Corrections to "Decay of Hankel singular values of analytic control systems"

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For (an adapted version) of the proof of [1, Corollaries 5 and 6] to work, the additional assumption that the semigroup generated by A is similar to a contraction semigroup should be made. We note that for example a Riesz spectral operator with spectrum in the closed left half-plane satisfies this additional assumption.

As shown in [3], [1, Corollaries 5 and 6] as stated are in fact correct. However, the proof given in [3] is completely different (and in fact gives a stronger result than what is stated in [1, Corollaries 5 and 6]).

In the proof of [1, Corollary 5] it was claimed that with $C = I$ the observability Gramian is bounded with a bounded inverse. This is in general false. With the above additional assumption, a C can be constructed such that with this C the observability Gramian is bounded with a bounded inverse; the remainder of the proof is then as in [1].

Proof. {Correction to proof of [1, Corollary 5]} We assume that A generates a contraction semigroup and show that a C can be constructed such that the observability gramian of the pair (A, C) equals the identity. In the general case this argument can be used on the transformed A and then after transforming the constructed C back, the observability Gramian in the original coordinates can be seen to be S^*S where S is the similarity transformation. As desired, S^*S is bounded with a bounded inverse.

Since A generates an analytic semigroup, $-A$ is sectorial in the sense of [2, Section V.3.10]. Therefore, $-\langle Au, v\rangle$ with domain $D(A)$ is a closable form according to [2, Theorem VI.1.27]. Denote the closure of this form by $\mathfrak{t}$ and the real part of $\mathfrak{t}$ by $\mathfrak{h}$ (see [2, Section VI.6.1] for the definition of the real part of a form). Since A generates a contraction semigroup, A is dissipative and therefore $\mathfrak{h} \geq 0$. By the second representation theorem [2, Theorem VI.2.23], there exists an operator $C : D(\mathfrak{h}) \to \mathscr{X}$ such that $2\mathfrak{h}[u, v] = \langle Cu, Cv\rangle$ for $u, v \in D(\mathfrak{h})$. Since $D(A) \subset D(\mathfrak{t}) \subset D(\mathfrak{h})$, this equality gives

$$-\langle Au, v\rangle - \langle u, Av\rangle = \langle Cu, Cv\rangle, \qquad u, v \in D(A),$$

which shows that the identity is a solution of the observation Lyapunov equation of the pair (A, C). Since A generates an exponentially stable semigroup, this Lyapunov equation has a unique nonnegative self-adjoint solution (which equals the observability Gramian). We conclude that the identity is the observability Gramian of the pair (A, C).

Since the above constructed C is an infinite-time admissible observation operator, it follows from [4, Theorem 1.4] (see also the paragraph below that theorem) that $C \in \mathcal{L}(\mathscr{X}_{1/2+\varepsilon}, \mathscr{X})$ for all $\varepsilon > 0$. It follows that the quadruple $(A, B, C, 0)$ satisfies the assumptions of [1, Theorem 3]. Therefore, the proof can be finished as in [1, proof of Corollary 5]. □

References

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