MA10230 Homework Hints

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Abstract

Hints for homework problem sheet questions for the MA10230 Methods and Applications course at the University of Bath, during the 2020/21 academic year.

Problem Sheet 8

This problem sheet focuses on how to solve separable and linear first order ODEs.

The general recipe for solving ODEs encountered on this problem sheet (you will encounter other methods for more general types of first order ODEs next week) is as follows:

- Ensure that the $\frac{dy}{dx}$ term is isolated, that is, has no coefficient/function multiplying it.
- Think about whether the ODE is separable (as this is usually a more straightforward solution to obtain) or if we must use the integrating factor method.
- Depending on your answer to the above, calculate an implicit/explicit *general* solution as detailed in the lecture notes.
- Apply any boundary/initial conditions to your general solution to determine any unknown constants and obtain an implicit/explicit *specific* solution.

Question 1

Part e)

We wish to solve the separable ODE

$$\frac{du}{dx} = x^2u + 1 + u + x^2$$

subject to the initial condition

$$u(0) = 0.$$

- The question explicitly says this is a separable ODE, so think about how you can write it in the form $\frac{du}{dx} = p(x)\, q(y)$

Question 2

We are told that the cooling process of an object at temperature T which is placed in a moving current of air with (constant) temperature T_A is described by the differential equation

$$\frac{dT}{dt} = k(T_A - T),$$

where $k > 0$ is a constant.

Part a)

If $T = T_0$ at $t = 0$, solve the equation for T

- Use the context of the question: since the object is cooling, what can we say about the relationship between T_A and T?

Part b)

A bunch of values are prescribed and we are asked to calculate k and hence find T when $t = 20$ minutes.

- Substitute everything into your general solution from a) and rearrange for k.
- So long as your are consistent with your units whenever you substitute values in, you can assume that t is measured in minutes (rather than seconds).

Question 4

Part d) and e)

We are asked to find the general solution of first-order linear differential equations.

- Be sure you substitute in the correct expression for the integrand in the integrating factor.
- When calculating the integrating factor remember that, even though we have an indefinite integral, we can take the constant of integration as 0 - see lecture notes for details as to why.