Traveling skyrmions and vortices in chiral antiferromagnets

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A chiral antiferromagnet

Consider a square lattice of spins $\boldsymbol{S}_{i,j}$ (on the plane) interacting via

- symmetric exchange, with energy

$$E_{\rm ex} = J \sum_{i,j} \boldsymbol{S}_{i,j} \cdot (\boldsymbol{S}_{i+1,j} + \boldsymbol{S}_{i,j+1}), \qquad J > 0,$$

- Dzyaloshinskii-Moriya (DM) interaction,

$$E_{\rm DM} = D \sum_{i,j} \left[\widehat{\mathbf{e}}_2 \cdot (\boldsymbol{S}_{i,j} \times \boldsymbol{S}_{i+1,j}) - \widehat{\mathbf{e}}_1 \cdot (\boldsymbol{S}_{i,j} \times \boldsymbol{S}_{i,j+1}) \right]$$

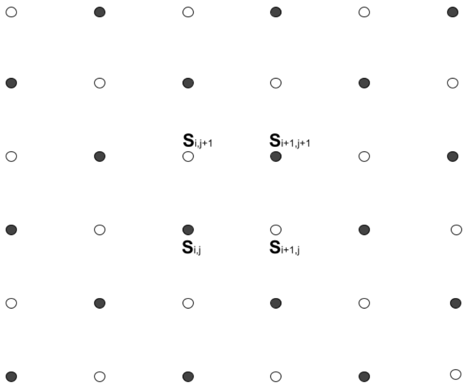
- and easy-axis (perpendicular) anisotropy

$$E_{\rm a} = -\frac{g}{2} \sum_{i,j} [(\boldsymbol{S}_{i,j})_3]^2.$$

From the Hamiltonian, we obtain the equation of motion

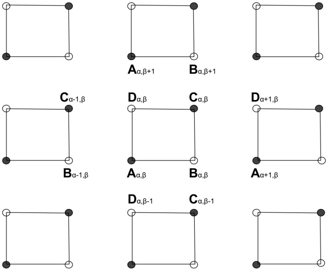
$$\frac{\partial \boldsymbol{S}_{i,j}}{\partial t} = -\boldsymbol{S}_{i,j} \times \frac{\partial E}{\partial \boldsymbol{S}_{i,j}}.$$

Two sublattices



The continuum approximation

Two sublattices (●, ○)



Consider a tetramerization of the lattice and define the variables ($s = |\boldsymbol{S}_{i,j}|$)

$$\begin{aligned}
\boldsymbol{m} &= \frac{1}{4s}(\boldsymbol{A}+\boldsymbol{B}+\boldsymbol{C}+\boldsymbol{D}) \quad \text{(Magnetization)}\\
\boldsymbol{n} &= \frac{1}{4s}(\boldsymbol{A}-\boldsymbol{B}+\boldsymbol{C}-\boldsymbol{D}) \quad \text{(Néel vector)}\\
\mathbf{k} &= \frac{1}{4s}(\boldsymbol{A}+\boldsymbol{B}-\boldsymbol{C}-\boldsymbol{D})\\
\mathbf{l} &= \frac{1}{4s}(\boldsymbol{A}-\boldsymbol{B}-\boldsymbol{C}+\boldsymbol{D}).
\end{aligned}$$

Assume that $\boldsymbol{m}, \boldsymbol{n}, \mathbf{k}, \mathbf{l}$ are slowly varying in space.

The continuum model is written entirely in terms of the Néel vector $\boldsymbol{n}$ (a nonlinear σ-model).

$$\boldsymbol{n} \times (\ddot{\boldsymbol{n}} - \boldsymbol{h}_{\text{eff}}) = 0,$$

$$\boldsymbol{h}_{\text{eff}} = \Delta \boldsymbol{n} + 2\lambda \epsilon_{\mu\nu} \hat{\mathbf{e}}_\mu \times \partial_\nu \boldsymbol{n} + n_3 \widehat{\mathbf{e}}_3, \qquad \lambda = \frac{D}{\sqrt{gJ}}.$$

Auxiliary variables

The derivation is based on the use of a small parameter $\epsilon = \sqrt{g/J}$

and we have the magnetization

$$\boldsymbol{m} = \frac{\epsilon}{2\sqrt{2}} \left(\boldsymbol{n} \times \dot{\boldsymbol{n}} \right)$$

and the auxiliary variables

$$\boldsymbol{k} = -\frac{\epsilon}{2} \partial_1 \boldsymbol{n}, \qquad \boldsymbol{l} = -\frac{\epsilon}{2} \partial_2 \boldsymbol{n}.$$

We need all four variables in order to reconstruct the spin lattice.

Domain walls and skyrmions in AFM

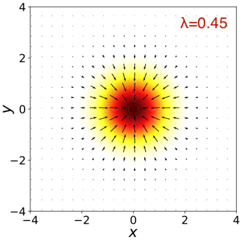
The time-independent σ-model coincides with the time-independent Landau-Lifshitz equation for a ferromagnet.

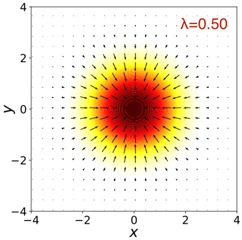
Domain walls, skyrmions, vortices etc that exist in a FM *(in the case that the magnetostatic field is not crucial)*, can also be found in an AFM.

For $\lambda < \frac{2}{\pi}$, the Néel is the ground state. DWs and skyrmions are excited states.

For $\lambda > \frac{2}{\pi}$, a phase transition occurs to the helical phase (spiral).

Increasing the DM parameter λ increases the skyrmion radius.





Vector plot for (n_1, n_2). Red means $n_3 < 0$ and white $n_3 > 0$.

Traveling domain walls

For a Néel wall $\boldsymbol{n} = \boldsymbol{n}_{DW}(x)$, with $n_2 = 0$, the DM term vanishes,

$$\boldsymbol{n} \times (\ddot{\boldsymbol{n}} - \boldsymbol{n}'' + \cancel{2\lambda \widehat{\mathbf{e}}_2 \times \boldsymbol{n}'} - n_3 \widehat{\mathbf{e}}_3) = 0.$$

A propagating DW is obtained by a Lorentz transformation

$$\boldsymbol{n}(x,t;\upsilon) = \boldsymbol{n}_{DW}\left(\frac{x-\upsilon t}{\sqrt{1-\upsilon^2}}\right), \qquad |\upsilon| < 1 \quad \text{(spinwave velocity)}.$$

- Propagation as a solitary wave (without force) is possible
  - in contrast to typical FM dynamics.
- The DW is **contracted** (in the propagation direction) when it is traveling.

The model with DMI is not Lorentz invariant.

Traveling domain wall and the spiral phase transition

For a traveling Néel wall, the DM term is rescaled upon a Lorentz transformation.

$$\boldsymbol{n} \times \left(\boldsymbol{n}'' - \frac{2\lambda}{\sqrt{1-v^2}} \hat{\mathbf{e}}_2 \times \boldsymbol{n}' + n_3 \widehat{\mathbf{e}}_3 \right) = 0$$

A propagating DW is (Lorentz transformed)

$$\boldsymbol{n}(x - vt) = \boldsymbol{n}_{DW} \left(\frac{x - vt}{\sqrt{1-v^2}} \right), \qquad$$ ~~$|v| < 1$~~ (spinwave velocity).

The propagating solution is valid only for the range of parameter values where the Néel state is stable

$$\frac{\lambda}{\sqrt{1-v^2}} < \frac{2}{\pi} \Rightarrow |v| < \sqrt{1 - \left(\frac{\pi\lambda}{2}\right)^2} \equiv v_c.$$

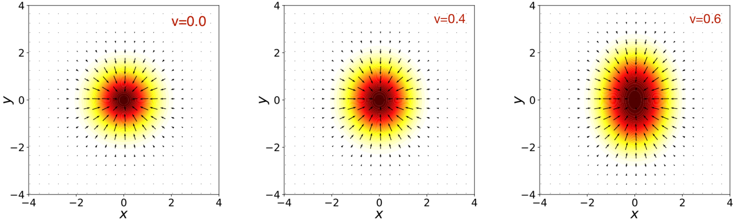
- The phase transition to the spiral state destabilizes the propagating DW for $v > v_c$.

Traveling skyrmions

Make the solitary wave ansatz $\boldsymbol{n} = \boldsymbol{n}(x - \upsilon t, y)$. It satisfies

$$\boldsymbol{n} \times \left(\boldsymbol{h}_{\text{eff}} - \upsilon^2 \partial_1^2 \boldsymbol{n} \right) = 0.$$

We find numerically (by a relaxation algorithm) skyrmions traveling along x, for a range of velocities υ.



The traveling skyrmions get more elongated with increasing velocity.

There is a maximum velocity $\upsilon_c(\lambda)$ where the skyrmion expands to infinity. For example, for $\lambda = 0.45$, we find $0 \leq \upsilon < \upsilon_c \approx 0.71$.

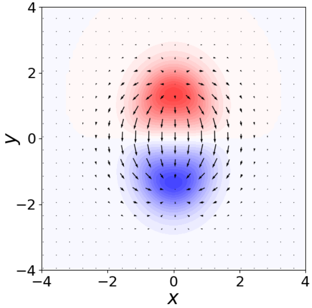
Magnetization of traveling skyrmions

The magnetization vector is given by

$$\boldsymbol{m} = \frac{\epsilon}{2\sqrt{2}} \boldsymbol{n} \times \dot{\boldsymbol{n}}.$$

The local magnetization

- is zero for a static AFM skyrmion.
- It is nonzero for a propagating one.



The magnetization vector $\boldsymbol{m}$ for a traveling skyrmion.
Red for $m_3 > 0$, blue for $m_3 < 0$.

The net magnetization

is nonzero for a propagating skyrmion.
It is increases with velocity.

Maximum velocity and the phase transition to the spiral

The key to understanding the behavior of the maximum velocity v_c is the numerical finding that the skyrmion expands in both the x and y directions as $v \to v_c$.

The equation contains derivatives in the two space directions x and y,

$$\boldsymbol{n} \times \left\{ [(1-v^2)\partial_1^2 \boldsymbol{n} - 2\lambda \widehat{\mathbf{e}}_2 \times \partial_1 \boldsymbol{n}] + [\partial_2^2 \boldsymbol{n} + 2\lambda \widehat{\mathbf{e}}_1 \times \partial_2 \boldsymbol{n}] + n_3 \widehat{\mathbf{e}}_3 \right\} = 0.$$

In the limit $v \to v_c$, the skyrmion is very elongated primarily in the y direction, and we may neglect the y derivatives.

Maximum velocity and the spiral phase transition

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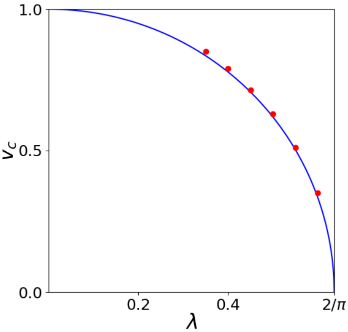
The 1D equation

$$\boldsymbol{n} \times \left[(1-v^2)\partial_1^2 \boldsymbol{n} - 2\lambda \widehat{\mathbf{e}}_2 \times \partial_1 \boldsymbol{n} + n_3 \widehat{\mathbf{e}}_3 \right] = 0$$

supports the Néel state only for dimensionless DM parameter smaller than $\frac{2}{\pi}$, or

$$\frac{\lambda}{\sqrt{1-v^2}} < \frac{2}{\pi} \Rightarrow |v| < \sqrt{1 - \left(\frac{\pi\lambda}{2}\right)^2} \equiv v_c.$$



Comparison with numerics

- Blue line shows

$$v_c = \sqrt{1 - \left(\frac{\pi\lambda}{2}\right)^2}.$$

- Red dots give the numerically calculated values (obtained when the traveling skyrmion size diverges).

The origin of the maximum skyrmion velocity is the topological phase transition from the Néel to the spiral state.

- For $\lambda \to 0$, the maximum velocity $v_c \to 1$, i.e., it goes to the value of the Lorentz invariant model (spinwave velocity).
- For $\lambda \to \frac{2}{\pi}$ (large DM), the maximum velocity is small, $v_c \to 0$.

Particle-like character of a skyrmion. The skyrmion mass.

Energy of a traveling skyrmion (solitary wave)

$$E = \frac{1}{2}\int \dot{\boldsymbol{n}}^2\, dxdy + E_0 = \frac{v^2}{2}\overbrace{\int (\partial_1 \boldsymbol{n}_0)^2\, dxdy}^{\mathcal{M}_0} + E_0$$

$\boldsymbol{n}_0$ a static skyrmion, E_0 its energy.
It is a Newtonian particle with mass $\mathcal{M}_0$

$$E(v) = \frac{1}{2}\mathcal{M}_0 v^2 + E_0, \qquad v \ll v_c.$$

Linear momentum

$$P = -\int \dot{\boldsymbol{n}} \cdot \partial_1 \boldsymbol{n}\, dxdy = v \underbrace{\int (\partial_1 \boldsymbol{n})^2\, dxdy}_{\mathcal{M}}.$$

For small v,

$$P = v\int (\partial_1 \boldsymbol{n}_0)^2\, dxdy = \mathcal{M}_0 v, \qquad v \ll v_c.$$

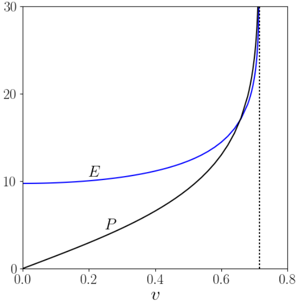
Energy-Momentum relation

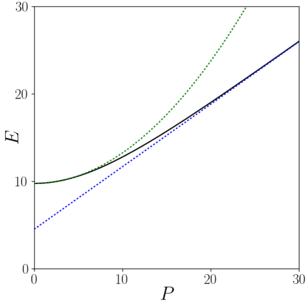
For small velocities

$$E \approx E_0 + \frac{P^2}{2\mathcal{M}_0}, \qquad \upsilon \ll \upsilon_c$$

For large momenta, we set $\upsilon \approx \upsilon_c$ in the group velocity relation

$$\upsilon = \frac{dE}{dP} \Rightarrow \upsilon_c \approx \frac{dE}{dP} \Rightarrow E \approx \upsilon_c P + E_c, \quad \upsilon \to \upsilon_c, \quad E_c \approx 4.5.$$





Solid lines show numerical data.

Energy and mass

Let us look into the energy-mass relation,

$$\begin{cases} E & \approx \upsilon_c P + E_c \\ P & = \mathcal{M}\upsilon \end{cases} \Rightarrow E \approx \mathcal{M}\upsilon_c^2 + E_c, \qquad \upsilon \to \upsilon_c.$$

From virial relations (omitted here), we find

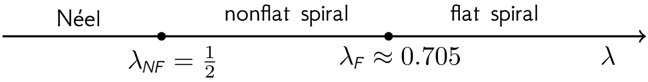
$$E = \mathcal{M} + \lambda \int \widehat{\mathbf{e}}_y \cdot (\partial_1 \boldsymbol{n} \times \boldsymbol{n}) dxdy.$$

Relativistic dynamics for chiral particles

- The dispersion relation is shifted by a constant in the relativistic limit.
- The Energy is shifted with respect to the mass by a chiral term.

DM interaction and easy-plane anisotropy

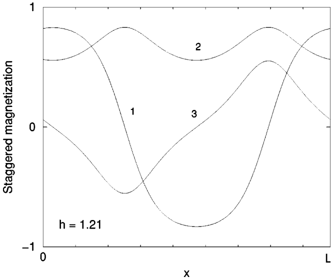
For a one-dimensional magnet, we have two phase transitions.



We have the respective ground states

- Néel state.
- A nonflat spiral (with energy lower than the flat spiral).
- The (usual) spiral with $n_2 = 0$ [a "flat" spiral on the (1 3) plane].

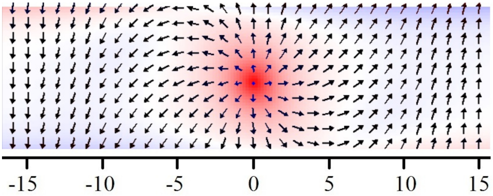
[Chovan, Papanicolaou, Komineas, PRB (2002)]



Vortex in a stripe

We consider a stripe geometry as this is suitable for shifting of magnetic information (it will also give rise to interesting effects).

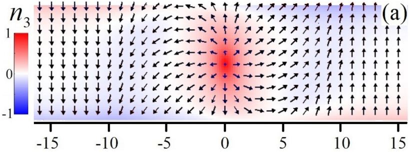
A hybrid vortex



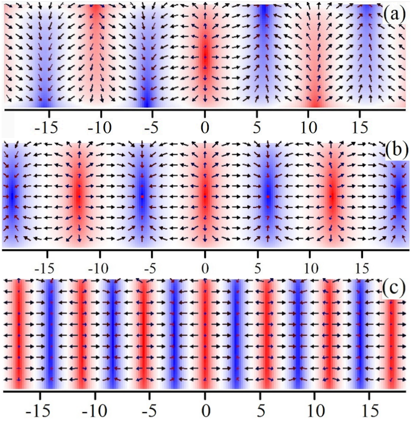
We have competition between

- boundary conditions (at $y = \pm L/2$) that favour canting of n_2,
- DM interaction that favours Néel type skyrmion.

Propagating vortex in a stripe



A propagating vortex (for $\lambda = 0.4, \; \upsilon = 0.60$)



A mixture of nonflat spiral configuration and edge vortices ($\upsilon = 0.78$).

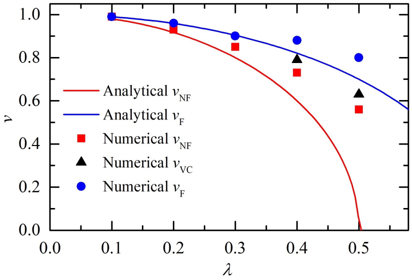
A vortex chain ($\upsilon = 0.79$).

A flat spiral ($\upsilon = 0.90$).

Dynamical phase transitions

We expect the following three phases (and two critical velocities).

- The Néel state for $\frac{\lambda}{\sqrt{1-\upsilon^2}} < \lambda_{NF}\,(=0.5) \Rightarrow \upsilon < \sqrt{1-\left(\frac{\lambda}{\lambda_{NF}}\right)^2} \equiv \upsilon_{NF}$.
- The flat spiral for $\frac{\lambda}{\sqrt{1-\upsilon^2}} > \lambda_F\,(\approx 0.7) \Rightarrow \upsilon > \sqrt{1-\left(\frac{\lambda}{\lambda_F}\right)^2} \equiv \upsilon_F$.
- The non-flat spiral for $\lambda_{NF} < \frac{\lambda}{\sqrt{1-\upsilon^2}} < \lambda_F \Rightarrow \upsilon_{NF} < \upsilon < \upsilon_F$.



Flat spiral
Vortex chain
Nonflat spiral

Summary

- Traveling domain walls, skyrmions and vortices exist in antiferromagnets.
- The dynamics of topological solitons in antiferromagnets is dramatically different than that of their ferromagnetic counterparts.
- Vortex phases arise due to dynamics in a stripe geometry.
- The maximum velocity for solitons is lower than the spinwave velocity.
- The topological phase transition from the Néel to the spiral phases is crucial for fast soliton dynamics.

References

- Komineas, Papanicolaou, SciPost 8, 086 (2020).
- Tomasello, Komineas, Phys. Rev. B 104, 064438 (2021).