Entrepreneur/Venture capitalist effort-shirking game.

Players: Risk-neutral VC and E: discount rate zero.

Timeline:

Date 1: Bargaining stage: VC makes a take-it-or-leave-it offer of equity to E; $\alpha \in [0,1]$.

Date 2: Both players exert effort: cost of effort is βe^2.

Success probability: $P = \gamma_E e_E + \gamma_{VC} e_{VC}$. If successful, venture provides income $R > 0$. If failure, venture provides zero income.

Therefore, value of the venture is $V = PR = (\gamma_E e_E + \gamma_{VC} e_{VC})R.$ (1)

We solve the game by backward induction:

Date 2 effort stage

We first take as given that the players have agreed the E's equity share $\alpha \in [0,1]$. and we solve for date 2 optimal effort levels.

We need to define the payoffs:

$$\Pi_E = \alpha V - \beta e_E^{\ 2} = \alpha(\gamma_E e_E + \gamma_{VC} e_{VC})R - \beta e_E^{\ 2}, \tag{2}$$

$$\Pi_{VC} = (1-\alpha)V - \beta e_{VC}^{\ 2} = (1-\alpha)(\gamma_E e_E + \gamma_{VC} e_{VC})R - \beta e_{VC}^{\ 2}. \tag{3}$$

In order to optimise, we solve $\dfrac{\partial \Pi_E}{\partial e_E} = 0,$ and $\dfrac{\partial \Pi_{VC}}{\partial e_{VC}} = 0.$

We thus obtain the optimal effort levels:

$$e_E{*} = \frac{\alpha \gamma_E R}{2\beta}, \quad e_{VC}{*} = \frac{(1-\alpha)\gamma_{VC} R}{2\beta}. \tag{4}$$

Substituting these optimal effort levels into (1), we obtain

$$V = \frac{\alpha \gamma_E^{\ 2} R^2 + (1-\alpha)\gamma_{VC}^{\ 2} R^2}{2\beta}. \tag{5}$$

Date 1: Equity offer stage.

Next, we substitute (4) and (5) into (2) and (3) to get payoffs in terms of α. We then solve the VC's optimal equity offer by solving $\frac{\partial \prod_{VC}}{\partial \alpha} = 0.$

We obtain

$$\alpha^* = \frac{\gamma_E^{\ 2} - \gamma_{VC}^{\ 2}}{2\gamma_E^{\ 2} - \gamma_{VC}^{\ 2}}. \tag{6}$$

Therefore, the VC's optimal equity offer depends on the E's and VC's relative abilities.

For example, if the VC provides "dumb-money" ($\gamma_{VC} = 0$) then $\alpha^* = \frac{1}{2}$.

RF