MN50324: April 2010.

Venture Capital/Entrepreneur exercise solutions:

Question 2:

First, note that the expected value of the firm is:

$$V = PR = (\gamma_E e_E + \gamma_{VC} e_{VC})R.$$

Solve by backward induction.

a) Optimal effort levels.

First, we need to define the players' payoffs:

$$\Pi_E = \alpha V - \beta e_E^{\ 2} = \alpha(\gamma_E e_E + \gamma_{VC} e_{VC})R - \beta e_E^{\ 2} \qquad (1)$$

$$\Pi_{VC} = (1-\alpha)V - \beta e_{VC}^{\ 2} = (1-\alpha)(\gamma_E e_E + \gamma_{VC} e_{VC})R - \beta e_{VC}^{\ 2} \qquad (2)$$

To find optimal effort levels, solve $\dfrac{\partial \Pi_E}{\partial e_E} = 0,$ and $\dfrac{\partial \Pi_{VC}}{\partial e_{VC}} = 0.$ We obtain:

$$e_E{}^* = \frac{\alpha \gamma_E R}{2\beta}, \text{ and } e_{VC}{}^* = \frac{(1-\alpha)\gamma_{VC} R}{2\beta}.$$

Note the effect of equity stakes and players' ability on the optimal effort levels. Also, note the conflict between offering more equity to one player at the expense of the other (ie if E has more equity, he works harder, but VC works less hard, and vice versa: note that Tykvova talks about this in her review paper).

b) To solve for the VC's optimal equity offer, we substitute the optimal effort levels into the payoff function (we only need to do this for the VC). First, it is useful to substitute the optimal effort levels into

$$V = PR = (\gamma_E e_E + \gamma_{VC} e_{VC})R.$$

We obtain

$$V = \frac{\alpha \gamma_E^{\ 2} R^2 + (1-\alpha)\gamma_{VC}^{\ 2} R^2}{2\beta}$$

Now substitute V and optimal effort levels into (2). We obtain

$$\Pi_{VC} = (1-\alpha)V - \beta e_{VC}{}^2$$
$$= \frac{(1-\alpha)\alpha\gamma_E{}^2 R^2 + (1-\alpha)^2\gamma_{VC}{}^2 R^2}{2\beta} - \frac{(1-\alpha)^2\gamma_{VC}{}^2 R^2}{4\beta}$$

=>

$$\Pi_{VC} = \frac{(1-\alpha)\alpha\gamma_E{}^2 R^2}{2\beta} + \frac{(1-\alpha)^2\gamma_{VC}{}^2 R^2}{4\beta}$$

=>

$$\Pi_{VC} = \frac{2(1-\alpha)\alpha\gamma_E{}^2 R^2 + (1-\alpha)^2\gamma_{VC}{}^2 R^2}{4\beta}$$

Finally, solve $\frac{\partial \Pi_{VC}}{\partial e_{VC}} = 0$. We obtain:

$$\alpha^* = \frac{\gamma_E{}^2 - \gamma_{VC}{}^2}{2\gamma_E{}^2 - \gamma_{VC}{}^2} \qquad (3)$$

e) When VC has no ability ($\gamma_{VC} = 0$) $\alpha^* = \frac{1}{2}$. (VC trades off giving equity incentives to the E against keeping some equity herself. (see diagram in the lecture slides).

f) As the VC's ability increases, the optimal offer falls.