MA10230 Homework Hints

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Abstract

Hints for homework problem sheet questions for the MA10230 Methods and Applications course at the University of Bath, during the 2020/21 academic year.

Problem Sheet 9

This problem sheet focuses on how to solve 0-degree homogeneous, Bernoulli, and exact differential equations.

Question 1

Part e)

We wish to find an implicit solution to the homogeneous differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^3 + y^3}{xy^2}$$

- Write the ODE in the form $\frac{dy}{dx} = g(\frac{y}{x})$
- Use the substitution $u = \frac{y}{x}$ to obtain a *separable* ODE in u.
- Solve for u and be sure to undo your substitution to get the desired solution y.

Question 2

Part c)

We are asked to find the solution to the first-order differential equation

$$x\,\frac{\mathrm{d}y}{\mathrm{d}x} + y = xy^2$$

- It is a Bernoulli equation so figure out what n (see lecture notes) should be in this case.
- Use the substitution $z = y^{1-n}$ to get a *linear* differential equation in z.
- Solve for z and be sure to undo your substitution to get the desired solution y.

Question 3

Part b)

We are asked to find the general solution of the first-order differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{x^2y^2 + y^5}$$

- Use the hint: swap the roles of x and y for *everything*. This means that whenever you look at theorems/methods in the lecture notes, you should keep in mind that x should be replaced by y and vice-versa.
- Determine what sort of differential equation it is: homogeneous, Bernoulli, or exact (remember x is swapped with y!).
- Since y (or equivalently x) is a function of a single variable, we have that $1/(\frac{\mathrm{d}y}{\mathrm{d}x}) = \frac{\mathrm{d}x}{\mathrm{d}y}$. Keep in mind that this would not be so straightforward if we had more than one variable, $y = y(x,t)$, say.

Question 4

Part c)

We are asked to find the general solution of the exact differential equation

$$y^2 - 2xy + e^y + (y - x^2 + xe^y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

- Perform a check for exactness.
- The general solution is given by $f(x,y) = c$, where $f = \int M\mathrm{d}x = \int N\mathrm{d}y$. Remember that for the indefinite integral of a multi-variable function, rather than a constant of integration '$+c$' appearing, we get an entire function in terms of the variable that we did not integrate with respect to.
- Play spot-the-difference to deduce what these 'functions of integration' should be and hence f (or use the equation derived in lectures, that is also fine).