly, it is possible to fit the curve in Fig. 5(a) with the

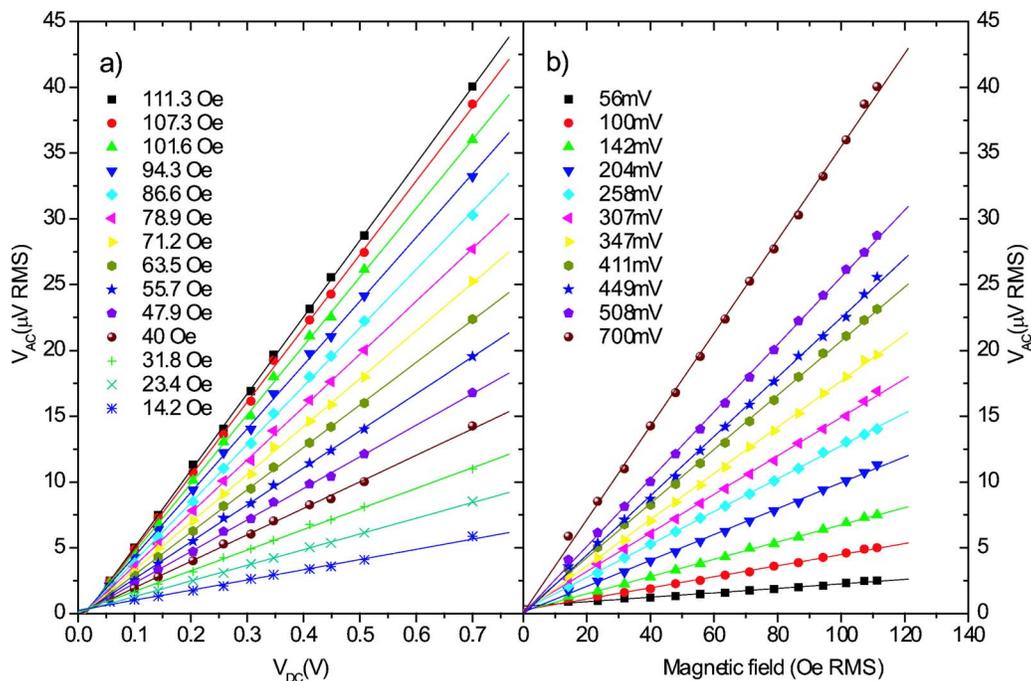


Fig. 4. (a) Lock-in signal V_{ac} as a function of the average light intensity V_{dc} for different magnetic fields generated at 125 Hz in 1 mm thick BK7 glass. (b) Lock-in signal V_{ac} as a function of the magnetic field at different average light intensities V_{dc} in 1 mm thick BK7 glass.

relation $\phi(V_{dc})=b/2+a/(2V_{dc})$; the ϕ value is $0.01523 \text{ m}^\circ/\text{Oe}$. This value is in good agreement with the result determined from Fig. 5(b), and represents the limit to which the curve in Fig. 5(a) converges for increasing average light intensity.

For a thickness of 1 mm BK7 glass, we find that the Verdet constant at 830 nm is $150 \pm 12^\circ/(\text{Tm})$, which corresponds well to the values in the literature (see Ref. 7 and references therein).

V. CONCLUSIONS

We have shown that important issues must be addressed when performing measurements of Faraday rotation with ac magnetic fields. In particular, the signal leakage from the instrument's voltage inputs can significantly contaminate the results when using a lock-in amplifier. Electronic interference from the voltage amplifier can be a large source of noise, potentially masking the real signal. The experimental procedure should be extended to take into account the average light intensity dependence of the measurements. The latter, although unexpected theoretically, is unavoidable because there is always some experimental noise. Accurate Verdet constants can be obtained by addressing these issues.

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¹We have designated the Faraday rotation angle by the symbol ϕ to avoid confusion with the lock-in phase θ .

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