MA10230 Homework Hints

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Abstract

Hints for homework problem sheet questions for the MA10230 Methods and Applications course at the University of Bath, during the 2020/21 academic year.

Problem Sheet 7

This problem sheet focuses on how to evaluate triple integrals and how to perform a change of variable into spherical coordinates.

Question 1

Part d)

We wish to evaluate the triple integral

$$\iiint_G xy^2 \cosh(xyz)\,\mathrm{d}V,$$

where G is a cuboid defined by $0 \leq x \leq 2,\ 0 \leq y \leq \frac{1}{2}\ln(3),\ 0 \leq z \leq 1$.

- Think about a good choice of order of integration (we are lazy and want to avoid doing integration by parts where possible, although ultimately it doesn't really matter).

Question 2

Part c)

We want to evaluate the triple integral

$$\iiint_G yz\,\mathrm{d}V,$$

where G is the xy-solid which has lower surface $z = g_1(x, y) = 0$, upper surface $z = g_2(x, y) = y$, and its projection onto $(x, y)-$plane is the region R which is the quarter disc of radius 2, centred at the origin, for which x and y are positive.

- Follow the recipe as given in Theorem 3.8 of lectures, that is, we will integrate with respect to z first.
- Think carefully about how to describe the region R in your limits.

Question 3

Part b)

By converting into spherical coordinates, we want to evaluate the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{0}^{\sqrt{1-x^2-y^2}} e^{-(x^2+y^2+z^2)^{3/2}}\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$$

- Think carefully about what the volume is that you are integrating over in the xyz-space and how you can describe this in spherical coordinates.
- We are performing a change of variables and so must multiply by (the absolute value of) the Jacobian!

Question 4

The shape of the Earth can be modelled by an oblate spheroid described by the equation

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1,$$

where $a \approx 6378.1370$ km, and $b \approx 6356.7523$. We want to use this information and a triple integral to estimate the volume of the Earth.

- Do not be scared by words
- Be inspired by theorem 3.8 of lecture notes, in particular:
  - What integrand do we need in order for the triple integral to give the volume?
  - What should the lower and upper surfaces g_1 and g_2 be in this question?
  - What should the 2D region R be?
- Be *very* careful when doing a change of variable in this question
- Be sure to actually answer the question, that is, at some point plug in values of a and b to give an approximate volume of Earth.