The Slutsky Equation
Seminar Handout

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ES20011 - Intermediate Microeconomics 1

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Problem 1. Assuming that only the price of good 1, p_1 changes prove that the Slutsky equation holds using the Cobb-Douglas utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$ where x_1 denotes the quantity of good 1 and x_2 denotes the quantity of good 2. Denote income as M and the price of good 2 as p_2.

Solution 1. Fill the blank spaces.

Find the demand functions for: (x_1, x_2):

From the maximisation problem:

$$Max_{x_1,x_2} U(x_1, x_2) =$$

s.t.: $p_1 X_1 + p_2 x_2 = M \quad \rightarrow x_2 =$

Using the substitution method we derive the functions

$$Max_{x_1} u(x_1) = \sqrt{x_1}\sqrt{(\qquad\qquad\qquad)}$$

Find the FOC: $\rightarrow \frac{du}{dx_1} = 0$

$\frac{du}{dx_1} =$

$x_1^* = \frac{M}{2p_1}$

We substitute x_1^* into our constraint:

$x_2 = \frac{M}{p_2} - \frac{p_1}{p_2}x_1 \rightarrow$ $\rightarrow x_2^* = \frac{M}{2p_2}$

$(x_1^*, x_2^*) = \left(\frac{M}{p_1}, \frac{M}{p_2}\right)$

We plug (x_1^*, x_2^*) into:

$u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

By substituting the into $u(x_1, x_2)$ we obtain the: $v(p_1, p_2, M) =$
Indirect utility function

Find the Hicksian demand functions (h_1, h_2)

From the minimisation problem we get:

$e(p_1, p_2, u) \equiv Min_{x_1,x_2}(M = p_1x_1 + p_2x_2)$

s.t. $\overline{u} =$

$x_2 =$

Using the substitution method from the constraint we derive the Hicksian demand function:

$Min_{x_1} e(x_1) = (p_1x_1 + p_2 \qquad\qquad) =$

Find the FOC:$\rightarrow \frac{de}{dx_1} = 0$

$\frac{\partial e}{\partial x_1} =$

$(h_1^*) =$

We substitute h_1^* into our constraint and we get:

$h_2^* =$

Now we plug (h_1^*, h_2^*) into $M = p_1 x_1 + p_2 x_2$ and we get the expenditure function:

The Slutsky's equation states:

$$\frac{\partial x_1}{\partial p_1} = \frac{\partial h_1}{\partial p_1} - \frac{x_1}{\partial M} x_1$$

Problem 2. Suppose an individual possesses a Cobb-Douglas utility function $u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$ and the price of good 1 changes from $p_1 = 4$ to $p_1 = 1$. Assume $M = 16$ and $p_2 = 1$. Numerically work out the total effect, the substitution effect and the income effect in terms of units of good 1. You can use the expressions derived from Exercise 1 above.

Solution 2. Fill the blank spaces.

$u(x_1, x_2) = x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$

$p_1 = 4 \rightarrow p_1^{'} = 1$

$p_2 = 1, \quad M = 16$

From the exercise 1 we know that the starting point is:

$x_1^* = \frac{M}{2p_1} = \qquad\qquad =$

$x_2^* = \frac{M}{2p_2} = \qquad\qquad =$

Find the effect when the price of good x_1 changes $\rightarrow p_1^{'} = 1$

$x_1^{'} =$

Total effect is

We plug the original price $p_1 = 4$ into the $v(p_1, p_2, M)$ and we get:

$v(p_1, p_2, M) = \frac{M}{2\sqrt{p_1 p_2}} = \qquad\qquad =$

Income required to achieve this utility level at price $p_1^{'}$, is found from:

$$\left(\frac{M'}{2(p_1^{'})}\right)^{\frac{1}{2}}\left(\frac{M'}{2(p_2)}\right)^{\frac{1}{2}} = 4 \qquad \Rightarrow M^{'} = 8$$

The demand for good 1 when $M^{'} = 8$ and price 1 is $x_1^{'} =$

Which is the income effect.

We know that the total effect is 6, then the substitution effect is equal to: