Active Audition Using Coupled Oscillators

Nick McCullen[⋆,∞]

with Marty Golubitsky[†] *and Tom Mullin*[#]*.*

[⋆]University of Leeds, [†]Ohio State University, [#]University of Manchester.
[∞]Experiments carried out at Manchester Centre for Nonlinear Dynamics.

PANDA Meeting on
"Patterns and Nonlinear Dynamics in Neuroscience"
University of Nottingham

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Active Audition Using Coupled Oscillators

Nonlinearity in Biological Systems

The Feed-Forward Network

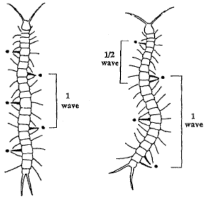
Experimental System

Other Filter Networks

Nonlinearity in Nature

Nature economises by using nonlinear phenomena:

- Turing patterns in morphogenesis;
- synchronisation of oscillations:
    - heart muscle,
    - fireflies;
- Central Pattern Generators (CPG) for locomotion[1]:



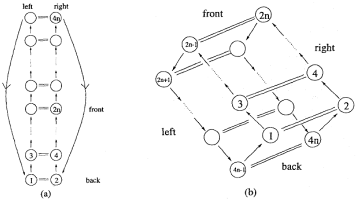


Fig. 1. (a) Schematic $4n$-cell network for gaits in $2n$-legged animals. Only cells $1, \ldots, 2n$ are connected to legs. (b) Folding up the network to eliminate long-range connections creates a structure with repeated modules, differing slightly at the two ends.

[1] "*A modular network for legged locomotion*", Golubitsky, Stewart, Buono, & Collins, Physica D (1998)

Nature Does It Better!

Animal visual and auditory systems have very good filtering characteristics:

- Can isolate signal from “noisy” background
- Able to discriminate specific frequencies
- Very good dynamic range (several orders of magnitude)
  - Difficult to achieve in devices
    $\Rightarrow$ Current amplifiers have linear response

Animal Auditory Systems

Active detection involved in mammalian hearing:

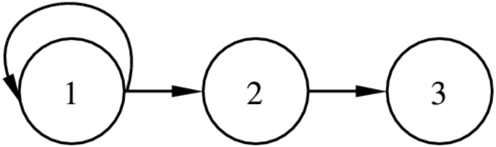
- Nonlinear growth
  $\Rightarrow$ large dynamic range

Involvement of Hopf bifurcation in insect hearing

- Active audition
- Coupling of limit cycles for small signal amplification

"Limit Cycles, Noise, and Chaos in Hearing."
R. Stoop, J.-J. V.D. Vyver, and A. Kern. *Microsc. Res. and Techn.*, **63:400–412**, 2004.

The Feed-Forward Network



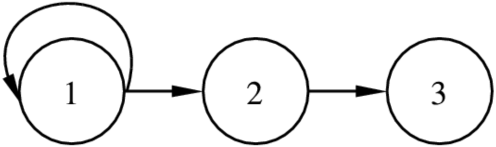
- Coupled systems have the form:

$$\begin{aligned}\dot{x}_1 &= f(x_1, x_1, \lambda)\\ \dot{x}_2 &= f(x_2, x_1, \lambda)\\ \dot{x}_3 &= f(x_3, x_2, \lambda)\end{aligned}$$

- λ is Hopf bifurcation parameter

"Some curious phenomena in coupled cell networks."
M. Golubitsky, M. Nicol, and I. Stewart. *J. Nonlin. Sci.*, **14(2):207–236**, 2004.

The Feed-Forward Network

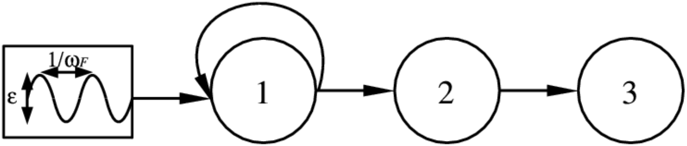


- Amplitude growth of unforced system:

$$\begin{aligned} A_1 &= 0 \\ A_2 &\sim \lambda^{\frac{1}{2}} \\ A_3 &\sim \lambda^{\frac{1}{6}} \end{aligned}$$

"Some curious phenomena in coupled cell networks."
M. Golubitsky, M. Nicol, and I. Stewart. *J. Nonlin. Sci.*, **14(2):207–236**, 2004.

With Periodic Input



- Forced network represented by:

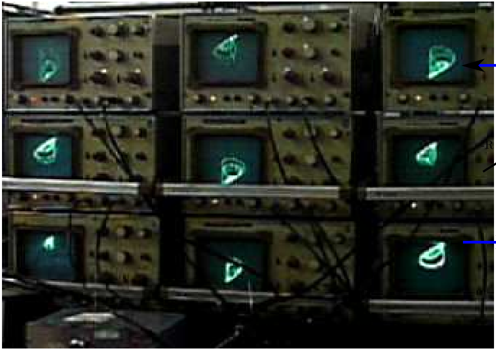
$$\begin{aligned} \dot{x}_1 &= f(x_1, x_1 + \varepsilon \cos(\omega_F t), \lambda) \\ \dot{x}_2 &= f(x_2, x_1, \lambda) \\ \dot{x}_3 &= f(x_3, x_2, \lambda) \end{aligned}$$

- λ held constant at Hopf Bifurcation

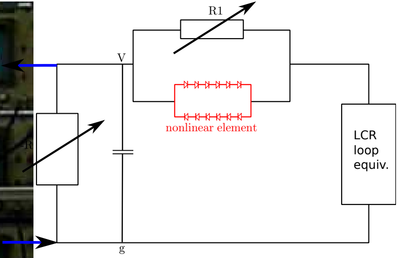
Experimental Electronic Oscillators

Modified *van der Pol* oscillators:

- LCR loop with nonlinearity from chain of diodes.



Circuit Schematic:



- Fixed-point response undergoes Hopf bifurcation before period-doubling cascade to chaos.
- Can connect units to make network of coupled oscillators.

Model of Oscillators

- 3 Degree-of-freedom system:

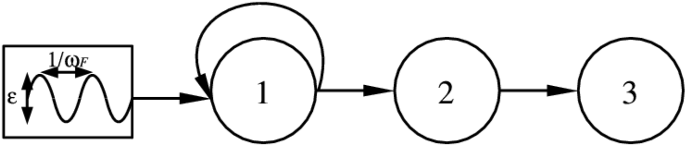
$$\begin{aligned}
\dot{x}_n &= \gamma[g(y_n - x_n) - \alpha_0 + \alpha_1(x_n - \sigma x_m)] \\
\dot{y}_n &= -z_n - g(y_n - x_n) \\
\dot{z}_n &= y_n - \rho z_n
\end{aligned}$$

- Has cubic nonlinearity:

  $g(V) = \beta_1 V + \beta_3 V^3$

"The origins of chaos in a modified Van der Pol oscillator."
J.J. Healey, D.S. Broomhead, K.A. Cliffe, R. Jones, T. Mullin. *Physica D*, 1991.

With (noisy) Periodic Input



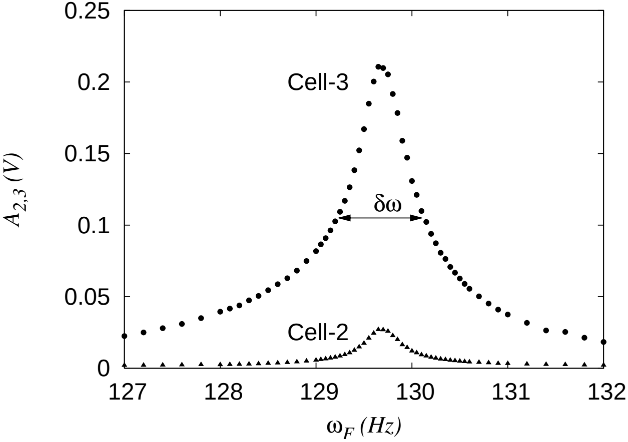
$\nu(t)+$

- Forced network represented by:

$$\begin{aligned}\dot{x}_1 &= f(x_1, x_1 + \varepsilon\cos(\omega_F t) + \nu(t), \lambda_1)\\ \dot{x}_2 &= f(x_2, x_1, \lambda_2)\\ \dot{x}_3 &= f(x_3, x_2, \lambda_3)\end{aligned}$$

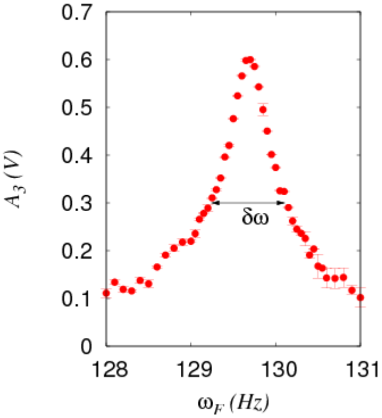
- λ held constant *near* Hopf Bifurcation ($\omega_1 \approx \omega_2 \approx \omega_3$)

Experimental Response



"Sensitive Signal Detection Using a Feed-Forward Oscillator Network"
N.J. McCullen, T. Mullin and M. Golubitsky, *Phys. Rev. Lett.*, **98, 254101** (2007)

Frequency Response



- Band-width:
  $\delta\omega \sim 1\% \ \omega_H$
  - $Q \sim 100$
- Narrow passband

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Signal Amplification & Noise Filtering

Amplification $(dB) = 20 \log_{10}(\frac{\texttt{Out}}{\texttt{In}})$

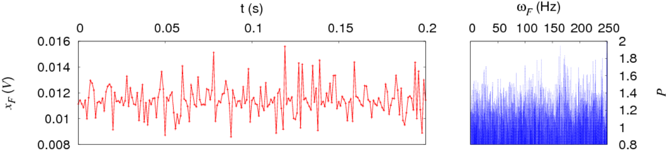
- Input: $\varepsilon \sim 5 \times 10^{-4} V$
  Output: $A_3 \sim 1V$

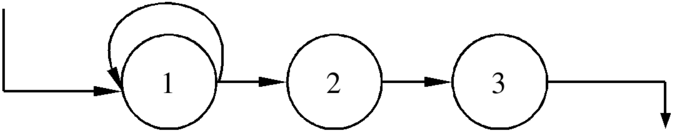
  $\approx 66dB$

- Signal recovery (*dynamic reserve*):
  Noise: $\nu \sim 5 \times 10^{-3} V$
  Signal: $\varepsilon \sim 5 \times 10^{-4} V$

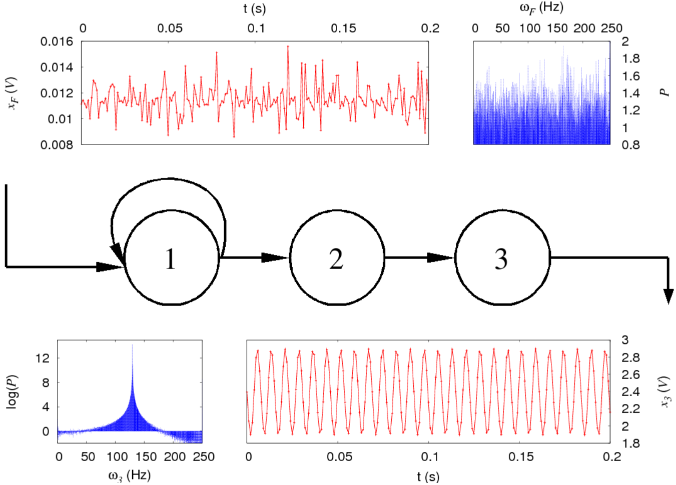
  Noise-Signal Ratio $\approx 20dB$

Noise Filtering & Signal Recovery

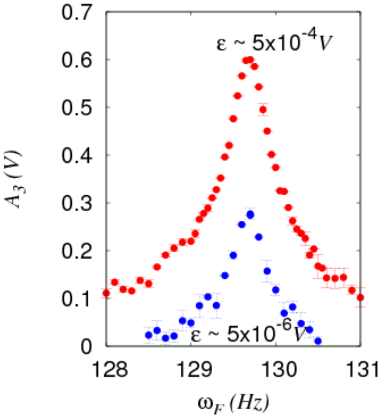




Noise Filtering & Signal Recovery



Small Signal Response



- $\varepsilon \sim 5 \times 10^{-4} V$

  $\nu \sim 5 \times 10^{-3} V$

  $SNR \approx 20dB$

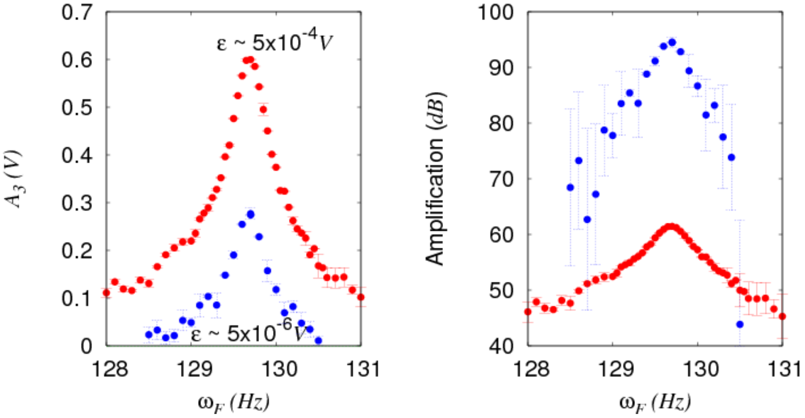
- $\varepsilon \sim 5 \times 10^{-6} V$

  $\nu \sim 5 \times 10^{-3} V$

  $SNR \approx 60dB$

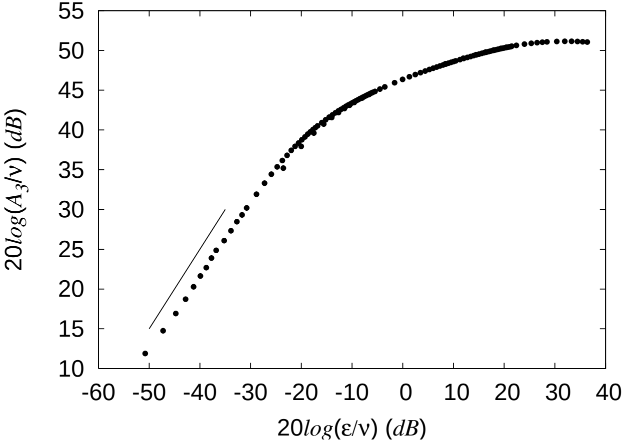
- Good signal recovery
- Nonlinear response – good dynamic compression

Non-Linear Amplification



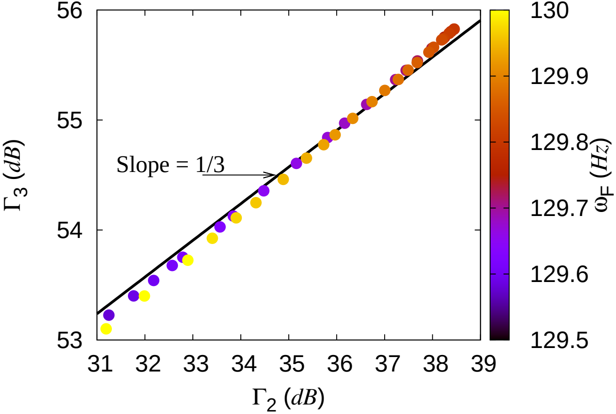
- Large dynamic range.

Dynamic Compression



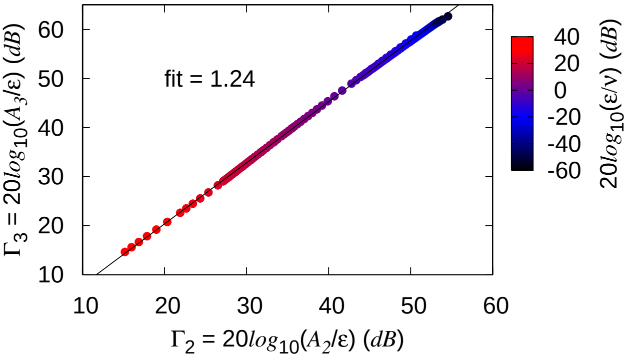
Amplitude Growth (1)

Amplification against Driving **Frequency**:

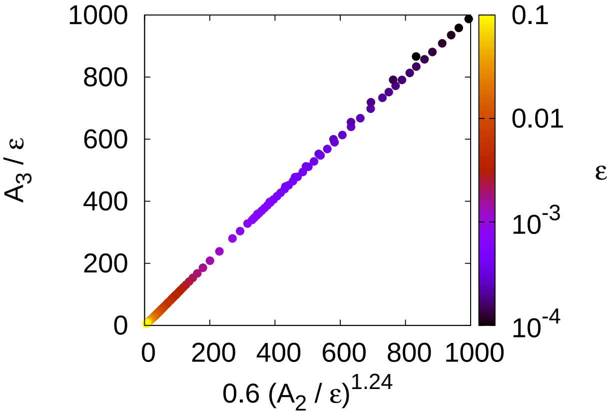


Amplitude Growth (2)

Amplification against Driving **Amplitude**:

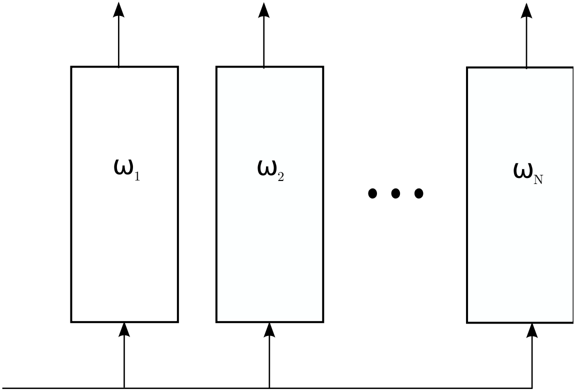


Amplification in Cells 2 and 3



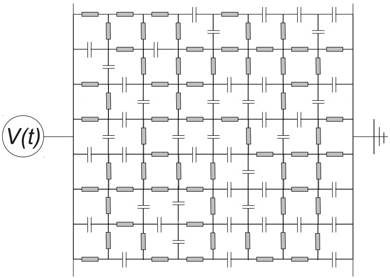
More Complex Filter Networks

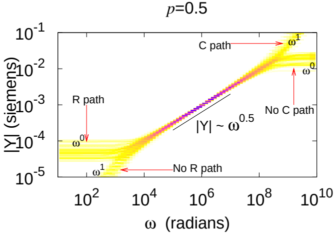
Multi-filter array:

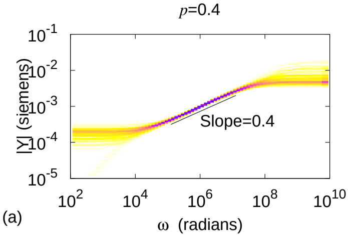


More Complex Filter Networks

Complex RC Networks:







"Emergent Behaviour in Large Electrical Networks"
Darryl P. Almond, Chris J. Budd and and Nick J. McCullen,
Approximation Algorithms for Complex Systems: Proceedings of the 6th International Conference on Algorithms for Approximation, Ambleside, Uk, 31st August-4th September 2009.
(Springer 2011)

Summary

- Nonlinear effects are found frequently in natural systems,
- synchronisation and resonance are particularly useful,
- application in neuroscience and signal detection.

- Well controlled experiments invaluable to:
  - study effect real world imperfections & noise,
  - both guide and confirm theoretical work.

- Many potential avenues to investigate with coupled oscillator systems.