2 Introduction to Networks

Networks: An Introduction,
M. E. J. Newman, Oxford University Press (2010), Chapter 6.

The structure and function of complex networks,
M. E. J. Newman, SIAM review (2003), Sec. I.

2.1 Network Basics

We've already seen that a complex system of multiple interacting components can be represented using a network and we can gain a good deal of understanding from the graphical representation alone.

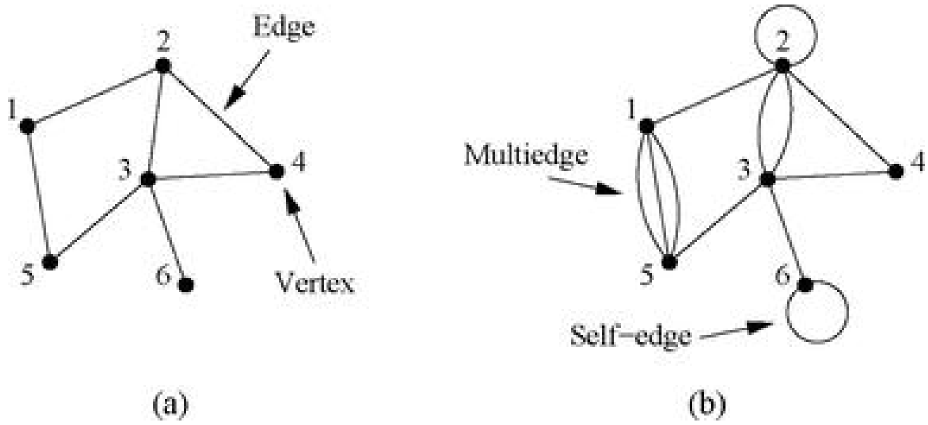

Figure 1: (a) A simple graph, i.e., one having no multiedges or self-edges. (b) A network with both multiedges and self-edges.

2.2 Network Definitions

Vertex: (Also *node*, *site*, *agent*): The fundamental unit of a network, used to represent the components (or individual agents) of a complex system. They may be assigned associated properties or variables.

Edge: The line connecting two vertices. Used to show the existence of a coupling link, connection or line of influence between two nodes.

Network	Vertex	Edge
Internet	Computer or router	Cable or wireless link
World Wide Web	Web page	Hyperlink
Citation network	Article, patent or legal case	Citation
Power Grid	Generating station or substation	Transmission line
Friendship network	Person	Friendship
Metabolic network	Metabolite	Metabolic reaction
Neural network	Neuron	Synapse
Food web	Species	Predation

Directed/undirected: An edge is said to be directed if it points in only one direction between two nodes, and undirected if it runs in both directions. A directed edge (or *arc*) from node i and j can be denoted by an arrow running from i to j, indicating the direction of influence. A network is said to be directed if all of its edges are directed.

Degree: The number of edges connected to a node. **Not** necessarily equal to the number of neighbouring vertices as there may be more than one edge between any two vertices. Sometimes referred to as "connectivity" but this term already has another meaning in graph theory. A directed graph has both an in-degree and an out-degree for each vertex, which are the numbers of in-coming and out-going edges respectively.

Connected Graph: There is a *path* of connecting edges between all pairs of vertices. In other words all nodes can reach, or be reached from, every other node in the network.

Component: The component to which a vertex belongs is that set of vertices that can be reached from it by paths running along edges of the graph. In a directed graph a vertex has both an in-component and an out-component, which are the sets of vertices from which the vertex can be reached and which can be reached from it.

Geodesic path: A geodesic path is the shortest path through the network from one vertex to another. Note that there is often more than one geodesic path between two vertices.

Diameter: The diameter of a network is the length (in number of edges) of the longest geodesic path which can be found in the network. You may sometimes see this term used to mean the average geodesic distance in a graph, although strictly this is a different quantity.

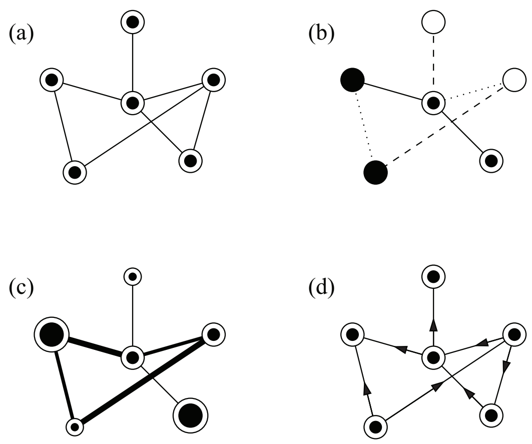

Figure 2: (a) an undirected network with only a single type of vertex and a single type of edge; (b) a network with a number of discrete vertex and edge types; (c) a network with varying vertex and edge weights; (d) a directed network.

2.3 Types of Graph

Even before looking at the mathematical aspects of networks or the statistical features of large networks we can see that they can be divided into different classes. The graphs in Figure 1 show two different types, a simple graph and one with multiple edges and self-edges. Figure 2 shows a few different small networks exhibiting different features. We can categorise many types of network using general features relating to the type of edges and nodes present, for example:

regular graphs All vertices having the same degree.
For example infinite or periodic regular lattices.

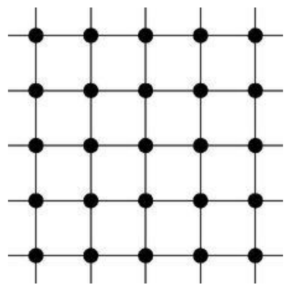

Figure 3: A 2D regular lattice.

Trees A connected, undirected network that contains no closed loops. E.g. models of branching processes.

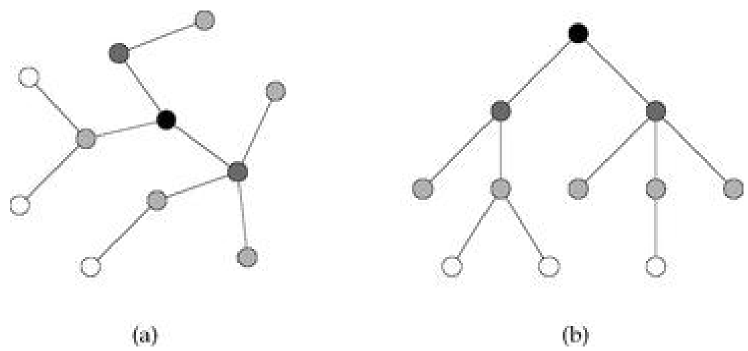

Figure 4: Two representations of the same tree, (a) shows branching from the centre outwards, (b) imposes a hierarchy.

Hypergraph A set of vertices in the graph can all be connected to each other. This is called a *clique*, and they can be all joined simultaneously with a *hyperedge*. A graph representing nodes connected by hyperedges is called a hypergraph. Examples of this are co-authors of books, actors in films, attendees at events and railway stations connected by train routes.

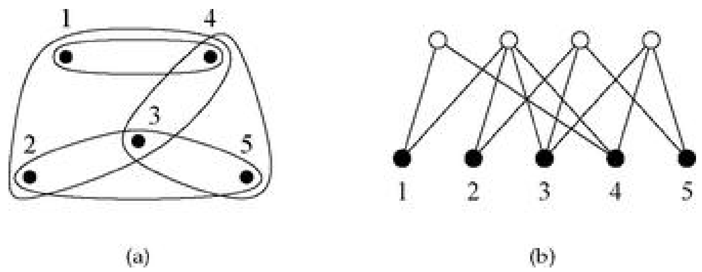

Figure 5: Five vertices in four different groups. (a) The hypergraph representation in which the groups are represented as hyperedges. (b) The bipartite representation in which four new vertices are introduced representing the four groups, with edges connecting each vertex to the groups to which it belongs.

Bipartite Also known as *affiliation* networks, these are often alternative ways of representing hypergraphs. Two sets of vertices are assigned, one representing the original vertices and the other representing the groups.

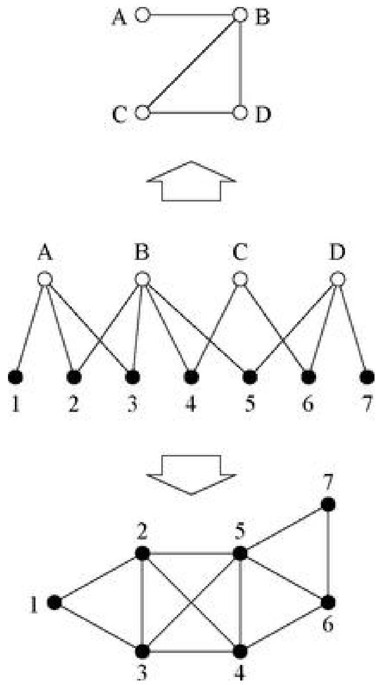

Figure 6: A bipartite network with seven of one type of one node joined to four of another type. The one-mode projections of the network are also shown.