Studying multiple parameter models of technology diffusion on complex networks

Nick McCullen[⋆]
Alastair Rucklidge[⋆], Tim Foxon[†], Frin Bale[⋄]
[⋆]*Mathematics*, [†]*Earth and Environment*, [⋄]*Energy Research Institute*
University of Leeds

British Applied Mathematics Colloquium
University College London
28 March 2012

Motivation: Energy and Complexity

- Model of uptake of energy technology,
- policy-makers interested in best strategy to roll-out etc.

Modelling diffusion of technology

- Behaviour of individuals is combination of factors:
  - personal + social benefit.
- Perceived personal benefits to individual.
- Social benefit: combination of both:
  - social norm (society in general),
  - personal social network – friends & neighbours.

Modelling technology adoption

- Different people have different relative priorities:

  α_i: weighting given to personal value to individual p_i,

  β_i: weighting of average of personal social contacts s_i,

  γ_i: weighting to adherence to mainstream social norm m.

  - $\alpha_i + \beta_i + \gamma_i = 1$: based on personality.

- Total *utility* to individual:

$$u_i = \alpha_i p_i + \beta_i s_i + \gamma_i m \tag{1}$$

- Define adoption state $x_i = 0, 1$.
- Decision made when perceived benefits $>$ costs:
  - purchase state flips to $x_i = 1$ when $u_i > \theta_i$ (one-way):

$$x_i' = x_i + (1 - x_i)\sigma(u_i - \theta_i), \tag{2}$$

  $\sigma(\cdot)$ is step function.

- Threshold model of adoption.

UNIVERSITY OF LEEDS

Summary of the model

- Assuming all i take same α, β, γ, p, θ:

$$\mathbf{u} = \alpha p + \beta A\mathbf{x}/\mathbf{k} + \gamma \bar{\mathbf{x}}, \tag{3}$$

$$\mathbf{x}' = \mathbf{x} + (1 - \mathbf{x})\, \sigma(\mathbf{u} - \theta). \tag{4}$$

$$s_i = \frac{1}{k_i} \sum_{\mathrm{nei}(i)} x_j = \frac{\sum_j A_{ij} x_j}{\sum_j A_{ij}}, \quad k_i = degree \text{ of } i; \quad m = \frac{1}{N} \sum_i^N x_i$$

Real-world social networks

- Real networks have many features, including:
    - local connections, distant ties, wide spread in degrees, community structure. . .

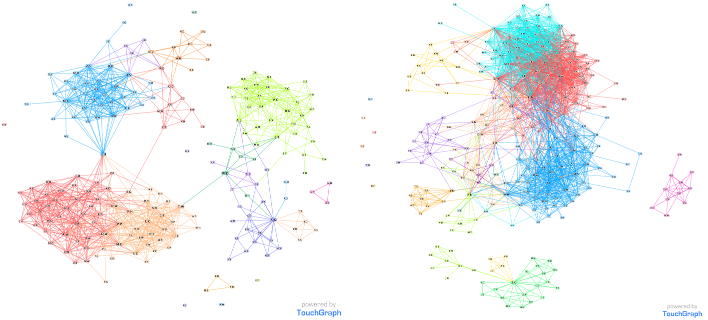
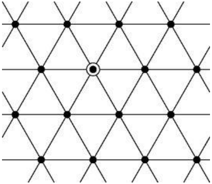


Figure: Inter-friend contacts on the *Facebook* website.

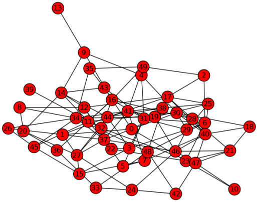
Network models

- Regular lattice:



+ e.g. city-like geography,
+ can have high *clustering*,
- long path-lengths $l \propto d^{1/D}$.

- Random (Erdős Renyí):



+ short path lengths $l \propto \frac{\log N}{\log \bar{k}}$,
- no *clustering* ($N \to \infty$).

UNIVERSITY OF LEEDS

"Complex" networks

- Different models reproduce different features.

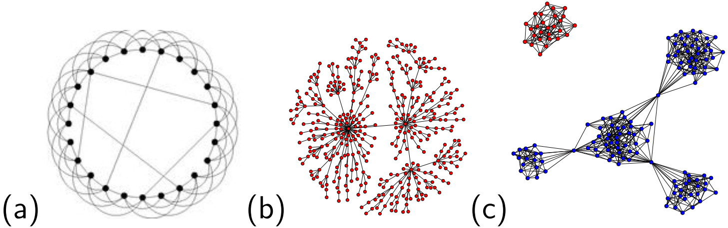


Figure: (a): A *small world* network with random *rewiring* of a regular lattice. (b): A preferential attachment graph which has a *scale-free* degree distribution. (c): A simple model of weakly-connected communities.

Numerical simulations

For a particular network and choice of model parameters:

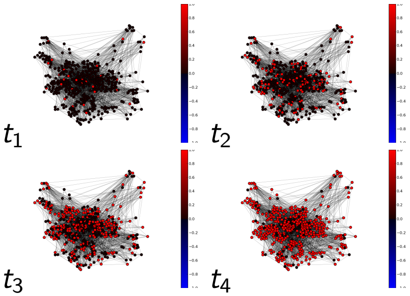
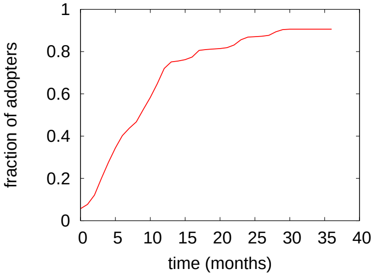


Figure: $t_{1,2,3,4} = 0, 4, 9, 27$

"Successful" uptake:

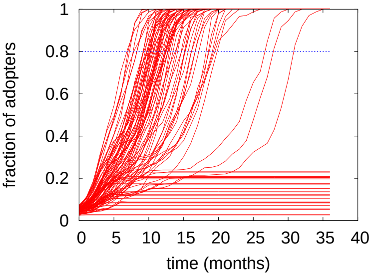


UNIVERSITY OF LEEDS

Sensitivity to model parameters

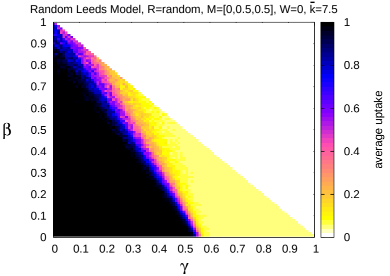
For each choice of parameters: e.g. here: $\theta = 0.25, p = 0.5$:

At each $\beta, \gamma, (\alpha = 1 - \beta - \gamma)$:
e.g. $\alpha = 0.05, \beta = 0.8, \gamma = 0.15$:



Take average of 100 realisations.

Repeat for all values:



- Can determine successful regions.

Random networks

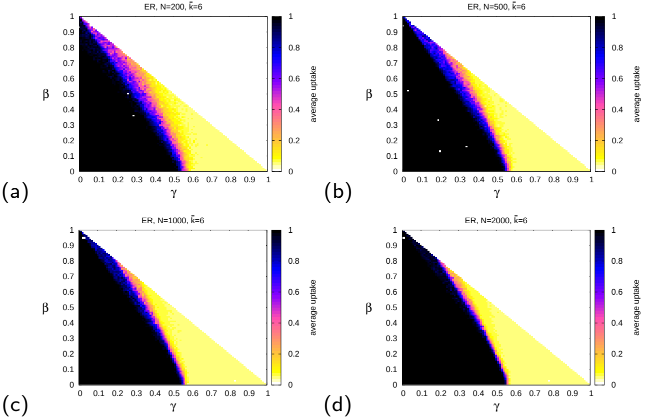


Figure: $\bar{k} = 6$. **(a)** $N = 200$, **(b)** $N = 500$, **(c)** $N = 1000$ **(d)** $N = 2000$.

Watts-Strogatz

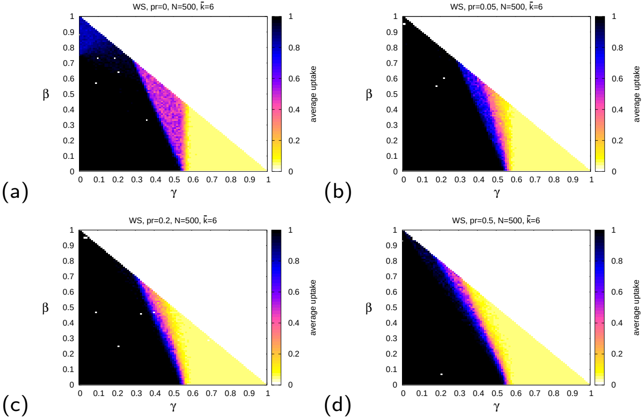


Figure: $\bar{k} = 6$, rewiring probability $p_r =$ **(a)** 0, **(b)** 0.05, **(c)** 0.2, **(d)** 0.5.

UNIVERSITY OF LEEDS

Analysis

- Simple cases:

$\alpha p > \theta$: Immediate uptake below $\beta = 1 - \gamma - \frac{\theta}{p}$,

$\alpha p + \gamma m_0 > \theta$: values below $\beta = 1 - \frac{\theta}{p} - \gamma \left(1 - \frac{m_0}{p}\right)$ successful.

- Simple *mean field*: assume average $\bar{s}_i = m$:

$$\begin{aligned} u = \alpha p + (\beta + \gamma) m_0 &\geq \theta, \quad \text{i.e.,} \\ p + (m_0 - p)(\beta + \gamma) &\geq 0; \quad \text{hence:} \\ \beta + \gamma &\leq \frac{\theta - p}{m_0 - p}, \end{aligned} \tag{5}$$

UNIVERSITY OF LEEDS

Local neighbourhood sensitivity

- Given individuals have a certain $\theta, p, \alpha, \beta, \gamma$ and m, require critical fraction of *active* neighbours:

$$s^* = \frac{\theta - \alpha p - \gamma m}{\beta}, \tag{6}$$

- $s^* > 1$: impossible,
- $s^* \leq 0$: immediate,
- $0 < s^* \leq 1$: required number of active contacts:

$$X_i \equiv \sum_j A_{ij} x_j \geq \lceil k_i s^* \rceil \equiv X_i^*, \tag{7}$$

- combining (6) and (7) gives X^* regions of β, γ plots. . .

UNIVERSITY OF LEEDS

Comparison with Watts-Strogatz

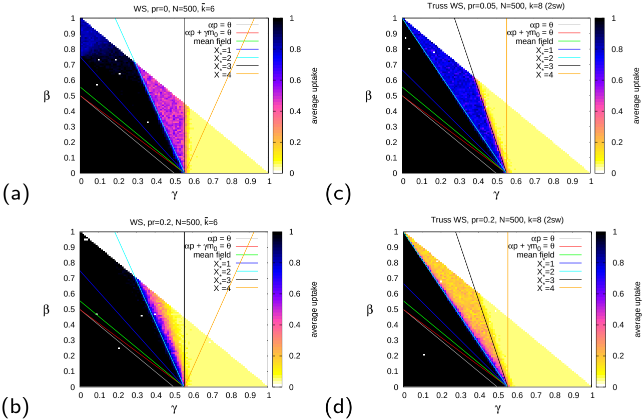


Figure: **(a)** 1D $\bar{k} = 6$, $p_r = 0$, **(b)** 1D $\bar{k} = 6$, $p_r = 0.2$;
(c) truss $k = 8$, $p_r = 0.05$, **(d)** truss $k = 8$, $p_r = 0.2$.

Chance and rate of uptake

- Number of active neighbours can be sufficient by chance with probability[1]:

$$P(X \geq X^*) = \sum_{n=X^*}^{k} \binom{k}{n} m^n (1-m)^{(k-n)}, \qquad (8)$$

  - $X^* = \lceil k(\theta - \alpha p - \gamma m)/\beta \rceil$.
- This is fraction of remaining $(1-m)$ of individuals to adopt, increasing overall average:

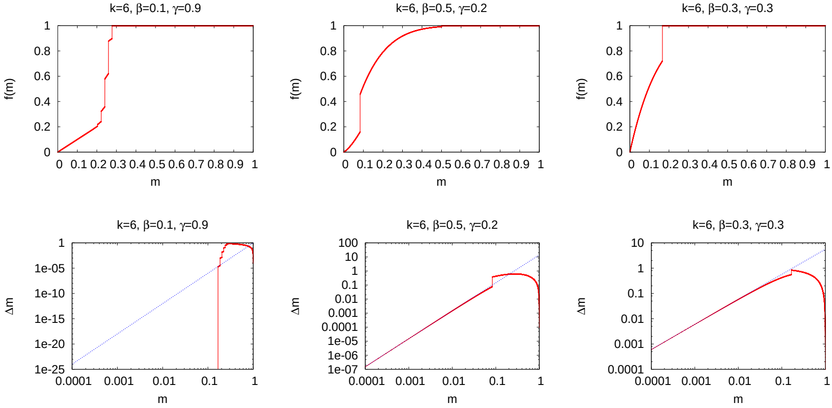
$$\Delta m = (1-m) P(X \geq X^*). \qquad (9)$$

[1] assume $k_i = k$ and random network.

UNIVERSITY OF LEEDS

Growth of initiated cluster

$$m' = m + (1-m)P(X \geq X^*) \equiv f(m).$$



UNIVERSITY OF LEEDS

Effect of initial seed size

$$\Delta m = (1-m)\sum_{n=X_i^*}^{k_i}\binom{k_i}{n} m^n (1-m)^{(k_i-n)},$$

- for small m:

$$\Delta m \sim \binom{k_i}{X_i^*} m^{X^*}. \qquad (10)$$

- For $X^* > 1$ disproportionate effect of low initial seed sizes (*"funding"*).
  - E.g. $k = 15$, $X^* = 4$, $\Delta m \sim 1365 m^4$.
    Half initial m_0 takes 8 times as long to reach target.

UNIVERSITY OF LEEDS

Other Networks

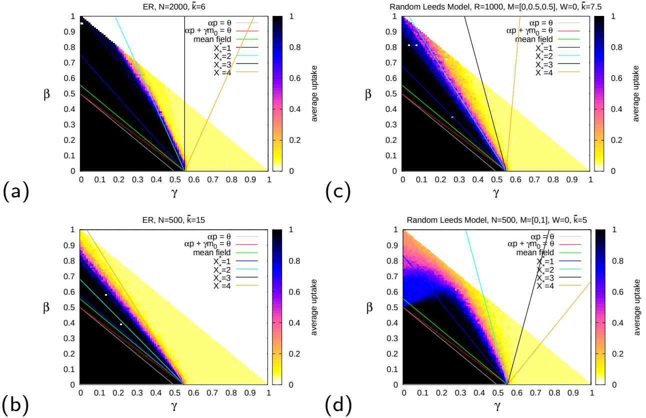


Figure: **(a)** Random $N = 2000$, $\bar{k} = 6$, **(b)** random $N = 500$, $\bar{k} = 15$, **(c)** geographic, connected communities, $\bar{k} = 7.5$, **(d)** disconnected communities, $\bar{k} = 5$.

Summary

1. A multi-parameter model of energy technology diffusion has been developed,
   - studied numerically at various parameters,
   - analytical treatment gives insight into numerical results,
   - implications for funding in initial stages.
2. Can use to compare possible interventions:
   - increase/focus initial funding,
   - reduce θ by providing incentives,
   - targeting communities (*critical mass*),
   - enhance social links using voucher schemes.