MA10230 Homework Hints

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Abstract

Hints for homework problem sheet questions for the MA10230 Methods and Applications course at the University of Bath, during the 2020/21 academic year.

Problem Sheet 6

This problem sheet focuses on how to perform a change of variable involving polar coordinates for double integrals. Before we get started, I want to reiterate a result from lectures which has a very subtle, but important, detail.

If $f(r,\theta)$ is continuous on a polar region R (see lecture notes for the notation used in describing R), then

$$\iint_R f(r,\theta)\,\mathrm{d}A = \int_\alpha^\beta \int_{r_1(\theta)}^{r_2(\theta)} f(r,\theta) r\,\mathrm{d}r\,\mathrm{d}\theta.$$

The main take away of this result is that even if the integrand is already in polar form and even if the region R is described in polar form, *we still need to multiply by the Jacobian r*. Since the LHS is just an integral over a region R, it means that when we prescribe the limits to the integrals ourselves we are technically still performing a change of variable and so the Jacobian needs to get involved. Essentially, this is because the double integral is only really defined for the Cartesian coordinate system (I am sure there is a much deeper explanation than that but I won't go into it here).

Question 4

Part c)

By converting to polar coordinates you wish to evaluate the double integral

$$\int_{-3}^{3} \int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}} \exp(-x^2 - y^2)\,\mathrm{d}x\,\mathrm{d}y.$$

- Geometrically, think about what the region of integration is, and how you can describe it using polar coordinates.
- In case you are unsure, $\exp(\cdot)$ is just exponential, that is, $\exp(a) = e^a$.
- Remember to multiply by the Jacobian when you perform the change of variable.

Question 5

Part d)

Using polar coordinates, you want to evaluate the double integral

$$\int\int 3r\,\mathrm{d}A, \quad \text{where } R \text{ is the region in the first quadrant bounded by } r = 1 \text{ and } r = \sin(2\theta).$$

- Think carefully about what the region of integration is, in order to prescribe the correct limits.
- Even though everything looks ready to go for evaluating the integral in polar coordinates, you are still performing a change of variable and so you *still* need to multiply by the Jacobian.

Question 6

Part a)

You want to show that

$$\left(\int_a^b f(x)\,\mathrm{d}x\right)\left(\int_c^d g(y)\,\mathrm{d}y\right) = \int_c^d \int_a^b f(x)g(y)\,\mathrm{d}x\,\mathrm{d}y.$$

- Expressions involving y can be treated as constant from the point of view of the integral with respect to x and vice versa.

Part b)

By computing I^2 and using part a), you want to evaluate the integral

$$I = \int_{-\infty}^{\infty} e^{-x^2/2}\,\mathrm{d}x.$$

- Notice that x is a dummy variable - you can replace it with any other variable name and the integral will still evaluate to the same value.