Complex Systems and Networks

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General References

1. *Networks: An Introducion*,
   M. E. J. Newman, Oxford University Press (2010).
2. *The structure and function of complex networks*,
   M. E. J. Newman, SIAM review (2003).
3. *Complex networks: Structure and dynamics*,
   Boccaletti *et al.*, Physics Reports 424 (2006).
4. *Statistical physics of social dynamics*,
   Castellano *et al.*, Rev. Mod. Phys. 81 (2009).

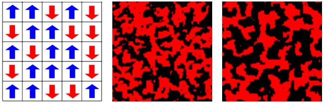
See references in: *Complex systems: A survey*,
M. E. J. Newman, Am. J. Phys. 79 (2011).

Basic Definitions (1)

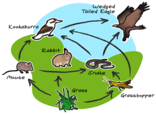
- Complex System:
  - System of many components;
  - relationships between components are important;
  - evolution of components' properties governed by *rules* of interaction;
  - behaviour of system at larger scales emerges naturally through interactions:
    - "*emergent properties*":

Examples of Complex Systems

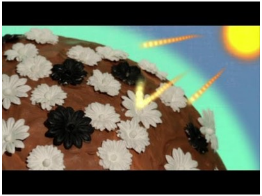
- Condensed Matter

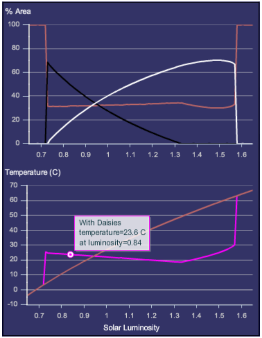


  (e.g. Ising model).
- Climate system
- Ecosystems



- Economies
- Cities





Emergent Properties

Segregation in Granular Materials:

Synchronisation of Fireflies:

The Trials of Life – A Natural History of Behaviour, narrated by David Attenborough.
Part 10 of 12 – "Talking to Strangers." ©BBC Bristol, MCMXC.

Theory and Modelling of Complex Systems

Simplified Models:

- Dynamical systems theory,
- networks,
- cellular automata,
- ...

attempt to represent important elements and find "universal" features.

Detailed Simulation:

- agent-based simulation,
- Monte Carlo simulation,
- ...

large-scale computer models, including many details.

Agent-Based Simulation

E.g. Modelling Flocking:

Simple computational flocking rules for "boids":

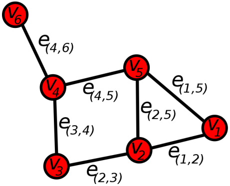
1. Collision Avoidance: short-range repulsion from local neighbours
2. Velocity Matching: steer towards average heading of neighbours
3. Flock Centring: steer towards center of mass of local neighbours

Conceptual Models of Complex Systems

- Higher level models abstracting the important features.
- Specify topology:
  - who interacts with whom:
  - the **network**.
- Specify dynamics:
  - behaviour of individual components,
  - interaction of components:
  - **coupled dynamical systems**.
- Try to investigate system and behaviours to gain insight:
  - analytical results,
  - numerical simulation,
  - statistical mechanics.

Basic Definitions (2)

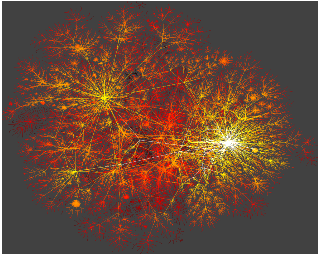
- *Networks* can be used to represent complex systems:
  - Graph, with components as *vertices*: $v_1 \ldots v_N$;
    (also *"nodes"*, *"sites"*, *"agents"*)
  - relationships shown as *edges*: $e_{(i,j)}$, connecting nodes.
    (also *"links"*, *"bonds"*, *"ties"*)



- Properties include: *node degree, path-length, centrality*...
- *Clustering* also important ($[V_1,\ V_2,\ V_5]$ above).

Examples of Networks

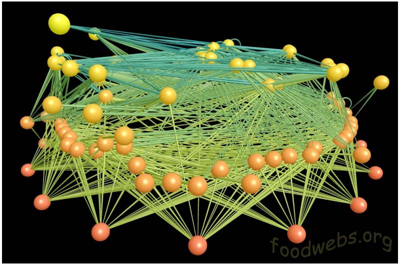
The Internet:



http://www.eee.bham.ac.uk/com_test/dsnl.aspx

- wide *degree distribution*.

Food webs:



Can also look at *robustness/resilience*:

- whether attacking nodes/edges could lead to system failure.

More Networks ("Communities")

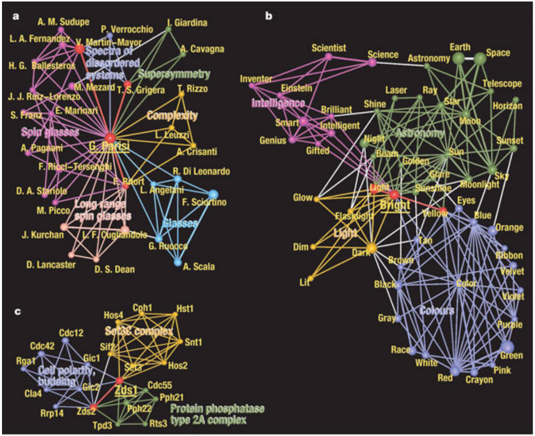
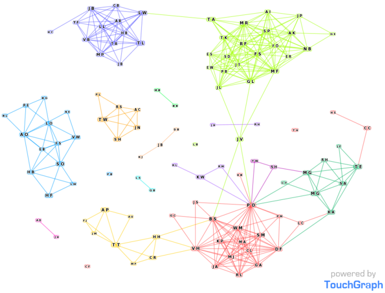


Figure: a, A co-authorship network. b, Word Association network. c, Protein–protein interactions in yeast. (From Palla &c., Nature.)

Human Social Networks

- Individuals can be interviewed about their own personal social network,
- *egocentric* networks constructed by asking an individual (the *ego*) about their contacts (the *alters*) and the links between them.



Dynamical Systems

Networks can then be used as the coupling links in dynamical systems. E.g. continuous-time dynamical systems:

$$\dot{x}_i = f(x_i) + \sum_{j \in K} \sigma_{i,j} g(x_j)$$

- Synchronisation of a chain of coupled nonlinear oscillators:

Spatio-Temporal Patterns

As well as complete synchronisation, can find other patterns:

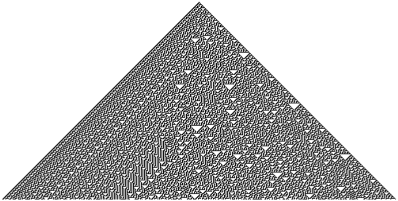
- Period two travelling waves:

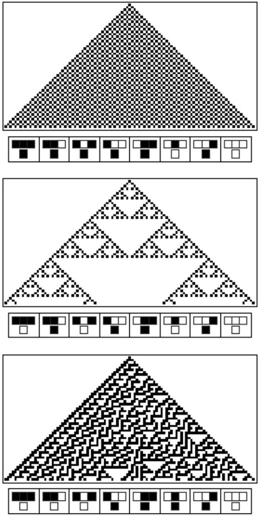
- Can study on complex networks as well as regular lattices.

Discrete Dynamics

E.g. Cellular Automata:

- each "site" has a *rule* based on own state and that of its neighbours,
- update at next time-step given by this rule.
- For 1D lattice shown, time is top to bottom.





[Wolfram (1983)]

Analytical Approaches

- How macroscopic laws be obtained from component interactions?
- use methods from statistical mechanics:
- e.g. thermodynamics: $m, p, v ... \rightarrow P, V, T$.
- Mean field arguments applied to networks:
- use "homogeneous mixing" assumption,
- try to obtain macro-variables and understand dynamical behaviour.

Mean Field Theories

E.g. SIR model of disease propagation:

$$S(\text{usceptible}) \xrightarrow{\lambda} I(\text{nfected}) \xrightarrow{\mu} R(\text{emoved})$$

Assuming homogeneous mixing (mean field):

$$\begin{aligned}
\dot{n}_S &= -\lambda \bar{k} n_I(t) n_S(t) \\
\dot{n}_I &= -\mu n_I(t) + \lambda \bar{k} n_I(t) n_S(t) \\
\dot{n}_R &= \mu n_I(t)
\end{aligned}$$

n_S, n_I, n_R are number density of S, I, R;
$\bar{k}$ is number of contacts per unit time.

- can determine critical ratio of rate constants for transmission, etc.