Learning a microlocal prior for limited-angle tomography

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1. Introduction

Limited-angle tomography is a highly ill-posed linear inverse problem. Reconstructions typically suffer from severe stretching of features along the central direction of projections, leading to poor separation between slices perpendicular to the central direction. A new method is introduced, based on machine learning and geometry, producing an estimate for interfaces between regions of different X-ray attenuation. The estimate can be presented on top of the reconstruction, indicating more reliably the true form and extent of features. The method uses directional edge detection, implemented using complex wavelets and enhanced with morphological operations. By using machine learning, the visible part of the wavefront set is first extracted and then extended to the full domain, filling in the parts of the wavefront set that would otherwise be hidden due to the lack of measurement directions.

2. Inverse Problem and Optimization

The inverse problem of reconstructing a tomographic image $\mathbf{f} \in \mathbb{R}^n$ based on X-ray measurements $\mathbf{m} \in \mathbb{R}^m$ is modeled by

$$\mathbf{m} = A\mathbf{f} + \epsilon, \tag{1}$$

where $A \in \mathbb{R}^{m\times n}$ is a linear forward operator and $\epsilon > 0$ models additive Gaussian noise. We consider regularized solutions to the inverse problems, achieved by minimizing the following functional:

$$\mathbf{f}_S = \left\{ \underset{\mathbf{f}\in\mathbb{R}^n, \mathbf{f}\geq 0}{\arg\min} \frac{1}{2}\|A\mathbf{f} - m\|_2^2 + \mu\|W_{\mathbb{C}}\mathbf{f}\|_1 \right\}, \tag{2}$$

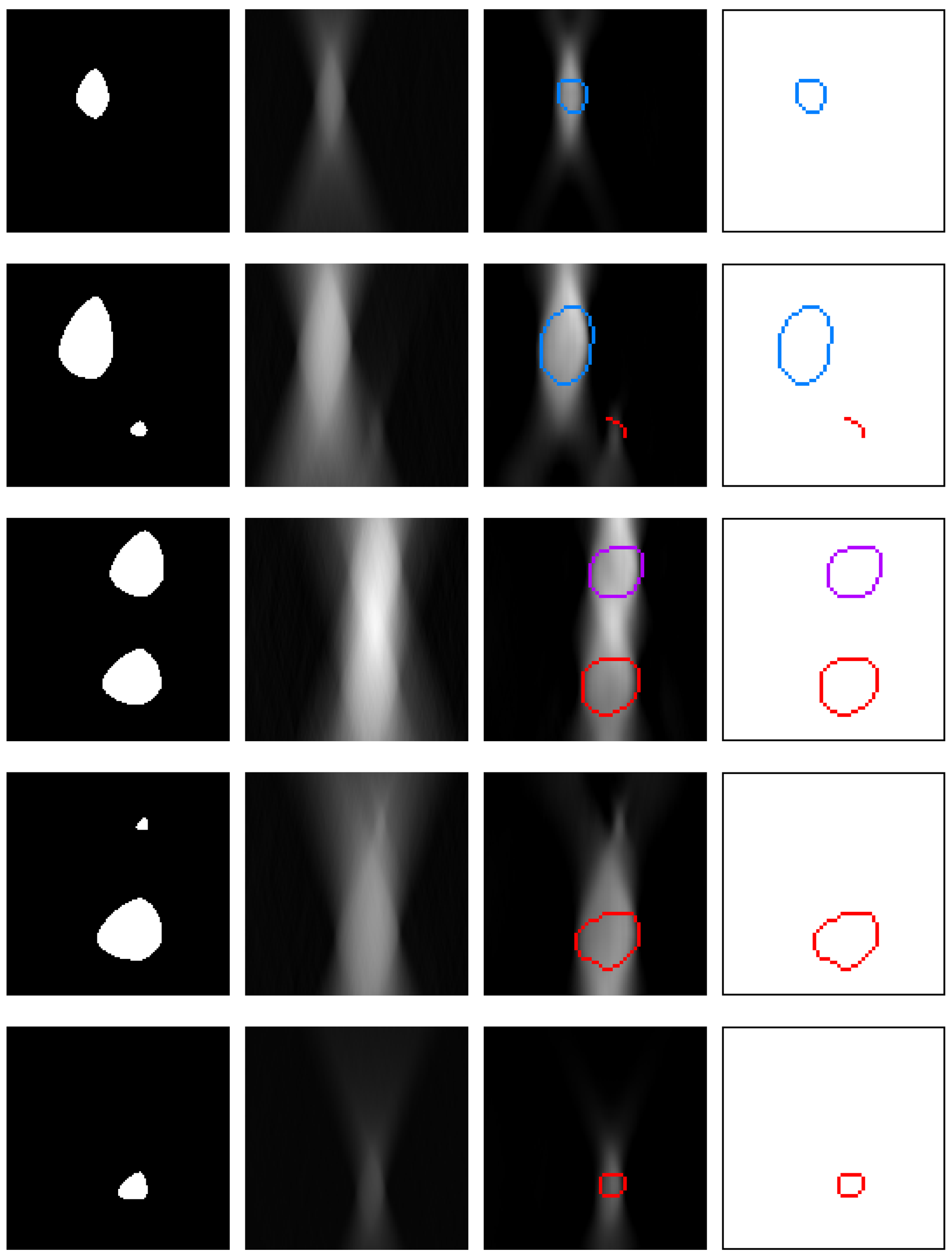
where $\mu > 0$ serves as a regularization parameter.

The PDFP algorithm [1] can be used to iteratively solve the above minimization problem:

$$\begin{cases} \mathbf{y}^{k+1} & = \mathbb{P}_C(\mathbf{f}^k - \tau \bigtriangledown \mathcal{G}(\mathbf{f}^{(k)}) - \lambda(W_{\mathbb{C}})^T\mathbf{v}^k), \\ \mathbf{v}^{k+1} & = (I - \mathcal{T}_{\mu\tau/\lambda})(W_{\mathbb{C}}\mathbf{y}^{k+1} + \mathbf{v}^k), \\ \mathbf{f}^{k+1} & = \mathbb{P}_C(\mathbf{f}^k - \tau \bigtriangledown \mathcal{G}(\mathbf{f}^{(k)}) - \lambda(W_{\mathbb{C}})^T\mathbf{v}^{(k+1)}), \end{cases}$$

where $\mathbb{P}_C$ is the Euclidean projection, τ and λ are positive parameters, $\mathcal{G}(\mathbf{f}) = \frac{1}{2}\|A\mathbf{f} - \mathbf{m}\|_2^2$, $\mu > 0$ represents the regularization parameter, and $\mathcal{T}$ is the soft-thresholding operator.

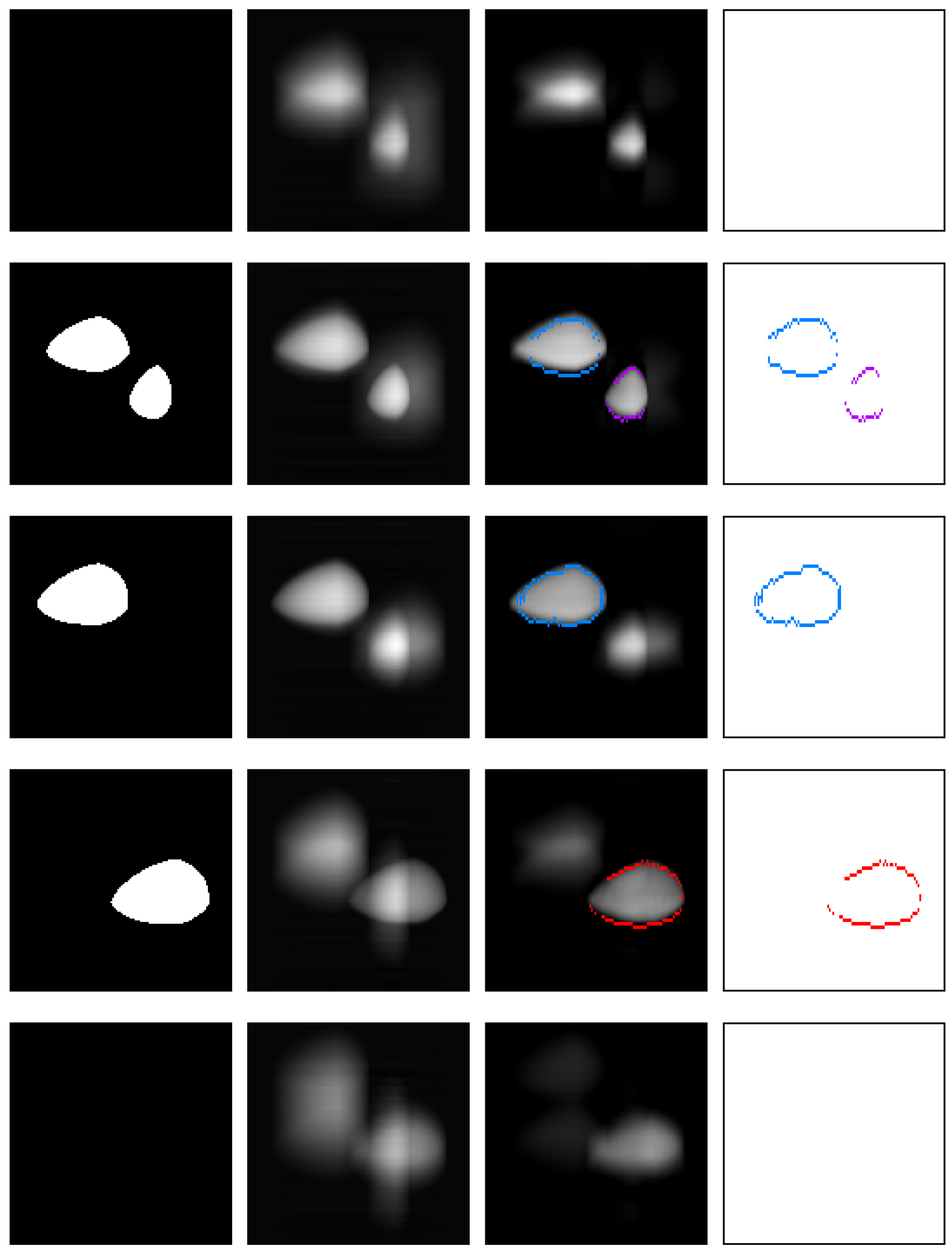
5. Results: xz-slices



(a) (b) (c) (d)

(a) Ground truth slice. (b) Tomosynthesis reconstruction. (c) PDFP reconstruction with learned boundary estimate. (d) Boundary estimate curve.

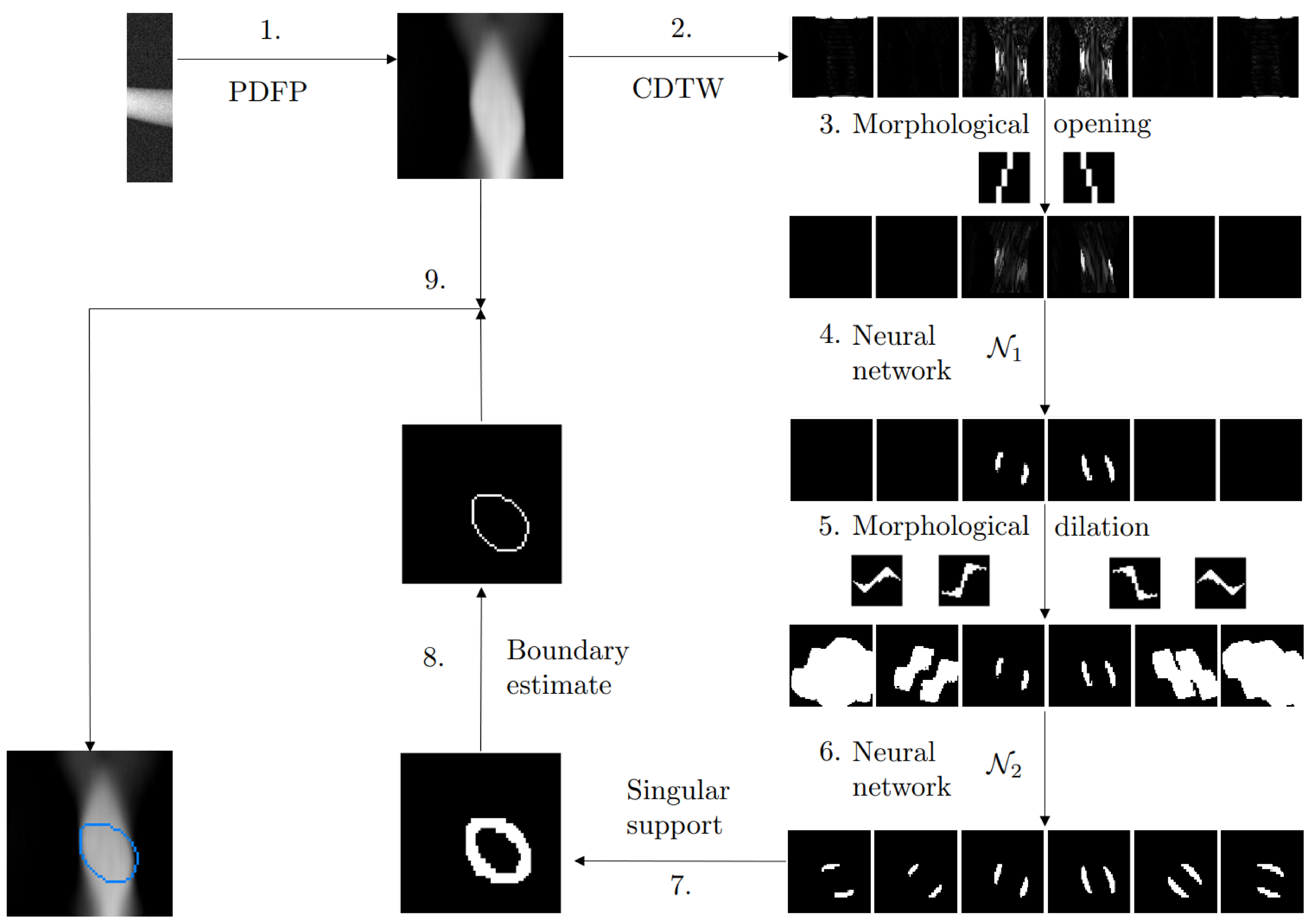
6. Results: xy-slices



(a) (b) (c) (d)

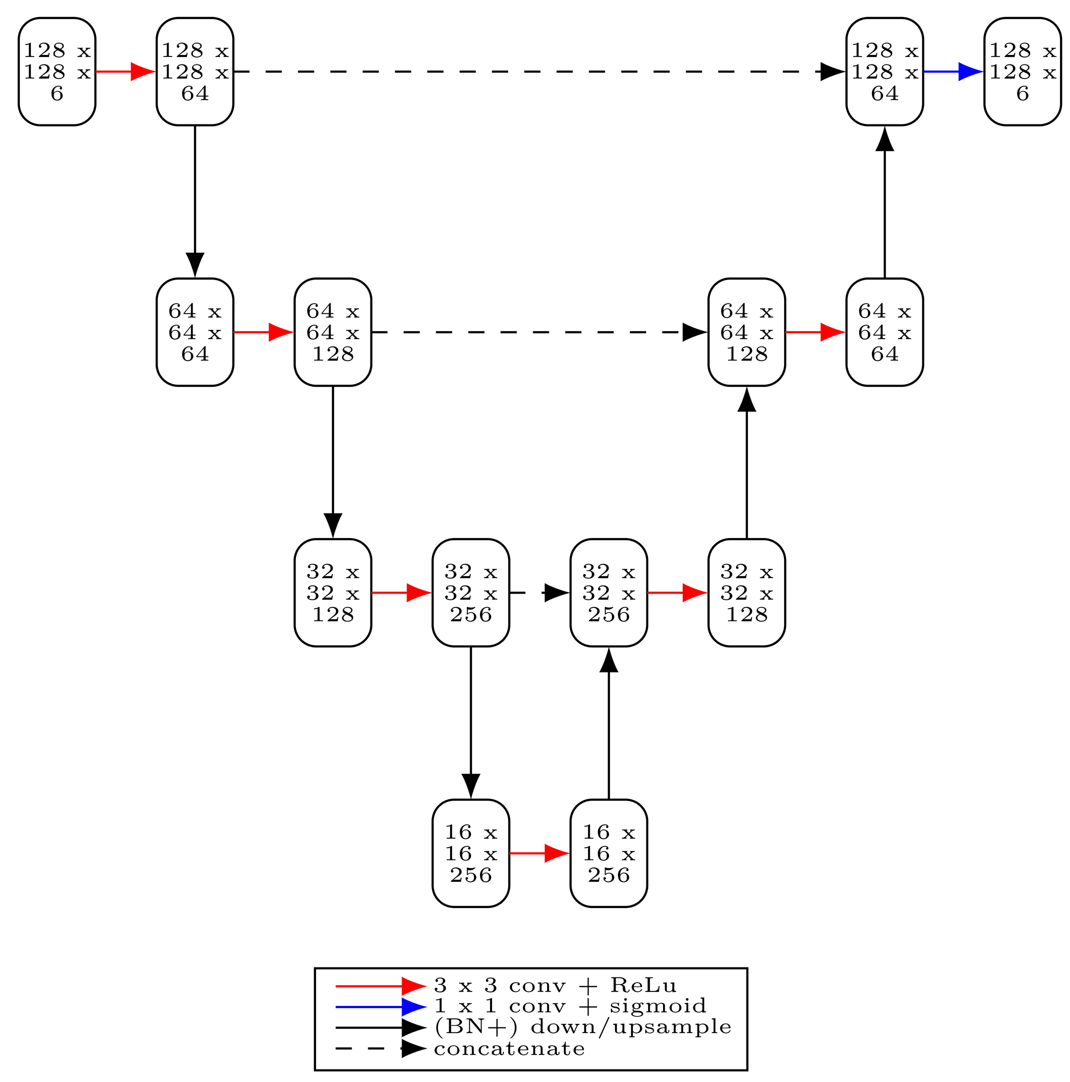
(a) Ground truth slice. (b) Tomosynthesis reconstruction. (c) PDFP reconstruction with learned boundary estimate. (d) Boundary estimate curve.

3. Method



1. Reconstruction using the PDFP algorithm with complex wavelet regularization. 2. Compute the finest scale complex wavelet coefficients [2] and take their absolute value. 3. Clean the coefficients using morphological opening with oriented line structuring elements. 4. Use a neural network to threshold the coefficients into a binary format. 5. Compute an initial guess of the microlocal prior, by dilating the binary subbands with custom-made structuring elements. 6. Use a network to predict the wavefront set in all six subbands. 7. Combine the information to form the singular support. 8. Compute the morphological skeleton of the singular support, estimating its boundary. 9. Add the learned boundary estimate on top of the reconstruction.

4. Neural Network Architecture



7. Main references

[1] Peijun Chen, Jianguo Huang, and Xiaoqun Zhang. A primal–dual fixed point algorithm for convex separable minimization with applications to image restoration. *Inverse Problems*, 29(2):025011, 2013.

[2] Ivan W Selesnick, Richard G Baraniuk, and Nick C Kingsbury. The dual-tree complex wavelet transform. *IEEE signal processing magazine*, 22(6):123–151, 2005.