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Modelling of Consumer Decision-Making Behaviour Using Dynamical Networks

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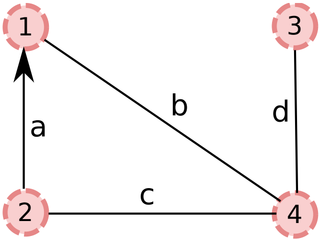
Models Meeting
7^{th} June 2010
University of Leeds

Outline

Network Models

- **Nodes** and **links** represent **individuals** and **interactions**.

Graph Representation:



Matrix Representation:

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}.$$

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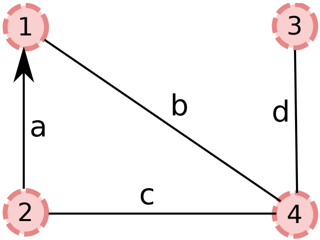

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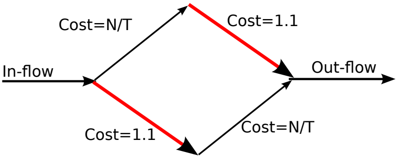
- Can measure properties such as relative importance of nodes/edges to whole system.

Braess' Paradox

- Finite capacity networks:



- Given free choice, agents use each route, giving average cost of 1.6.

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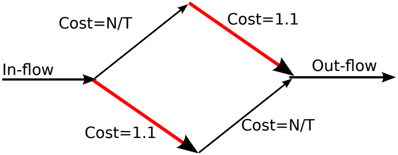
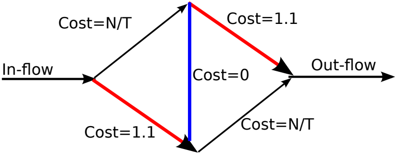


- Low-cost short-cut added:



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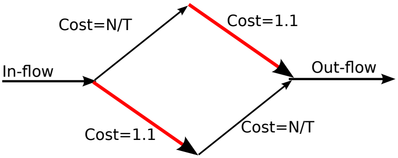

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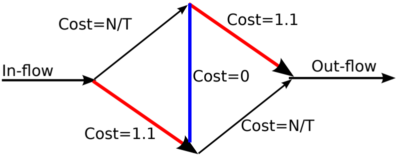

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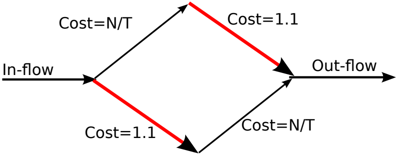

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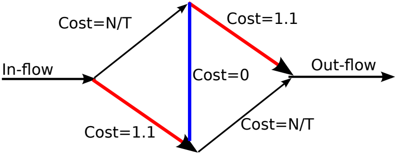

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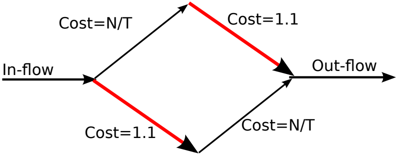

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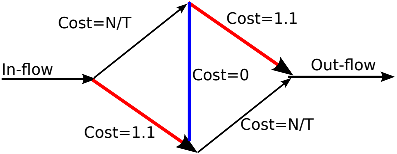
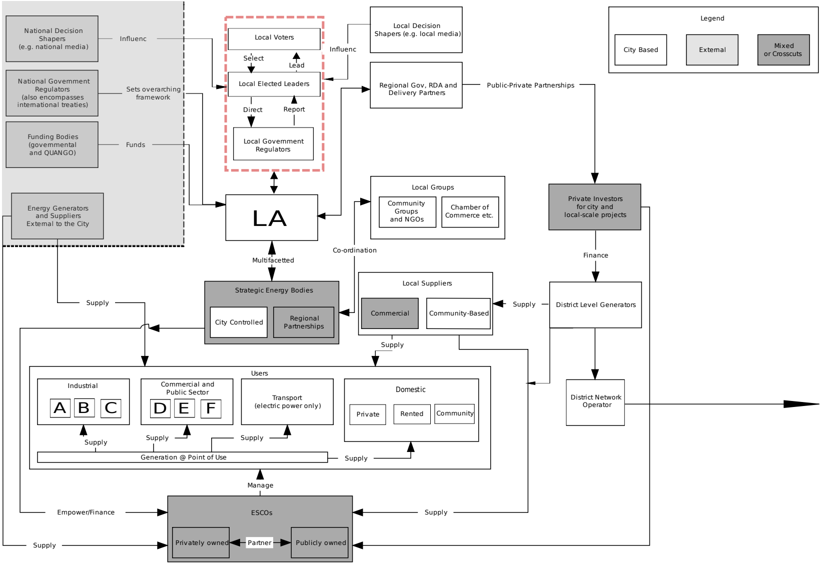


- Given free choice, agents use each route, giving average cost of 1.6.
- All agents end up using short-cut and average cost goes up to 2!

- Reveals counter-intuitive phenomenon.
- Many real-life examples exist.



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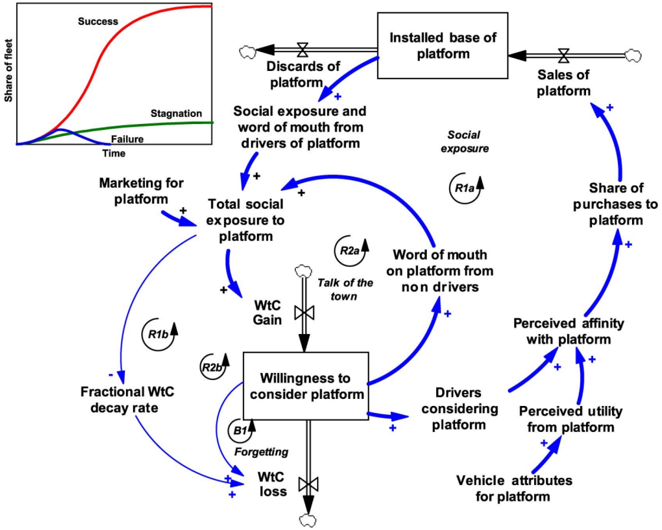
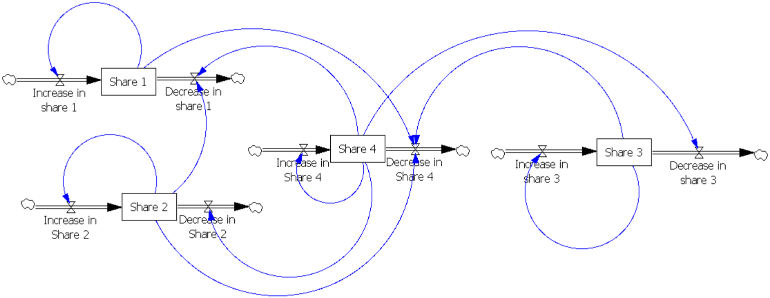
Dynamical Systems

"System Dynamics" representation:



Struben and Sterman. Environment and Planning B (2008).



Dynamical System:

$$
\begin{aligned}
\frac{dx_1}{dt} &= a(x_1 - x_2) + b(x_1 - x_4) \\
\frac{dx_2}{dt} &= c(x_2 - x_4) \\
\frac{dx_3}{dt} &= d(x_3 - x_4) \\
\frac{dx_4}{dt} &= b(x_4 - x_1) + c(x_4 - x_2) + d(x_4 - x_3)
\end{aligned}
$$

Coupling Matrix:

$$
M = \begin{pmatrix} (a+b) & -a & 0 & -b \\ 0 & c & 0 & -c \\ 0 & 0 & d & -d \\ -b & -c & -d & (b+c+d) \end{pmatrix}
$$

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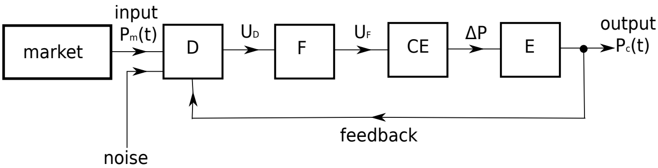
- This “fair” price is formed with reference to the consumers’ own estimate and the information coming from the other consumers.

Introduction to Control Systems

- Start with a simple model characterized by a single variable, namely, the energy price, specific for each consumer and the market in general.
- Making the decision whether to buy some goods (or whether to continue buying) the consumers compare the market price with some "reasonable price" in their own mind.
- This "fair" price is formed with reference to the consumers' own estimate and the information coming from the other consumers.
- This leads to the dynamical network model of the consumers' decision-making.

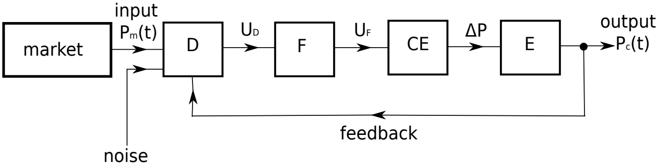
Control Systems Approach

Automatic control systems with feedback conveniently embody the above properties.



- The estimator (E) is the controlled object.
- $P_c(t)$ is current reasonable estimation of price of goods from the consumer (output signal).
- $P_m(t)$ is market price of the goods (input reference signal).
- $P_c(t)$ and $P_m(t)$ are compared by the discriminator (D)according to some function U_D.

Formulating the Model



- The customer would average the short time-scale price fluctuations on the market.
  - Model this by passing the signal from the D output through the filter (F).

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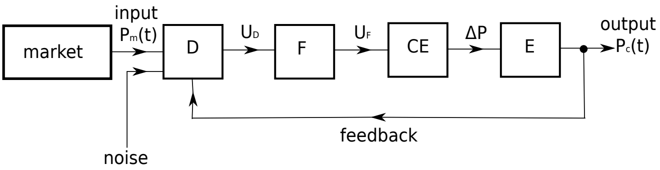

- The customer would average the short time-scale price fluctuations on the market.
  - Model this by passing the signal from the D output through the filter (F).
- Then the signal from the F output U_F is supplied to the control element (CE), directly changing the operated consumer estimation price in a manner such as to approach the reference market price.

Mathematical Representation of Consumer

Equation for the estimator E:

$$P_c = (P_c)_i + \Delta P,$$

- $(P_c)_i$ is the consumer price estimation in the initial moment
- ΔP is the change in the consumer estimation price controlled by CE.

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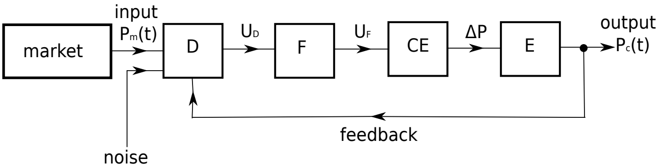

Equation for CE:

$$\Delta P = -SU_F,$$

where S is the slope of the CE characteristics.
Equation for F:

$$U_F = K(p)U_D,\ p \equiv \frac{d}{dt},$$

where $K(p)$ is the filter transmission factor.

Form of the Function for the Discriminator

$$U_D = E\Phi(P_c - P_m),$$

where E is the maximum at the D output, $\Phi(P_c - P_m)$ is the discriminator nonlinear characteristics.

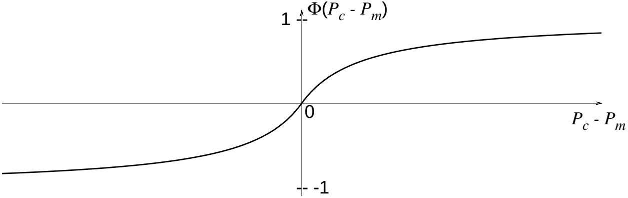

Figure: Discriminator nonlinear characteristics.

Analysis

- Current price deviation $P(t) = P_c(t) - P_m(t)$,
- $\sigma = SE$ is the greatest error to be corrected,
- $X = \dfrac{P}{\sigma}$ is dimensionless price-deviation,
- $\gamma = \dfrac{P_i}{\sigma}$ is initial dimensionless price-deviation.

Obtain the following equation for the consumer (MC):

$$X + K(p)\Phi(X) = \gamma, \; p \equiv \frac{d}{dt}.$$

Analysis

Assuming the market price remains constant, and using the simplest filter ($K(p) = \frac{1}{1+ap}$):

$$\frac{dX}{d\tau} + X + \Phi(X) = \gamma.$$

- This is a first order ordinary differential equation.
  - Dynamics can be simulated on a computer to find steady states etc.

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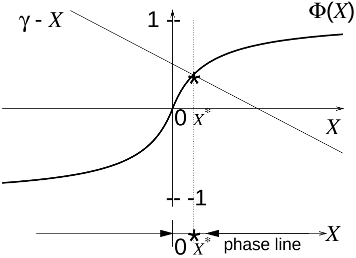
Analysis of Single Consumer

- Equilibrium states for single consumer can be found from:

$$\gamma - X = \Phi(X).$$



(a)

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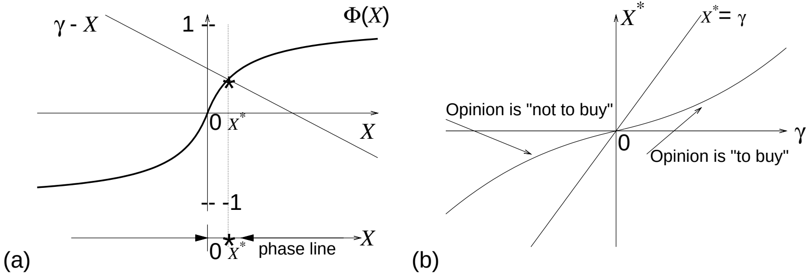

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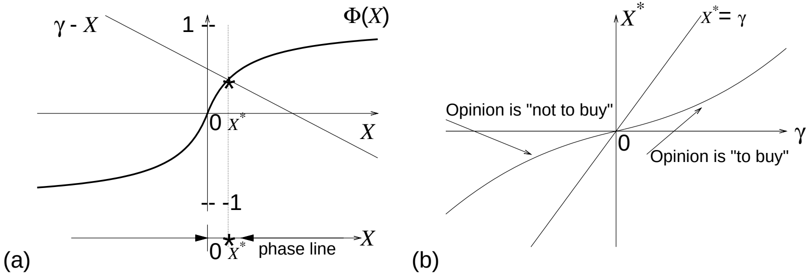

- Initial estimation (γ) is positive then the decision is “to buy”.
- Initial estimation is negative then the consumer decides “not to buy”.

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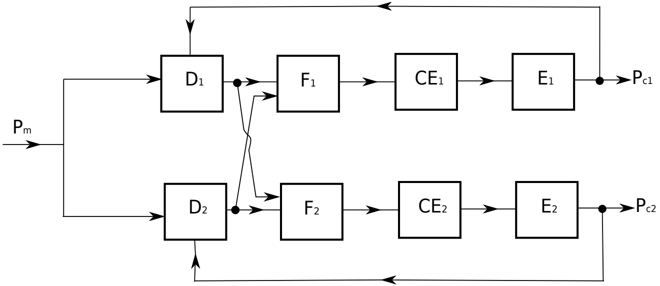
Model of two coupled consumers



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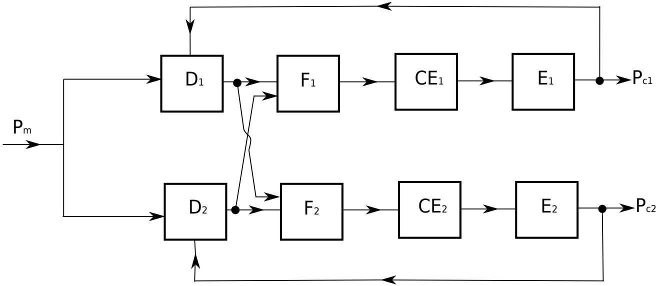

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\begin{aligned}
\frac{dX_1}{d\tau} + X_1 + \Phi(X_1) &= \gamma_1 + \kappa\Phi(X_2), \\
\frac{dX_2}{d\tau} + X_2 + \Phi(X_2) &= \gamma_2 + \delta\Phi(X_1).
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Real-Life Interpretation of Coefficients

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- If δ is positive (also κ), this can be interpreted as a cooperative type of coupling where the consumers are likely to do the same as their neighbours.
- Negative coupling coefficients represent an “antagonistic” type of interaction, where the neighbours disagree.

Networks of Consumers

For simplicity consider only the lattice topology with nearest-neighbour coupling.

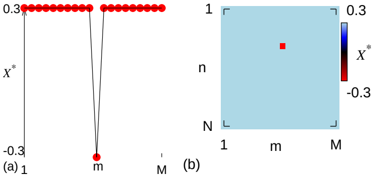
Chain of coupled MCs.

$$\frac{dX_n}{d\tau} + X_n + \Phi(X_n) = \gamma_n + \delta\Phi(X_{n-1}) + \kappa\Phi(X_{n+1}),$$

Two Dimensional Lattice.

$$\begin{aligned}\frac{dX_{n,m}}{d\tau} + X_{n,m} + \Phi(X_{n,m}) = \gamma_{n,m} \quad &+ \quad \delta\Phi(X_{n-1,m}) + \kappa\Phi(X_{n+1,m}) \\ &+ \quad \delta\Phi(X_{n,m-1}) + \kappa\Phi(X_{n,m+1}),\end{aligned}$$

Simple Initial Condition



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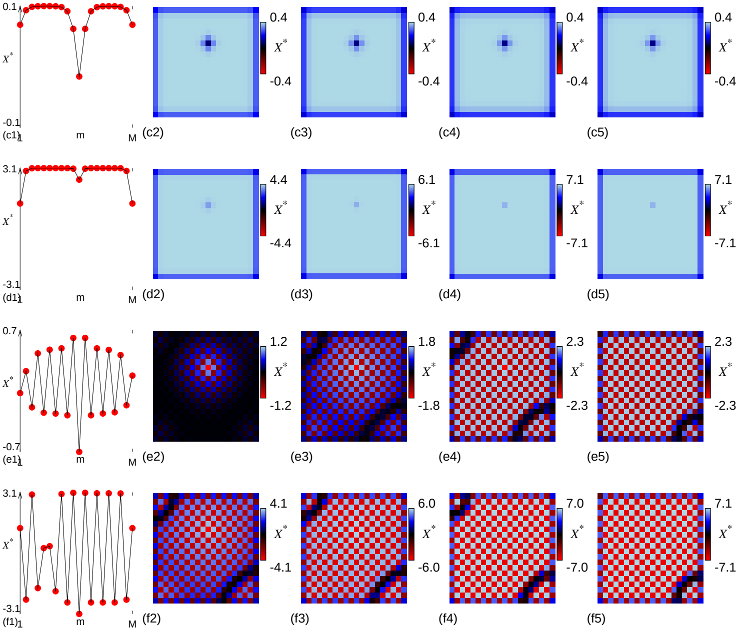

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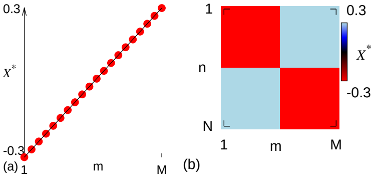
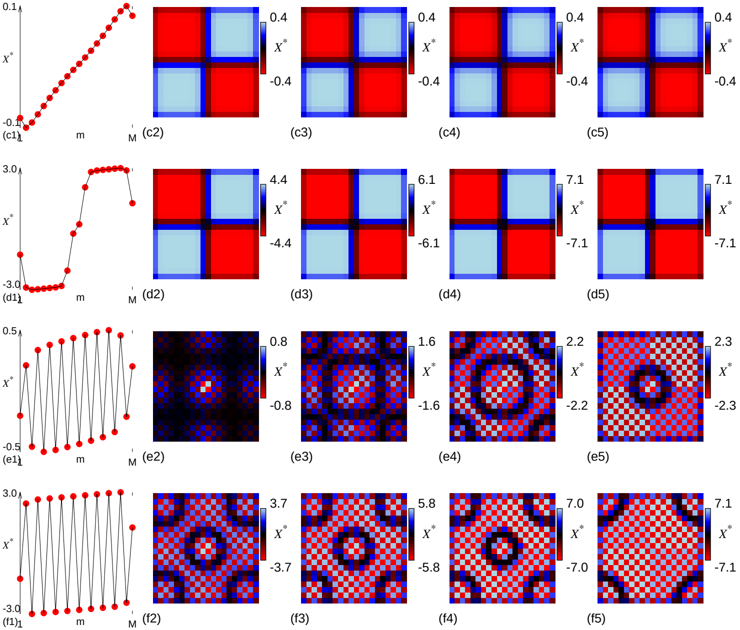
More Complicated IC




0.1
X^*
-0.1
(c1) 1 m M
(c2) 0.4 X^* -0.4
(c3) 0.4 X^* -0.4
(c4) 0.4 X^* -0.4
(c5) 0.4 X^* -0.4
3.0
X^*
-3.0
(d1) 1 m M
(d2) 4.4 X^* -4.4
(d3) 6.1 X^* -6.1
(d4) 7.1 X^* -7.1
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0.5
X^*
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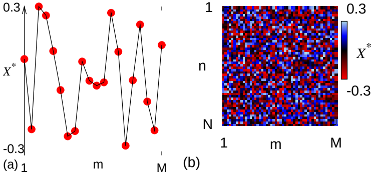
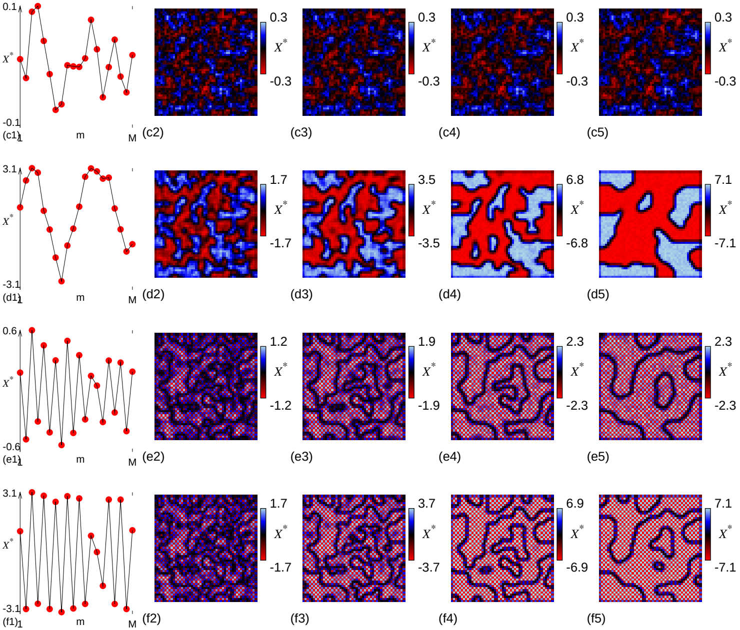
(e1) 1 m M
(e2) 0.8 X^* -0.8
(e3) 1.6 X^* -1.6
(e4) 2.2 X^* -2.2
(e5) 2.3 X^* -2.3
3.0
X^*
-3.0
(f1) 1 m M
(f2) 3.7 X^* -3.7
(f3) 5.8 X^* -5.8
(f4) 7.0 X^* -7.0
(f5) 7.1 X^* -7.1

Randomised Initial Configuration





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  - Antagonistic information exchange leads to the loss of the reasonable decision-making among consumers.

Where From Here?

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  2. Dynamical network, control-systems approach and ABS?
  3. Networks? Dynamical systems? Policy models?

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- Evaluate type of data required quantity to formulate (and verify) models. Build models while gathering data.