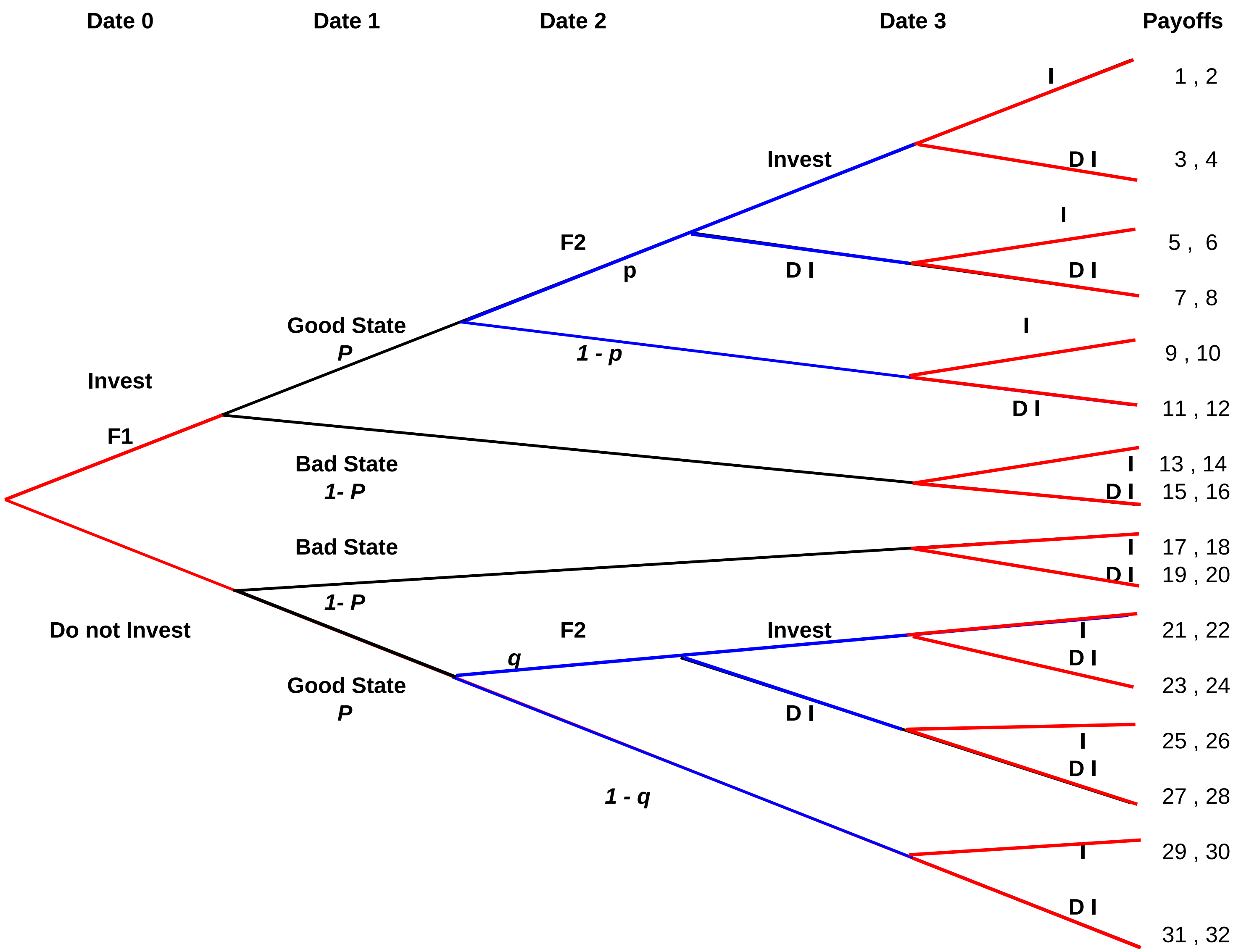
Extensive Form Game



Real Options: Payoffs

$-2I + R + R(1-\gamma), -F + R(1-\gamma)$	(1, 2)
$-I + R, -F + R$	(3, 4)
$-2I + 2R, 0$	(5, 6)
$-I + R, 0$	(7, 8)
$-2I + 2R, 0$	(9, 10)
$-I + R, 0$	(11, 12)
$-2I, 0$	(13, 14)
$-I, 0$	(15, 16)
$-I, 0$	(17, 18)
0,0	(19, 20)
$-I + R(1-\gamma), -F + R(1-\gamma)$	(21, 22)
$0, -F + R$	(23, 24)
$-I + R, 0$	(25, 26)
0,0	(27, 28)
$-I + R, 0$	(29, 30)
0,0	(31, 32)

We solve the game by backward induction. First, we take as given that firm 1 has invested at date 0, the good state of nature has occurred, firm 2 has appeared at date 2, and firm 2 has invested at date 3. Firm 1 makes its final decision (I, DI) by comparing payoffs (1) and (3). By assumption, $I > R(1-\gamma)$. Therefore (3) > (1). Therefore, firm 1 chooses DI.

Next, take as given that firm 1 has invested at date 0, the good state of nature has occurred, firm 2 has appeared at date 2, but firm 2 has chosen not to invest at date 3. Firm 1 makes its final decision (I, DI) by comparing payoffs (5) and (7). Since $R > I$, Firm 1 chooses I.

In summary, if firm 1 has invested at date 0, the good state of nature has occurred, and firm 2 has appeared at date 2, then, in the product market sub-game, if firm 2 invests, firm 1 does not, while if firm 2 does not invest, then firm 1 does.

Next, move back to consider firm 2's decision (I, DI), given that firm 1 has invested at date 0, the good state of nature has occurred, and firm 2 has appeared at date 2. Firm 2 correctly anticipates that, if it chooses I, then firm 1 will react by choosing DI, whereas if it chooses DI, firm 1 will react by choosing I. Therefore, firm 2 makes its decision by comparing (4) and (6). Since $R > F$, firm 2 will choose I at date 3 (to deter firm 1 from entering).

Next, take as given that firm 1 has invested at date 0, and that the bad state of nature has occurred. Therefore, firm 2 does not appear. Firm 1 simply compares (13) and (15). Therefore, firm 1 chooses not to invest.

Next, take as given that firm 1 has not invested at date 0, the good state of nature has occurred, firm 2 has appeared at date 2, and firm 2 has invested at date 3. Firm 1 makes its final decision (I, DI) by comparing payoffs (21) and (23). By assumption, $I > R(1-\gamma)$. Therefore (23) > (21). Therefore, firm 1 chooses DI.

Next, take as given that firm 1 has not invested at date 0, the good state of nature has occurred, firm 2 has appeared at date 2, but has chosen not to invest at date 3. Firm 1 makes its final decision (I, DI) by comparing payoffs (25) and (27). Since $R > I$, Firm 1 chooses I.

Moving back to firm 2's choice, firm 2 compares (24) and (26). since $R > F$, firm 2 chooses I.

Next, take as given that firm 1 has not invested at date 0, and the bad state of nature has occurred. Since firm 2 does not appear, firm 1 makes its second decision by simply comparing (17) and (19). Therefore, firm 1 does not invest.

Finally, we move back to firm 1's date 0 decision. Firm 1's expected payoff from investing is

$$\begin{aligned} &P[p(-I+r)+(1-p)(-2I+2R)]-(1-P)I \\ &= P(2-p)(R-I)-(1-p)I \end{aligned} \tag{1}$$

Firm 1's expected payoff from not investing (delaying) is

$$P(1-q)(R-I). \tag{2}$$

Firm 1 therefore invests early if (1) $\geq$(2), that is if

$$PR + P(q-p)(R-I) \geq I \tag{C1}$$

Otherwise, firm 1 delays investing.

In summary, the equilibrium is as follows.

a) If $PR + P(q - p)(R - I) \geq I$, firm 1 invests early (at date 0). If the good state of nature occurs, firm 2 appears with probability p, in which case firm 2 invests at date 2, and firm 1 does not re-invest. With probability $1 - p$, firm 2 does not appear, and firm 1 then re-invests. If the bad state occurs, firm 2 does not appear, and firm 1 does not re-invest at date 3.
b) If $PR + P(q - p)(R - I) < I$, firm 1 delays investing until date 1. If the good state of nature occurs, firm 2 appears with probability $q > p$, in which case firm 2 invests at date 2, and firm 1 does not re-invest. With probability $1 - q$, firm 2 does not appear, and firm 1 then re-invests. If the bad state occurs, firm 2 does not appear, and firm 1 does not re-invest at date 3.