Modelling the Dynamics of Decision-Making on Networks

Nick McCullen,
Alastair Rucklidge, Tim Foxon, William Gale,[1]
Mikhail Ivanchenko, Vladimir Shalfeev,[2]
[1] *University of Leeds, United Kingdom.*
[2] *Nizhny Novgorod State Technical University, Russia*

SIAM Conference on Dynamical Systems
May 22–26, 2011

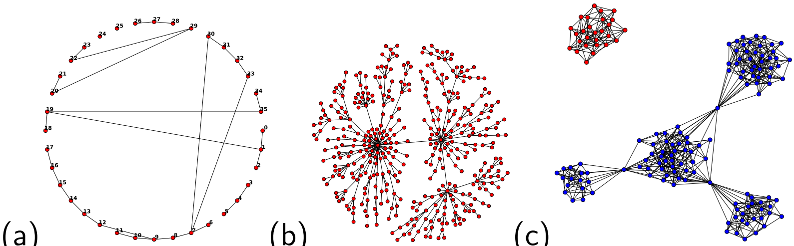
Energy and Complexity

Aim of research:

- Aid city government decision-making on energy policy,
  - e.g. strategy for reducing consumption and fuel poverty.
- Model households as dynamical systems, connected via social network.
- Dissemination of technology or ideas can be studied using models of diffusion on networks.
- Simulate interventions related to adoption of new technology or energy use strategies.
  - e.g. incentives for home insulation.

Types of Model Network

- Model networks constructed to a give different qualitative features of real world interactions:



(a): *Small world* network with 20% rewiring of a regular lattice.

(b): Preferential attachment graph with a *scale-free* degree distribution.

(c): Simple model of weakly-connected communities.

UNIVERSITY OF LEEDS

Real-World Social Networks

- Different types of social connection exist; these include:
    - geographical neighbours, distant friendships, family trees, *communities*.

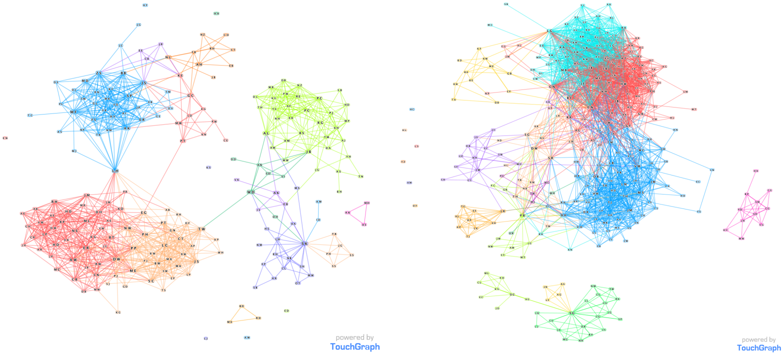
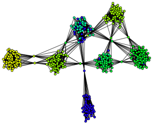
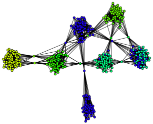


Figure: Inter-friend contacts on the *Facebook* website.

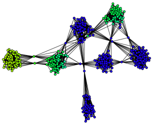
Example Uptake Results



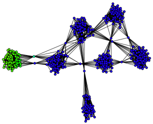
t_1



t_2

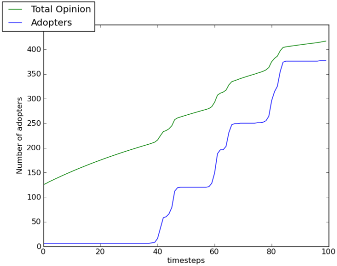


t_3



t_4

t_1=45, t_2=60, t_3=75, t_4=90



UNIVERSITY OF LEEDS

Model of Social Influences

Individual's preference weighted relative to social influence[1]:

- *Utility* (benefit) of product to individual:

$$U = \alpha p + (1 - \alpha)s$$

p: personal utility: intrinsic value of product to individual, Intrinsic to product and individual, could depend on:

- potential savings,
- relative or absolute,
- pay-back time;
- environmental credentials (may change),
- negative effects of barriers to adoption.

[1] S.A. Delre, W. Jager, T.H.A. Bijmolt, and M.A. Janssen. *Will it spread or not? The effects of social influences and network topology on innovation diffusion* (2010).

Model of Social Influences

$$U = \alpha p + (1 - \alpha)s$$

s: social utility: fraction of other individuals with technology,

- Data suggests individuals assign different relative value to personal contacts and society[2].
  - someone buys when adoption within society and contact network are above respective thresholds,
  - individuals classed as early, majority or late adopters.

α: relative weighting of personal to social value.

[2] T.W. Valente. *Social network thresholds in the diffusion of innovations* (1996).

Model Specifications

- Purchase state variable $x_i = 0, 1$:

  0: not purchased, 1: purchased,

- At each time-step evaluate *utility* of product:

$$U_i = \alpha p_i + \beta s_i + \gamma m$$

$$s_i = \sum_j^{K_i} \rho_{ij} x_j / \sum_j^{K_i} \rho_{ij}, \; m = \sum_{k=1}^{N} \sigma_k x_k / \sum_k^{N} \sigma_k .$$

- If U_i crosses a threshold θ_i then $x_i \rightarrow 1$.

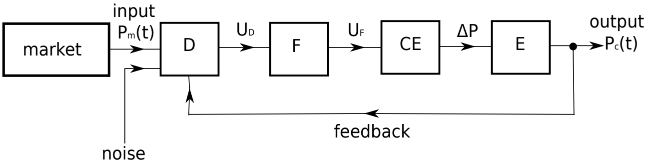
UNIVERSITY OF LEEDS

Dynamics of Opinion Formation

- Time-scales for updating opinion (τ_1) and making purchases (τ_2) may be different:
  - τ_1 opinion updated after interacting with friends and taking in media (e.g. daily, weekly),
  - τ_2 purchase decisions made less frequently (motivated by monthly pay-day, weather, prices, breakages etc.).
- Need to model intermediate dynamics of opinion in between decisions.

Modelling Consumer Opinion Formation

- Processes modelled as elements in a control system:



With constant market price and simplest filter, can obtain following[3]:

$$\dot{X} = \gamma - X - \Phi(X).$$

X current dimensionless price-deviation,

γ initial dimensionless price-deviation.

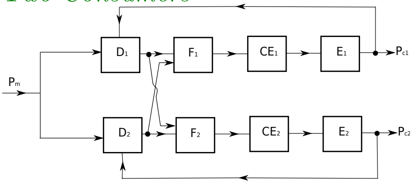
$\Phi(X)$ nonlinear function of deviation.

[3]McCullen *et al.*, to appear in IJBC (2011).

UNIVERSITY OF LEEDS

Interacting Consumers

Two Consumers

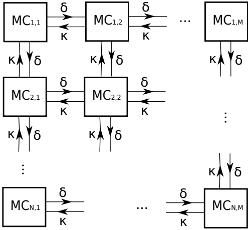


The equations can then be written:

$$\dot{X}_1 + X_1 + \Phi(X_1) = \gamma_1 + \kappa\Phi(X_2),$$

$$\dot{X}_2 + X_2 + \Phi(X_2) = \gamma_2 + \delta\Phi(X_1).$$

Lattice of Consumers



With equations:

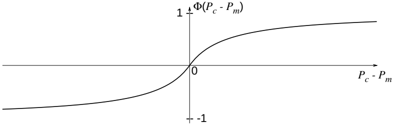
$$\dot{X}_{n,m} + X_{n,m} + \Phi(X_{n,m}) = \gamma_{n,m} + \delta\Phi(X_{n-1,m}) + \kappa\Phi(X_{n+1,m}) + \delta\Phi(X_{n,m-1}) + \kappa\Phi(X_{n,m+1}).$$

UNIVERSITY OF LEEDS

Analytical Results

Form of the Discriminator

Choose $\Phi(X)$ to limit large deviations:



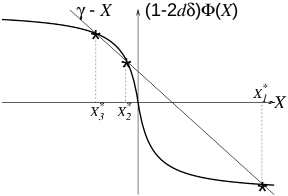
e.g.: $\Phi(X) = \dfrac{\beta X}{1 + |\beta X|}$

Single Consumer

Equilibrium states found from:

$$\gamma - X = \Phi(X).$$

Two Interacting Consumers



UNIVERSITY OF LEEDS

Clustering of Opinion Over Time

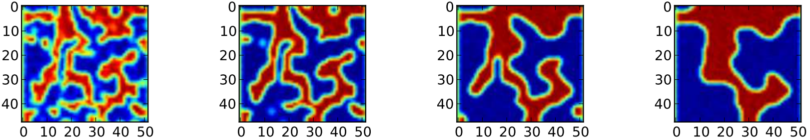
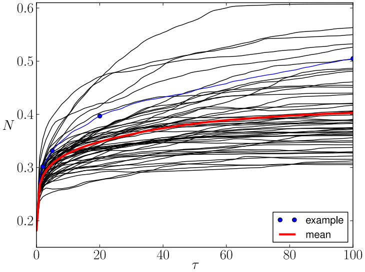


Figure: $\delta = 2; \tau = 2, 5, 20, 100$



Measure using *Mix-Norm*[a]:

$$N^2 = \sum_{k,l} \frac{|a_{k,l}|^2}{\sqrt{1 + k^2 + l^2}},$$

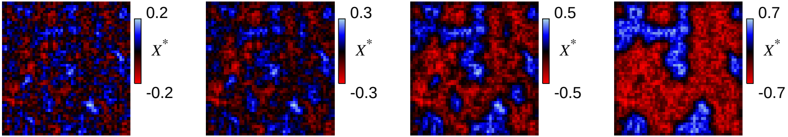
$a_{k,l}$: Fourier transform coefficients;

N larger for coarser structure.

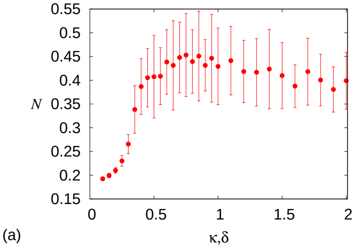
[a] Mathew *et al.*, Physica D (2005)

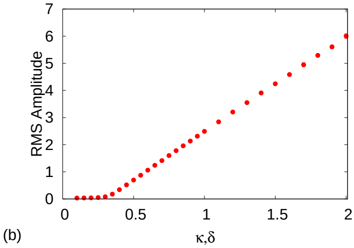
UNIVERSITY OF LEEDS

Clustering and Coupling Strength



δ = 0.25 δ = 0.3 δ = 0.35 δ = 0.4

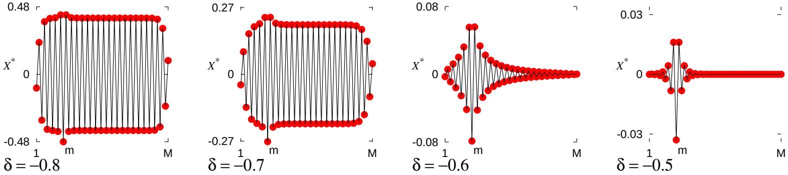


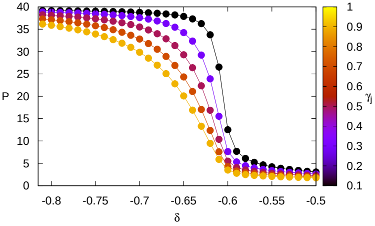


(All for $\tau = 100$)

UNIVERSITY OF LEEDS

Localized Patterns (1D lattice)





Use *participation number*:

$$P = \frac{1}{\sum z_n^2},$$

with $z_n = \frac{|X_n|}{\sum_n |X_n|}$.

UNIVERSITY OF LEEDS

Summary and Future Work

- Spread of energy technologies can be modelled as diffusion on networks.
  - Need to choose correct network model(s).
  - Also dynamics of individual decisions.
- Model exchange of opinions as a coupled dynamical system:
  - find clustering of opinions over time,
  - depends on the strength of opinion exchanges.

Next step:

- Combine opinion dynamics model with discrete purchase decision.

UNIVERSITY OF LEEDS