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THE CASE FOR SUBDIVISION SURFACES IN BUILDING DESIGN

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ABSTRACT

In his seminal 2010 IASS Journal paper titled "New Challenges for the Structural Morphology Group", Andrew Borgart outlined the need for radical new techniques for the design, engineering and construction of complex geometry structures in order to continue to produce innovative and beautiful design solutions in the current economic and environmental climate. He concluded that unorthodox solutions were needed, and that these would only be provided through transfer of technologies from a wide range of disciplines. This paper rises to Borgart's challenge, by demonstrating through a case study project that the adoption of Subdivision Surfaces as a new modeling framework for engineering design would go a long way towards addressing these problems and would reinvigorate the Shell and Spatial Structure design community.

Keywords: *Subdivision Surface, Structural Morphology, Multi-objective Optimization, Digital Architectonics.*

1. INTRODUCTION

Since computers began to appear in the design office back in the 1960s, engineers have been searching for ways in which they can be used to streamline the design process. However, whilst in the fields of aerospace and automotive computing power was harnessed to optimize the design itself, in the building industry, computer aided design (CAD) was seen simply as an electronic version of paper, used for its ease of editing, storage and printing, rather than as a tool for analysis in itself. Whilst engineers in other industries were innovating through 3D solid and parametric modeling, building construction industry drawings were being created manually in the same way as had previously been done with a pencil and drawing board.

1.1 Motivation

This paper begins by accepting the "new challenges" in the field of Structural Morphology identified by Borgart [1], and the "radical new techniques for the design, engineering and construction of complex geometry structures" that follow. It highlights some of the current inefficiencies in the way computers are used in the design of complex geometry buildings such as shells and spatial-structures, and goes on to explore what CAD software might be like if it were invented from scratch today, rather than being built on a tradition of drawing boards and pencils. By reporting a case-study project built with Subdivision Surface technologies, it shows that Subdivision can provide a useful platform on which to combine creative building design and intelligent engineering, producing aesthetically pleasing designs against a financial and environmentally constrained agenda.

1.2 Current state-of-the-art

CAD began in two dimensions, defining geometry with straight lines and later incorporating Bezier curves and Splines. When CAD moved into 3D surface representation, it took these Splines and arrayed them into grids to make Spline surfaces and NURBS. They required a 2D parameterization of space in which to array two pseudo-orthogonal sets of Splines, and this limitation sometimes led to the need to break down a desired surface into separate four-sided patches. Such an approach made three dimensional solid modeling Boolean operations such as intersection and union possible, but computationally expensive, approximate, and often introduced problems of discontinuities in tangent (causing creases) or in rate-of-change of tangent (distorting reflections).

Spline surfaces, and their ubiquitous progeny NURBS, cannot easily be fabricated for building construction, and doubly-curved, architecturally-driven surfaces are often either post-rationalized by the engineer or contractor into singly-curved or flat panels, or converted into a triangulated mesh in order to be built. The use of patches, and of post-rationalizing the geometry for construction, is time-consuming and inevitably leads to a compromise between the surface desired by the architect, the surface representation of the chosen CAD program, and the need for a constructible solution. Whilst recent work by others has focused on methods linking numerical and physical models as a means to derive complex forms [2], this paper proposes Subdivision Surfaces as a tool particularly useful for Structural Morphology design.

2. SUBDIVISION SURFACES

Subdivision surfaces were developed throughout the 1980s for applications in 3D computer graphics and have seen a recent focus on development in the digital entertainment industries for computer animation and gaming [3]. They represent a smooth 3D surface using a polygonal mesh defined by a set of vertices and an underlying topology. The mesh can be constructed from triangles [4] or quadrilaterals [5] or a combination of both [6]. For simplicity of explanation this paper will focus on triangular meshes, but the descriptions extend to quadrilateral meshes also.

2.1 Description

A mesh is a very simple object to construct and manipulate, but generally has a crease along every edge and therefore cannot describe a smooth surface as such. However, a mesh can be made finer through a process of subdivision. That is to say that each triangular face (as shown in Figure 1a) can be split into four smaller triangles by introducing a new "child" vertex along each edge (shown in red in Figure 1b) and joining each of these child vertices to the other two with a new edge. In itself, this purely topological subdivision does not actually change the surface geometry, since each planar face is subdivided into four smaller, but still co-planar, faces. The key to a Subdivision Surface representation is that the child vertices are not simply placed along the original edge, but have their position carefully calculated as a weighted average of the positions of all the surrounding vertices. In this sense, the averaging of the coordinates has the effect of reducing the discontinuities, and the resulting mesh has four times as many triangles but is much smoother. There are many different methods for weighting the positions of the surrounding vertices, each known as a Subdivision Scheme. Some only place the child vertices at these weighted positions (interpolating schemes) and others also move the original "parent" vertices to new positions (approximating schemes), as shown in Figure 1c and d.

For example, one commonly used interpolating Subdivision Scheme for triangular meshes is the "Loop" Scheme [7]. This scheme places newly created vertices at a weighted average of the positions of the vertices of the faces either side of the edge being divided. The two vertices at the ends of the edge being divided are given a weighting three times higher than the two other face vertices, as shown in the left of Figure 2. Similarly, existing "parent" vertices are moved to the weighted position as shown on the right of Figure 2, where its own previous position is weighted against

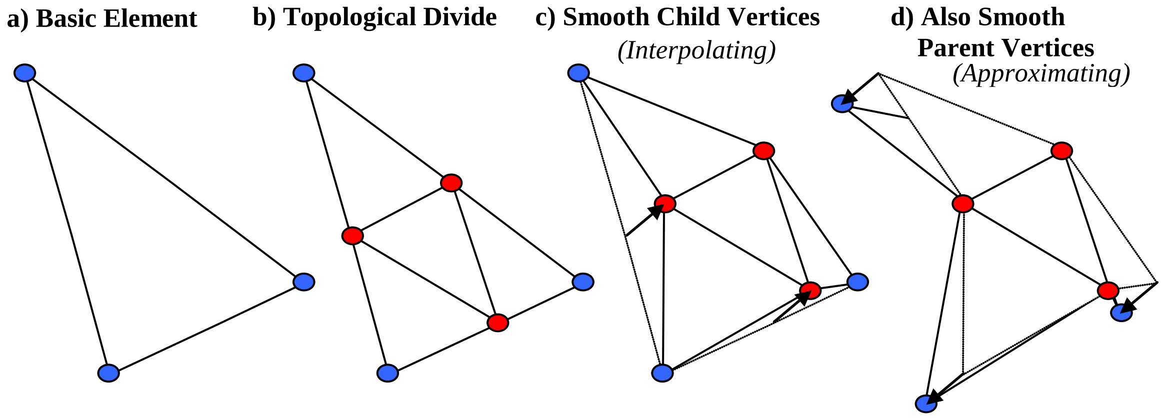
a) Basic Element **b) Topological Divide** **c) Smooth Child Vertices** *(Interpolating)* **d) Also Smooth Parent Vertices** *(Approximating)*

Figure 1. *Example subdivision of triangles*

the average position of its surrounding vertices in a ratio of 5-to-3. Other weightings are used for special cases such as vertices which have only three edges touching them or those at the mesh boundary.

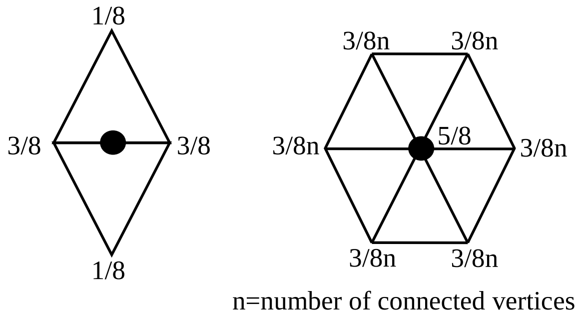

Figure 2. *Loop subdivision scheme weights for child (left) and parent vertices (right) adapted from [8].*

Once a mesh has been subdivided, it can be subdivided again and again in a recursive manner. Each "generation" gives a finer and finer mesh with four times as many triangles as the previous but with a smoother geometry. As more and more subdivisions are carried out, the mesh converges closer and closer to an underlying "limit surface", which is guaranteed not to have any creases and in general has C2 continuity (continuous rate-of-change of tangents) [8]. Whilst a detailed explanation of the various different schemes and their underlying mathematical construction is beyond the scope of this paper, the interested reader is directed to the excellent introduction by Zorin et al [8].

The advantages provided by this hierarchical series of meshes were capitalized upon by the computer graphics industries, whereby the same object (represented by the limit surface) could be displayed at various levels of detail as required. An object such as a game character far away from the viewer might only be drawn using a few hundred triangles, whereas the exact same character would be subdivided more when in the foreground and drawn with thousands of triangles. This method was very efficient with computing resources and allowed games to run faster and with more objects, textures and sounds.

2.2 Advantages for the building industry

Using subdivision limit surfaces to define building geometries offers many benefits to the shell and spatial structure design community. From an architectural point of view, their guaranteed smoothness creates aesthetically pleasing doubly-curved surfaces. These surfaces can be easily manipulated in real-time using the same techniques as in the computer graphics industries, namely they can be edited, and even stored, at a low level of subdivision and then rendered later at higher levels of subdivision to produce accurate drawings or images.

Unlike NURBS, which need a local two-dimensional U-V coordinate system to be defined, no parameterization of a Subdivision surface is necessary. This means that a complex surface does not need to be split into four-sided patches, something which is often the source of errors and discontinuities along seams in standard CAD modeling. However, if such a parameterization is desired, it can be incorporated as discussed below.

Subdivision can be thought of as a recursive process of coordinate smoothing, where each "child" (and possibly "parent") vertex inherits a coordinate smoothly interpolated from its neighboring vertices. This process need not be limited solely to a vertex's coordinates, and other associated properties can also be smoothed. For example, in a louvered façade, each vertex in the control mesh might be assigned a value of louver opening angle. In this case, as the mesh representing the façade is subdivided, the louver opening angles of the new vertices would be smoothly interpolated from those of the original vertices, giving the resulting façade design a smooth and aesthetically pleasing appearance. The same principle could be applied to any property associated with each mesh vertex, such as color, transparency or environmental performance (e.g. acoustic or thermal properties), with the underlying principle of subdivision ensuring that the properties are smoothly distributed over the resulting surface mesh. Of particular interest to façade design is the ability to assign a local U-V coordinate system to each vertex, as is done in texture-mapping and is a constraining requirement of NURBS as discussed above. Successively subdivided meshes then smoothly distribute this texture-map across a surface, allowing, for example, a façade panelization scheme to be mapped over a complex surface with minimal distortion.

Engineers often have to convert complex surface geometries into simplified meshes for finite element (FE) or computational fluid dynamics (CFD) analyses. With a subdivision surface representation, the hierarchical level of detail can be used to generate a mesh at the required density for any given analysis application. A CFD analysis could be performed on a relatively coarse mesh and an FE analysis on a finer mesh (or vice versa as required), but each mesh would be a representation of the same limit surface. Subdivision schemes are available which can locally subdivide the mesh more in some areas than others, giving the control necessary to create quality engineering analysis models without any extra effort by the user.

It is often also the case that a complex surface needs some sort of mesh to represent its support structure. A surface defined using subdivision could be designed using the standard tools such as cutting with sections or draping a grid over it. But Subdivision Surfaces also come with their own inherent triangulated structural grid, which can also be sampled at various levels of detail to give a sensible panel or member size. Figure 3 shows, for example, what the roof of the British Museum Great Court could have looked like if it had been constructed using a subdivision surface representation.

Since subdivision surfaces are created using a relatively coarse initial control mesh, they lend themselves very well to optimization, and a relatively complex surface can be defined by only a few control vertices. This opens up possibilities of carrying out multi-objective optimization to assess a proposed structure for any number of structural or environmental performance criteria, and using the results to feed-back and define new positions for the control mesh vertices. Since the control mesh has very few degrees of freedom, any optimization will be fast and could easily provide real-time feedback to a designer on the performance of the current proposal, as the case-study in the following section demonstrates.

In order to be of use in building construction, subdivision surfaces need to be constrained such that they can be forced to respect a given boundary. In the case of the British Museum Great Court described above, with its rectangular shape on-plan, a standard subdivision scheme's goal of smoothing the geometry would result in the corners being rounded off. Clearly this would not be acceptable in this context and full control is needed to specify where the subdivision can occur and where a given constraint has to be respected – usually at least around the boundary. Subdivision schemes can be adapted to achieve this; with the price paid being a lack of C2 continuity around these constraints. Tangent continuity is preserved however, so no creases appear, and the effect is localized, so this is viewed as an acceptable compromise.

3. CASE STUDY

In November 2008 an architectural competition was launched to generate designs for the construction of a new tropical hothouse as an extension to the existing 1969 glass-house in the botanical garden of the University of Aarhus in Denmark. As part of a team including architects C.F. Moller, who designed the original glass-house, and engineers Soren Jensen, the authors were able to apply subdivision surface modeling techniques for the first time on a real-life design project. The aim was to validate the integrated analysis and optimization approach to building design, and to test the Subdivision Surface framework, whilst at the same time develop an innovative and efficient design proposal for the architectural competition.

The design development was split into two phases, firstly to establish a surface geometry representing the proposed building envelope, and secondly to develop a supporting structure for this envelope. Ideally, these two tasks would have been performed in parallel, to allow a holistic solution which would be optimized in terms of both environmental and structural performance. However, since the timescale of the design competition was very tight,

Figure 3. *British Museum Great Court roof options using various levels of subdivision*

this was not practical and the two studies were performed sequentially.

Since its purpose was to house tropical plants, the thermal performance of the building was critical and drove the design exploration. The building envelope geometry was therefore derived based on an optimization of the solar gain of the building subject to multiple, and sometimes conflicting objectives. When a shape had been decided upon, a structural grid capable of supporting this building envelope was derived through geometrical and structural performance evaluation.

3.1 Geometry definition

The architects required a dome-like structure and the engineers were keen to absorb sunlight in winter to reduce heating requirements, without overheating during the summer. Whilst these two aims might seem contradictory, it was believed that the high latitude of the site location might mean that a building shape could be found which would capture the low winter sun more efficiently than the high-in-the-sky summer sun. This investigation lent itself very well to a parametric study, combined with embedded performance measures and an automated optimization loop, all of which can be efficiently delivered within a Subdivision Surface modeling framework.

As Figure 4 shows, a smooth dome-like building envelope can be represented by a very coarse control mesh with only seven vertices (six arranged in a hexagon around the base and one above at the apex). A very simple study could then be made using a single degree-of-freedom, the position of the apex vertex along the north-south axis. This would have the effect of leaning the dome to the south or north, exposing more or less of its surface to the southerly sun, and the solar gain performance of each dome was assessed.

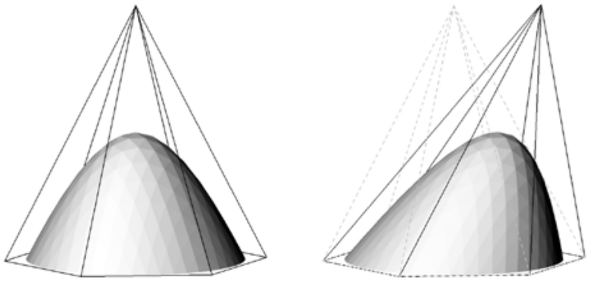

Figure 4. *Coarse control mesh with subdivision surface*

In practice, a slightly more complex parametric model was created, using a handful of parameters to control a mesh subdivided once from Figure 4, therefore having twelve vertices. The parameters controlled aspects such as stretching and curving on-plan as well as leaning (see Figure 5), with the footprint area being kept constant to allow sensible comparisons on performance to be made.

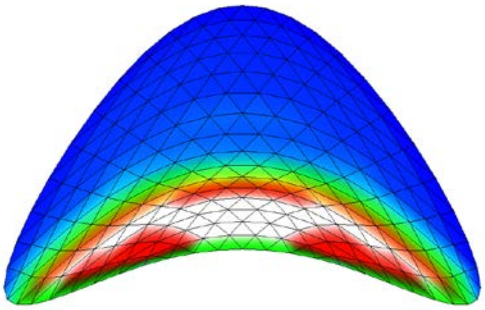

Figure 5. *Parametrically defined model*

It should be noted that the subdivision surface framework allows a handful of parameters to define a C2 smooth underlying limit surface as a potential building envelope. The mesh shown in Figures 3-7 is chosen simply for rendering and does not mean that the building itself has to be faceted in this way (although it could).

3.2 Performance analysis and optimization

The same Subdivision framework is also useful in automating the process of assessing the thermal performance of each candidate dome. In the case of the Aarhus Botanical Garden project the location of the site was known, and so the position of the sun at any time of day for any day of the year could easily be calculated [9] within the same software environment. As Figure 6 shows, this was also combined with use of the Fresnel equations for light transmittance and ray-tracing algorithms for shadow calculations, and integrated to give the solar gain during a full day. It allowed an accurate estimate of the likely daily winter and summer solar gain of any potential geometry to automatically be found and fed back to the user. This feedback was both graphical, for example the summer solar gain is shown in Figure 5 using a color scale of white/red indicating areas of high solar gain and blue showing areas of low-gain, and numerical as a single overall "performance rating" for use in optimization.

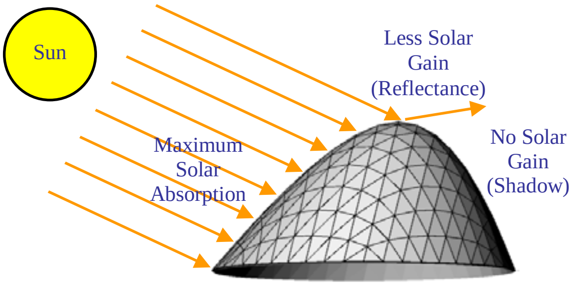

Figure 6. *Solar gain calculation*

The mesh was sampled at a given level of subdivision to provide a sufficiently accurate representation of its geometry for solar gain calculations without being unnecessarily complicated (and therefore slow to calculate). If a different study were being performed, for example using CFD calculations or structural buckling behavior, a different level of subdivision, involving more or less triangles, could be used to represent the same geometry at a sensible level of detail.

For the Aarhus project, the authors incorporated a simulated annealing optimization algorithm after Kirkpatrick et al [10] which was able to explore the design space of different dome shapes, one such shape being shown in Figure 7. In this way, the combination of geometry definition parameters which gave the best overall performance against a predefined weighting of various indicators such as winter and summer solar gain and enclosed volume could be quickly identified.

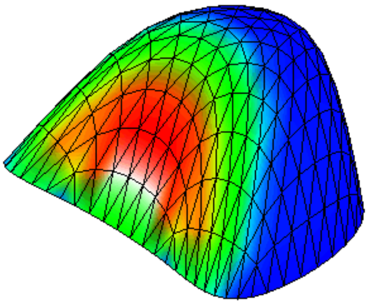

Figure 7. *Simulated annealing iteration snapshot*

3.3 Structure

Once a particular building shape had been decided upon, a structural grid to support the building envelope had to be developed. Once again, the subdivision modeling framework had an inherent benefit in that it suggested a number of triangulated grids, one for each level of subdivision, and one could be chosen which made a suitable compromise between fewer members, but each having sensible lengths as far as buckling and fabrication were concerned. Obviously the subdivision mesh does not have to be constructed in its entirety, nor from a single section. Subsets of the mesh edges or, as shown in the left of Figure 8, a hierarchy of primary and secondary structure are both sensible design options.

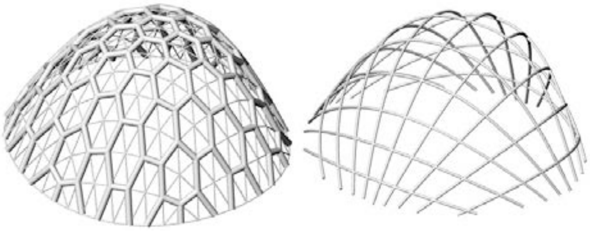

Figure 8. *Possible (left) and chosen (right) structure*

During the structural design phase the Aarhus Botanical Garden project moved towards an inflated PTFE cushion solution. This therefore favored a quadrilateral structural solution and the final design used the subdivision limit surface cut by rotated planes to give singly-curved steel members. The resulting structural frame is shown in the right of Figure 8, and is in itself the product of a parametric study on the spacing of planes and their centre of rotation. A quadrilateral Subdivision Scheme (Catmull-Clarke) was then used to generate a quad-based mesh to allow the formfinding of the PTFE cushions using dynamic relaxation. Further detail on the design development and optimization of this case study is outside the scope of this paper, but has been published elsewhere by the authors [11].

3.4 Results

The resulting design proposal encompassed an environmentally optimized form within an aesthetically desirable, smooth subdivision surface geometry (Figure 9). It was the result of a tight collaboration between academia, engineers and architects and was submitted into the design competition as one of six internationally acclaimed and specially invited teams of building professionals. These factors are believed to have been fundamental in the design winning the competition. This is believed to be the first Subdivision Surface building to be constructed, is currently on-site in Aarhus, and is due for completion in 2012. This demonstrates the success of taking a collaborative and integrated approach to building design, and confirms that a parametric subdivision surface framework has great potential as a design tool for the construction industry.

Figure 9. *Competition winning design - image courtesy of C. F. Moller*

4. FUTURE DIRECTION

The Digital Architectonics Research Group, and its associated MPhil program, in the Department of Architecture & Civil Engineering at the University of Bath, UK, of which the authors are members, is pioneering the use of subdivision surfaces in the building design process. And whilst subdivision surfaces provide many advantages over traditional methods of building modeling, the focus of their development to date has been towards computer animation and gaming applications and there are still some challenges to be addressed if they are to be implemented seamlessly into the design of shell and spatial structures.

The main current limitation on the use of subdivision surfaces is the question of intersections. Currently no elegant mechanism has been developed to calculate the line of intersection between two subdivision limit surfaces. Intersection is the basis of all Boolean operations such as Union and Difference, and such functionality will need to be accessible if subdivision surfaces are to be used on real live building projects.

An early attempt by the authors to develop such functionality can be seen in Figure 10 with the intersection of the limit surfaces shown in red. This uses a recursive intersection algorithm based on convex-hull bounding boxes adapted from Kobbelt [12], and solves the intersection to within a pre-defined tolerance in much the same way that NURBS are intersected. However, subdivision surface intersections suffer more than NURBS from the problem of how to represent the intersection within the same framework.

This need not be a problem for practical design problems, since the final as-built structure will almost certainly undergo some sort of rationalization into straight or singly curved sections. For example, whilst the steelwork for the Aarhus Botanical Garden shown in Figure 8 was based on the intersection of planes with the subdivision limit surface, for construction it was idealized as a series of best-fit circular arcs to allow fabrication from singly-curved sections. Nevertheless, if a robust Subdivision Surface modeling framework is to be used for structural morphology design, the challenge of Boolean operations will need to be addressed, and is the subject of continuing research by the authors.

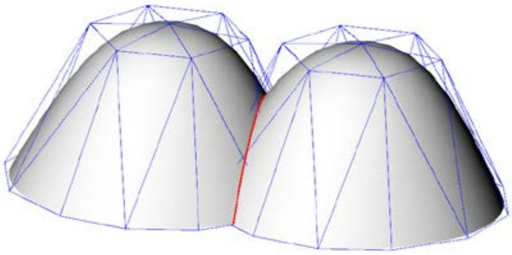

Figure 10. *Subdivision intersection*

There are also some issues with the inherent one-directionality of subdivision surfaces which will require a shift in the way architects design their buildings. Subdivision Surfaces are defined using a coarse control mesh, and are subdivided to find the limit surface on which a design proposal would be based. Currently architects have a clear idea of where they want their surface to be, and would wish to work backwards from this, to discover the coarse control mesh which will result in their desired surface. Whilst software can be developed to back-calculate a control mesh from a given target surface, it is suggested that a more radical change in approach is needed. If a designer only has a coarse control mesh to define, they might think more carefully about where the surface should be and why, and maybe use other feedback mechanisms such as structural or environmental calculations to help make their decision. If a building envelope can be thought of as the result of a process of design and optimization, rather than a desired aesthetic vision to be post-rationalized, then subdivision modeling can be harnessed to its full potential and the resulting buildings will be responsive to their environment and more easily and efficiently constructed.

5. CONCLUSIONS

The majority of current CAD software is effectively an electronic version of paper. But 3D building design complexity has increased such that this is no longer sufficient. In parallel, computer speed and memory has increased such that it is no longer necessary.

Borgart's call for "radical new techniques for the design, engineering and construction of complex geometry structures in order to continue to produce innovative and beautiful design solutions in the current economic and environmental climate" [1] can be addressed by adopting a Subdivision Surface representation for complex geometry structures. Subdivision limit surfaces are aesthetically desirable, and their recursive levels of representation have advantages in terms of providing a wide range of analysis meshes and options for support structure from a single base model.

Easily combined with parametric modeling, subdivision surfaces can quickly generate many design options. They lend themselves well to integrated analysis tools, which can use a carefully selected level of detail to calculate performance against a wide range of criteria. This can provide an interactive design environment, which does not stifle creativity, but rather provides instant feedback on design performance so that the designer can make informed decisions based on knowledge of the repercussions. Such software can also be allowed to control the design exploration, incorporating multi-objective optimization algorithms to help the user to quickly identify a particularly promising design direction.

Despite some limitations, which are currently under investigation, the benefits that subdivision surfaces offer to the building design community are still vast, and building design practitioners should learn to use them or risk losing out. Their adoption might mean changing the way building design is approached. It might also require a change in the building design process itself, moving away from a linear top-down work-flow to one with inherent design collaboration between architect and engineer right from the start. But these changes are seen as welcome, if not necessary, precursors to meeting the challenges posed by modern day shell and spatial structure projects.

It is time for a *Tabula Rasa* in terms of how software is used to design complex geometry structures. And subdivision surfaces, fully integrated with performance modeling and optimization, can provide just the catalyst that Borgart called for.

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