Studying multiple parameter models of technology diffusion on complex networks

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& everyone on the Energy-Complexity project.

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Motivation: Energy and Complexity

- Diffusion on networks used to model various phenomena:
  - Spread of diseases, ideas, beliefs...
- uptake of technology.
- E.g. Smart-phones:
  - visible and socially desirable,
  - mediated by social contacts between individuals.
- Energy technologies:
  - usually hidden (e.g. loft insulation),
  - decisions based on individual benefit.
- Policy-makers interested in uptake of energy efficiency measures,
  - success of various measures to encourage uptake.

Modelling diffusion of technology

Behaviour of individuals influenced by many factors:

- Intrinsic benefits and costs to individual.
- Social benefit:
    - information/influence exchanged between individuals,
    - follow social trend,
- combination of both:
    - social norm (society in general),
    - personal social network – friends & neighbours.
- Diffusion on social network.
- Combination of factors:
    - personal + social benefit (e.g. solar panels).

Modelling technology adoption

- Initially purchase state $x_i = 0$ (except for initial *seed*).
- Decision made when perceived benefits $>$ costs:
  - purchase state flips to $x_i = 1$ when $u_i > \theta_i$ (one-way):

$$x_i' = x_i + (1 - x_i)\sigma(u_i - \theta_i), \tag{1}$$

  $\sigma(\cdot)$ is step function.
- Threshold model of adoption.

Model parameters

- Different people have different relative priorities:

  α_i: weighting given to personal value to individual p_i,
  β_i: weighting of average of personal social contacts s_i,
  γ_i: weighting to adherence to mainstream social norm m.

  - $\alpha_i + \beta_i + \gamma_i = 1$: based on personality.

- Total *utility* to individual:

$$u_i = \alpha_i p_i + \beta_i s_i + \gamma_i m \tag{2}$$

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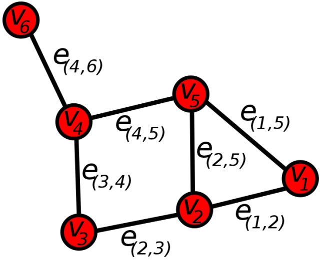
Modelling Social Interactions

- Individuals are considered as *nodes* on a network.
  - Properties of nodes are associated with variables:
    - adoption state $x_i \in [0, 1]$
    - ability to buy (low θ: able; high θ: unable),
    - motivation to buy (u_i).
- *Links* ('edges') are drawn between connected individuals.
  - Information/influence passed via edges.



Adjacency matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

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Summary of the model

- Assuming all i take same α, β, γ, p, θ:

$$\mathbf{u} = \alpha p + \beta \frac{A\mathbf{x}}{A\mathbf{1}} + \gamma \frac{|\mathbf{x}|_1}{N}, \tag{3}$$

$$\mathbf{x}' = \mathbf{x} + (1 - \mathbf{x})\, \sigma(\mathbf{u} - \theta). \tag{4}$$

$$s_i = \frac{1}{k_i} \sum_{\mathrm{nei}(i)} x_j = \frac{\sum_j A_{ij} x_j}{\sum_j A_{ij}}, \quad k_i = \textit{degree} \text{ of } i; \quad m = \frac{1}{N} \sum_i^N x_i$$

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Real-World Social Networks

- Real networks have many features, including:
  - local connections, distant ties, wide spread in degrees, community structure...

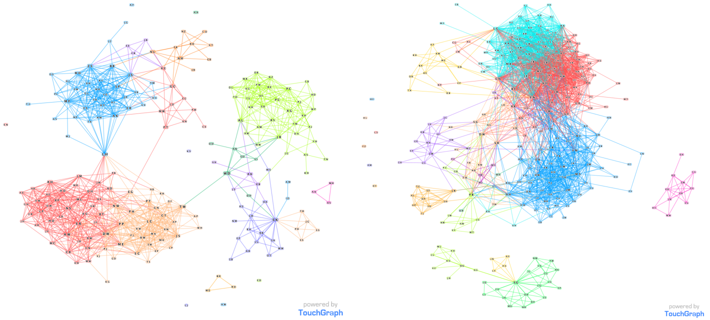
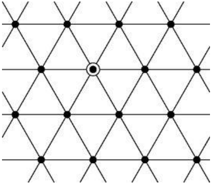


Figure: Inter-friend contacts on the *Facebook* website.

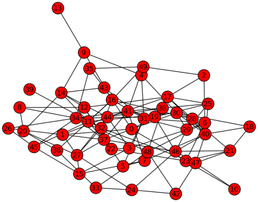
Network Models

- Regular lattice:



+ e.g. city-like geography,
+ can have high *clustering*,
- long path-lengths $l \propto d^{1/D}$.

- Random (Erdős Renyí):



+ short path lengths $l \propto \frac{\log N}{\log \bar{k}}$,
- no *clustering* ($N \to \infty$).

"Complex" Networks

- Different models reproduce different features.

(a) (b) (c)

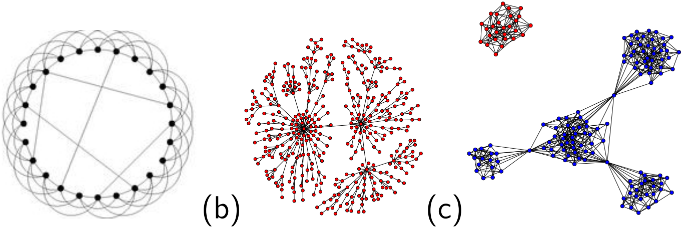


Figure: (a): A *small world* network with random *rewiring* of a regular lattice. (b): A preferential attachment graph which has a *scale-free* degree distribution. (c): A simple model of weakly-connected communities.

Numerical Simulations

For a particular network and choice of model parameters:

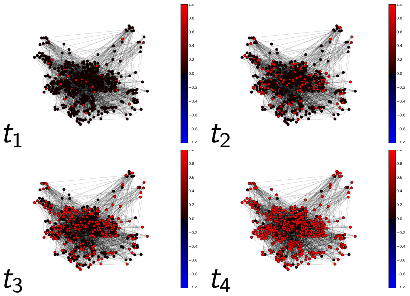
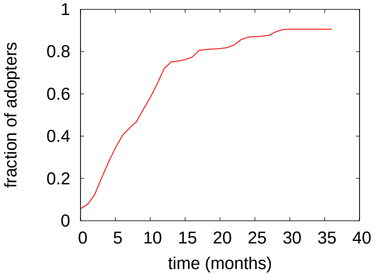


Figure: $t_{1,2,3,4} = 0, 4, 9, 27$

"Successful" uptake:

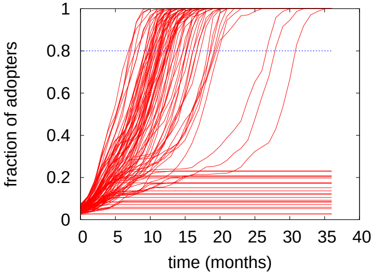


Sensitivity to Initial Conditions

For a particular network and choice of model parameters:

100 realisations:

- Same network can give different results,
- sensitive to details of network and initial seed,
- need to study *ensemble averages*.



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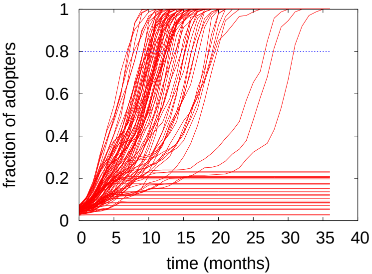
Sensitivity to Model Parameters

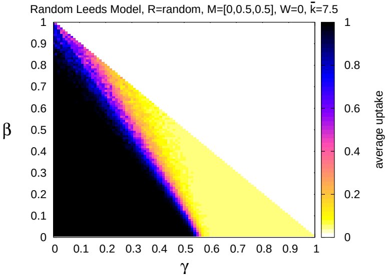
For each choice of parameters: e.g. here: $\theta = 0.25, p = 0.5$:

At each $\beta, \gamma, (\alpha = 1 - \beta - \gamma)$:
e.g. $\alpha = 0.05, \beta = 0.8, \gamma = 0.15$:



Take average of 100 realisations.

Repeat for all values:



- Can determine successful regions.

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Random Networks

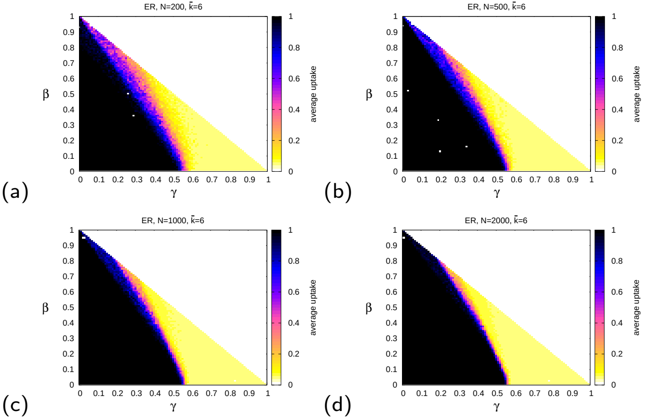

Figure: $\bar{k} = 6$. **(a)** $N = 200$, **(b)** $N = 500$, **(c)** $N = 1000$ **(d)** $N = 2000$.

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Watts-Strogatz

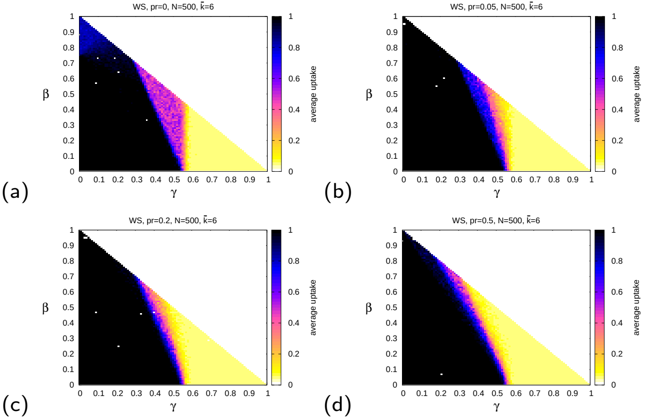

Figure: $\bar{k} = 6$, rewiring probability $p_r =$ **(a)** 0, **(b)** 0.05, **(c)** 0.2, **(d)** 0.5.

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Analysis

- Simple cases:

$\alpha p > \theta$: Immediate uptake below $\beta = 1 - \gamma - \frac{\theta}{p}$,

$\alpha p + \gamma m_0 > \theta$: values below $\beta = 1 - \frac{\theta}{p} - \gamma\left(1 - \frac{m_0}{p}\right)$ successful.

- Simple *mean field*: assume average $\bar{s}_i = m$:

$$\begin{aligned} u = \alpha p + (\beta+\gamma) m_0 &\geq \theta, \quad \text{i.e.,} \\ p + (m_0 - p)(\beta + \gamma) &\geq 0; \quad \text{hence:} \\ \beta + \gamma &\leq \frac{\theta - p}{m_0 - p}, \end{aligned} \tag{5}$$

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Local neighbourhood sensitivity

Since αp and γm are the same for all i, $x_i \rightarrow 1$ when:

$$\frac{1}{k_i}\sum_j A_{ij}x_j > \frac{\theta - \alpha p - \gamma m}{\beta}, \qquad (6)$$

For each α, β, γ, require X^* neighbours with $x_i = 1$ to cross θ:

$$\beta = \frac{\gamma(m_0 - p) + p - \theta}{p - X^*/k_i}. \qquad (7)$$

Can plot lines for each X^*, showing required social push.

Watts-Strogatz

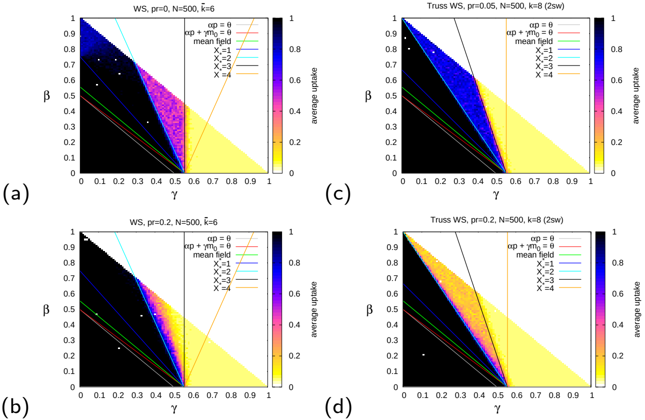

Figure: **(a)** 1D $\bar{k} = 6$, $p_r = 0$, **(b)** 1D $\bar{k} = 6$, $p_r = 0.2$; **(c)** truss $k = 8$, $p_r = 0.05$, **(d)** truss $k = 8$, $p_r = 0.2$.

Other Networks

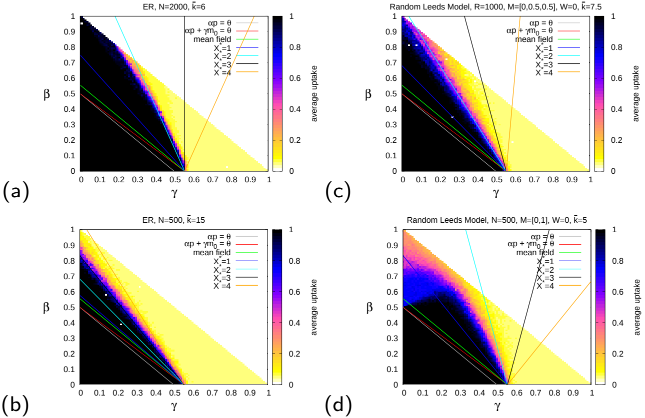

Figure: **(a)** Random $N = 2000$, $\bar{k} = 6$, **(b)** random $N = 500$, $\bar{k} = 15$, **(c)** geographic, connected communities, $\bar{k} = 7.5$, **(d)** disconnected communities, $\bar{k} = 5$.

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Further Analysis

The condition in (6) is the probability that:

$$X_i \geq \left\lceil k_i \left(\frac{\theta - \alpha p - \gamma m}{\beta} \right) \right\rceil \equiv X_i^*, \tag{8}$$

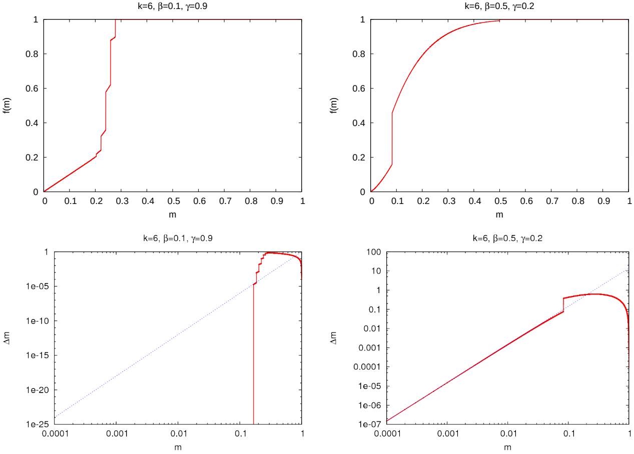
i.e.:

$$P(X_i \geq X_i^*) = \sum_{n=X_i^*}^{k_i} \binom{k_i}{n} m^n (1-m)^{(k_i - n)}. \tag{9}$$

- Assuming independence of neighbours.

Growth of initiated cluster

$$m_{t+1} = m_t + (1 - m_t)P(X \geq X^*) \equiv f(m_t). \qquad (10)$$



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Effect of initial seed size

$$\Delta m = (1-m)\sum_{n=X_i^*}^{k_i}\binom{k_i}{X^*} m^n (1-m)^{(k_i-n)}, \qquad (11)$$

for small m:

$$\Delta m \sim \binom{k_i}{n} m^{X^*} \qquad (12)$$

E.g. $k = 15$, $\beta = 1$: $X^* = \lceil k\theta \rceil = 4$, $\Delta m \sim 1365 m^4$.

- Half initial m_0 takes 8 times as long to reach desired level.

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Summary

1. A multi-parameter model of technology diffusion has been developed,
   - studied numerically at various parameters,
   - analytical treatment gives insight into numerical results,
   - implications for funding in initial stages.
2. Can use to compare possible interventions:
   - reduce θ by providing incentives,
   - targeting communities (*critical mass*),
   - enhance social links using voucher schemes.

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