Fractals on the Complex Plane

Introduction

Benoit B. Mandelbrot coined the term "fractal" to refer to a new class of mathematical shapes whose uneven contours could mimic the irregularities found in nature. Mandelbrot traced his work on fractals to a question he first encountered as a young researcher: how long is the coast of Britain? The answer, he was surprised to discover, depends on how closely one looks. On a map an island may appear smooth, but zooming in will reveal jagged edges that add up to a longer coast. Zooming in further will reveal even more coastline. The key to fractals are that they are self-similar – zooming in on one area shows a similar shape to the whole.



How to Generate the Mandelbrot Set

Arguably the most famous fractal is the Mandelbrot Set, which is plotted on the complex plane as follows:

For each point on the complex plane, i.e. for each complex number C:

- Take the current number, square it and add the starting number. i.e. $z \rightarrow z^2+C$
- Repeat until the modulus (|Z| or √ZZ*) of the current number is larger than a given limit value, say until |z|>2.

All the points where the modulus does not escape to infinity are in the Mandelbrot Set and are traditionally coloured black. The points which do escape to infinity are coloured white or are given some amount of brightness / colour-scale proportional to how quickly their modulus grew bigger than the limit.



Mandelbrot chose his own middle initial, but it doesn't stand for anything...or does it?