Oligopoly
Notes

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Question 1. Write down a homogeneous good duopoly model of quantity competition. Using your model, explain the following:
(a) the reaction function of the Stackelberg follower;
(b) the Stackelberg equilibrium. Show that the leader's profit cannot be lower in the Stackelberg equilibrium than in the Cournot equilibrium.

We consider a market for a homogeneous good with two firms, that do not act simultaneously: firm 1 is the *leader*, and sets the quantity q_1 first, while firm 2, the *follower*, that sets its quantity q_2 only after observing q_1.

In our model we assume a linear inverse demand function:

$$P(Q) = a - bQ = a - b(q_1 + q_2) \tag{1}$$

and zero production costs, so that $C_1(q_1) = C_2(q_2) = 0$.

After observing firm 1 setting q_1, firm 2 maximises its profits, which are a function of its decision variable q_2, and where the quantity q_1 appears as a parameter:

$$\Pi_2(q_2; q_1) = [a - b(q_1 + q_2)]q_2 \tag{2}$$

(a) The first order condition is:

$$MR_2(q_2; q_1) = MC \tag{3}$$

Remember that the marginal revenue function is $MR = P(Q) + QP'(Q)$:

$$MR = a - b(q_1 + q_2) + (-b)q_2 \Rightarrow= a - bq_1 - bq_2 - bq_2 \tag{4}$$

$$\frac{\partial \Pi}{\partial q_2} = a - bq_1 - 2bq_2 = 0 \tag{5}$$

and defines firm 2's reaction function:

$$q_2 = f_2(q_1) = \frac{a}{2b} - \frac{1}{2}q_1. \tag{6}$$

Firm 1 has an advantage, because it can set its decision variable q_1 knowing how firm 2 will react.
In mathematical terms, this accounts to maximise its profits where in place of q_2 there is firm 2's reaction function:

$$\Pi_1(q_1) = [a - b(q_1 + q_2)]q_1 = [a - b(q_1 + \frac{a}{2b} - \frac{1}{2}q_1)]q_1 = \frac{a}{2}q_1 - \frac{b}{2}q_1^2 \tag{7}$$

Notice how this is a function of q_1 without q_2 as parameter, and its maximisation implies the first order condition:

$$MR_1(q_1) = \frac{a}{2} - bq_1^s = 0 \tag{8}$$

(b) This condition gives the Stackelberg equilibrium quantities both for firm 1 and also firm 2, through the reaction function:

$$q_1^s = \frac{a}{2b} \qquad q_2^s = f_2(q_1^s) = \frac{a}{4b}. \tag{9}$$

Plugging the value q_1^* in $\Pi_1(q_1^s)$, we get the profit of firm 1 in equilibrium:

$$\Pi_1(q_1^s) = \frac{a}{2}q_1^s - \frac{b}{2}(q_1^s)^2 = \frac{a}{2}\frac{a}{2b} - \frac{b}{2}\left(\frac{a}{2b}\right)^2 = \frac{a^2}{8b}, \tag{10}$$

Plugging q_1^* and q_2^* in $\Pi_2(q_1^s, q_2^s)$, we get firm 2 equilibrium profit:

$$\Pi_2(q_2^s; q_1^s) = a\frac{a}{4b} - b\frac{a}{2b}\frac{a}{4b} - b\frac{a^2}{16b^2} = \frac{a^2}{16b}. \tag{11}$$

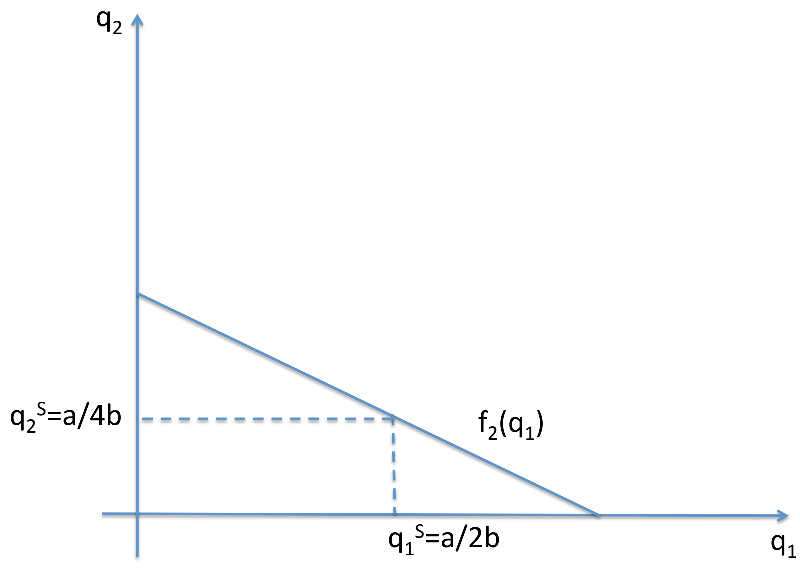

Figure 1: Stackelberg's model reaction function $f_2(q_1) = \frac{a}{2b} - \frac{1}{2}q_1 = \frac{a}{4b}$

Let's now consider a Cournot market, where no firm has a first mover advantage and quantities are set simultaneously.

Firm 1 solves its profit maximisation problem of setting the optimal quantity q_1 by taking the quantity q_2 of firm 2 as given. At the same time Firm 2 solves its profit maximisation problem of setting the optimal quantity q_2 by taking the quantity q_1 of firm 1 as given.

This means that each firm is equipped with a reaction function defined by its own first order condition, which is the same that we have seen for the follower firm in the Stackelberg market:

$$q_1 = f_1(q_2) = \frac{a}{2b} - \frac{1}{2}q_2; \tag{12}$$

$$q_2 = f_2(q_1) = \frac{a}{2b} - \frac{1}{2}q_1. \tag{13}$$

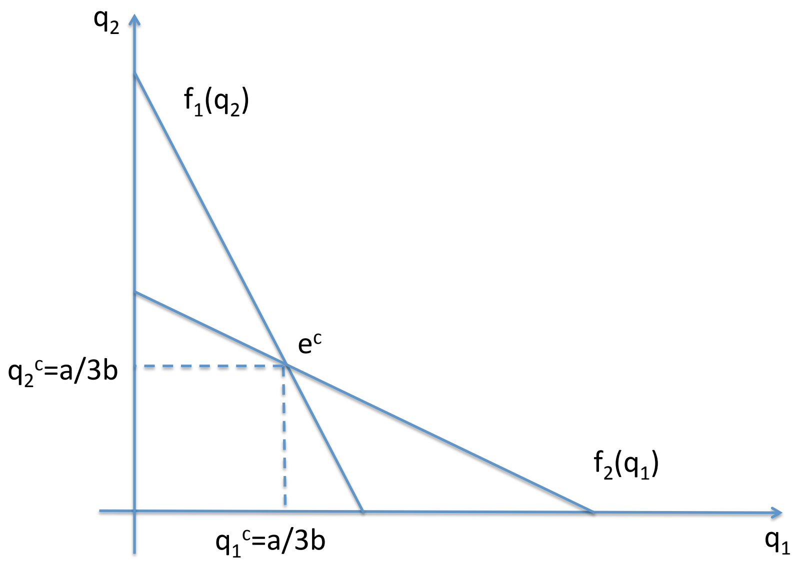

Figure 2: Reaction functions $f_2(q_1) = \frac{a}{2b} - \frac{1}{2}q_1$ and $f_1(q_2) = \frac{a}{2b} - \frac{1}{2}q_2$ of firm 2 and firm 1, respectively, in this Cournot market.

There is only one point (q_1^c, q_2^c) that satisfies both equations, and this is the Cournot equilibrium:

$$q_1^c = \frac{a}{3b} \qquad q_2^c = \frac{a}{3b}. \tag{14}$$

Due to the symmetry of the market, the equilibrium quantities produced by firm 1 and firm 2 are equal, and equal are the profits:

$$\Pi_2^c = [a - b(q_1^c + q_2^c)]q_2^c = [a - b(2q_2^c)]q_2^c = [a - 2bq_2^c]q_2^c = [a - 2b\frac{a}{3b}]\frac{a}{3b} = \frac{a^2}{9b} = \Pi_1^c.$$

Notice how this value is (slightly) lower than the leader's profit in the Stackelberg market, but larger than the Stackelberg follower's profit.

Additional questions

(c) Verify what happens if the Stackelberg's leader chooses the same quantity adopted by Cournot 's firms.

Let's see what happens if we plug the Cournot equilbrium quantity into the Stackelberg reaction function. From the equation (6), we know the firm 2's reaction function:

$$q_2 = f_2(q_1) = \frac{a}{2b} - \frac{1}{2}q_1$$

Plugging the Cournot equilibrium quantity $q_1 = \frac{a}{3b}$, we get:

$$q_2 = f_2(q_1) = \frac{a}{2b} - \frac{1}{2}\frac{a}{3b} = \frac{a}{2b} - \frac{a}{6b} = \frac{a}{3b}.$$

If the leader in Stackelberg model goes to the market with the Cournot equilibrium quantity, the follower will choose exactly the same quantity of the leader. This means that the two firms will share the market, selling the same quantities of good ($q_1^s = q_2^s$)
Let's calculate the profit for both firms:

$$\Pi_1(q_1^s*) = [a - b(q_1^s * + q_2^s*)]q_1 = [a - b(\frac{a}{3b} + \frac{a}{3b})]\frac{a}{3b} = \frac{a^2}{3b} - \frac{2a^2}{9b} = \frac{a^2}{9b} = \Pi_2(q_2^s*)$$

Comparing with the previews Stackelberg model, the leader's profit now is smaller, while the profit of the follower is bigger, in particular:

$$\Pi_1(q_1^s*) \leq \Pi_1(q_1^s) \qquad \Pi_2(q_2^s*) \geq \Pi_2(q_2^s)$$

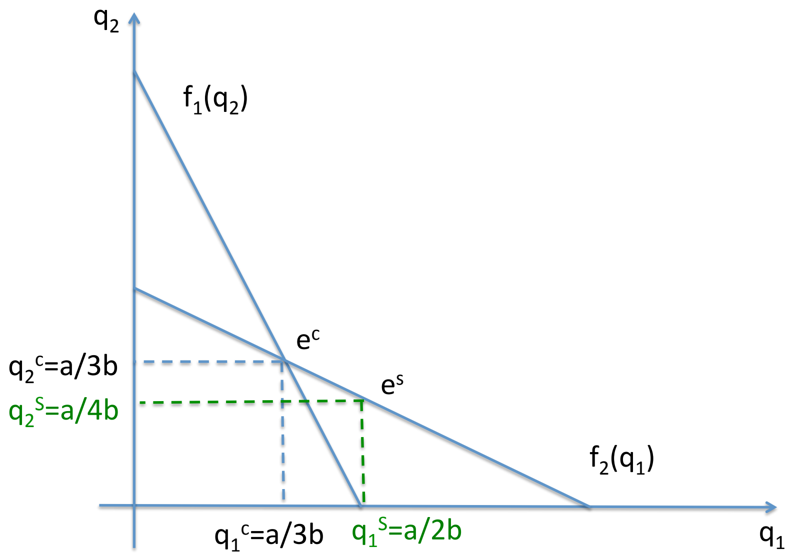

Figure 3: Cournot and Stackelberg equilibrium quantities

(d) Verify what happens in terms of equilibrium quantity and profit if the Cournot's firms face different cost functions.

Let's see assume constant marginal costs $c_1 \neq c_2 \neq 0$.

The linear inverse demand function is:

$$P(Q) = a - bQ = a - b(q_1 + q_2) \tag{15}$$

Remember that total revenue is $qP(Q)$, then Firm's 1 total revenue is:

$$q_1 P(Q) = aq_1 - bq_1^2 - b\bar{q_2}q_1$$

where $\bar{q_2}$ is firm 2 quantity, which is fixed for firm 1.

Firm 1 solves its profit maximisation problem, setting $MR_1 = MC_1$ (necessary condition).

$$a - 2bq_1 - b\bar{q_2} = c_1$$

Then, the reaction function of firm 1 is:

$$q_1 = f_1(q_2) = \frac{a - c_1}{2b} - \frac{1}{2}\bar{q_2}$$

Similarly firm 2's reaction function is:

$$q_2 = f_2(q_1) = \frac{a - c_2}{2b} - \frac{1}{2}\bar{q_1}$$

Plugging the reaction function of firm 2 within $\bar{q_2}$ and the reaction function of firm 1 within $\bar{q_1}$ we get the equilibrium quantities:

$$q_1 = \frac{a - 2c_1 + c_2}{3b}$$

$$q_2 = \frac{a - 2c_2 + c_1}{3b}$$

The equilibrium quantities produced by firm 1 and firm 2 depend on their own cost function and on the cost function of the other firm.

In particular we observe that:

$$if \quad c_1 > c_2 \Rightarrow \quad then \quad q_1 < q_2 \Rightarrow \quad \Pi_1(q_1) < \Pi_2(q_2)$$

and the other way around if $c_1 < c_2$.

Numerical Exercises with solutions

Exercise 1

Suppose we have an inverse demand function $P = 100 - 2Q$, and zero marginal costs, so that $MC = 0$. Calculate optimal quantities, prices and profits when we consider:
(a) perfect competition,
(b) monopoly,
(c) duopoly - Cournot,
(d) duopoly - Bertrand,
(e) duopoly - Collusion.

Solution 1

(a) In perfect competition the price is equal to marginal cost:

$$p = MC \Rightarrow \quad 100 - 2Q = 0 \quad \Rightarrow Q_c = 50, \quad p_c = 0 \quad \Pi_c = 0$$

(b) In monopoly the price is determined by setting marginal revenue equal to marginal cost:

$$MR = MC \Rightarrow 100 - 4Q = 0 \Rightarrow Q_m = 25 \quad p_m = 50 \quad \Pi_m = 1250$$

(c) In a Cournot duopoly firms set quantities Q_1 and Q_2 simultaneously. Both seek to maximize profits by taking the quantity of the other firm fixed: in particular, they set marginal revenues equal to zero (because marginal costs are zero here):

$$\begin{cases} MR_1 = MC \Rightarrow 100 - 2Q_2 - 4Q_1 = 0 \\ MR_2 = MC \Rightarrow 100 - 2Q_1 - 4Q_2 = 0 \end{cases}$$

Thus, we have $Q_1 = Q_2 = 16.6$, $p = 33.3$, $\Pi_1 = \Pi_2 = 555,5$.

(d) In the Bertrand duopoly the firms set prices simultaneously. If the market is equally shared and since the firms have the same marginal cost, the two firms in order to maximize the profit fix the price equal to marginal cost.
$Q_1 = Q_2 = 25$, $p = 0$, $\Pi_1 = \Pi_2 = 0$.

(e) Firms in the industry collude and form a *cartel.* The industry profit is shared in some pre-determined way. In this particular case, the two firms share equally the market, and the result is the same quantity produced in the monopoly.

$Q_1 = Q_2 = 12.5$, $p = 50$, $\Pi_1 = \Pi_2 = 625$.

A very general rule in term of firm profits and prices is:
Perfect competition $\leq$ Bertrand $\leq$ Cournot $\leq$ Monopoly

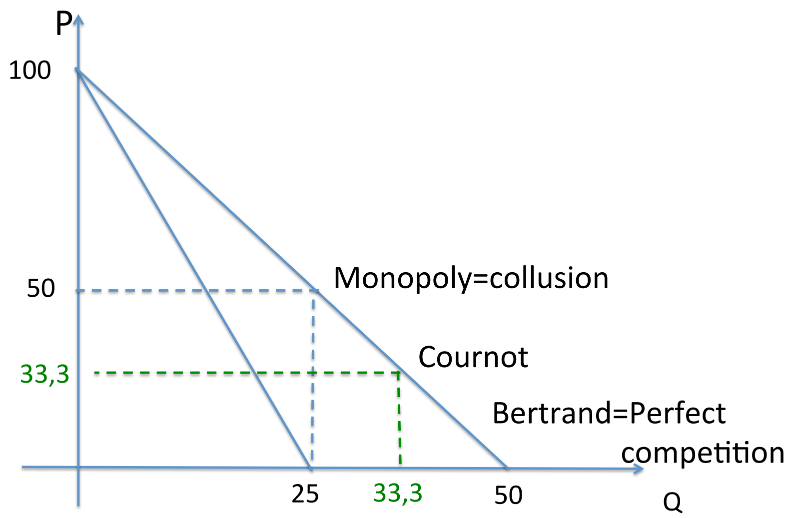

Figure 4: Perfect competition, Bertrand, Cournot, Monopoly equilibrium quantities