Global trends in IPO methods: Book building versus auctions with endogenous entry

Ann E. Sherman*

Department of Finance, University of Notre Dame, Notre Dame, Indiana 46556, USA

Received 2 July 2004; received in revised form 23 August 2004; accepted 7 September 2004

Available online 18 July 2005

Abstract

The U.S. book-building method has become increasingly popular for initial public offerings (IPOs) worldwide over the last decade, whereas sealed-bid IPO auctions have been abandoned in nearly all of the many countries in which they have been tried. I model book building, discriminatory auctions, and uniform price auctions in an environment in which the number of investors and the accuracy of investors’ information are endogenous. Book building lets underwriters manage investor access to shares, allowing them to reduce risk for both issuers and investors and to control spending on information acquisition, thereby limiting either underpricing or aftermarket volatility. Because more control and less risk are beneficial to all issuers, the advantages of book building’s allocational flexibility could explain why global patterns of issuer choice are surprisingly consistent. My models also predict that offerings with higher expected underpricing have lower expected aftermarket volatility; that an auction open to large numbers of potential bidders is vulnerable to inaccurate pricing and to fluctuations in

* I would like to thank Ravi Jagannathan, Sheridan Titman, Jay Ritter, Tim Loughran, Paul Schultz, Robert Battalio, Bill Wilhelm, Alexander Ljungqvist, Don Hausch, Larry Benveniste, David Goldreich, Sudipto Dasgupta, Ed Prescott, V. V. Chari, Jack Kareken, Tom Gresik, and seminar participants at the U.S. Securities and Exchange Commission Office of Economic Analysis, the Federal Reserve Bank of Minneapolis, University of Minnesota, University of Notre Dame, and University of Wisconsin-Madison for useful comments and ideas. This work began when I was at Hong Kong University of Science and Technology, the support of which is gratefully acknowledged.

*Tel.: +1 574 631 3373; fax: +1 574 631 6416.

E-mail address: asherman@nd.edu.

0304-405X/$ - see front matter © 2005 Elsevier B.V. All rights reserved.
the number of bidders; and that both book-built and auctioned IPOs will exhibit partial adjustment to both private and public information.

© 2005 Elsevier B.V. All rights reserved.

**JEL classification:** G24; G28; G32

**Keywords:** Initial public offerings; Ipos; Auctions; Book building; Regulation; Entry; Uniform price; Discriminatory

1. **Introduction**

Book building is the primary initial public offering (IPO) method in the United States, but for decades it has generated controversy because it allows shares to be preferentially allocated. Investors complain that they are shut out of the allocation process, calling for changes that will give everyone a fair chance. After controversies about spinning, laddering, and other questionable IPO allocation practices, the New York Stock Exchange/National Association of Securities Dealers IPO Advisory Committee, headed by Geoffrey Bible, recommended in 2003 that possible regulatory impediments to IPO auctions be eliminated. Others have gone further, arguing that the use of IPO auctions should be required (Economist, 2002). In 2004, the well-known search engine company Google announced that it would go public through an auction, instead of the traditional U.S. route.

The ability to control access to IPO shares makes book building (the advance gathering of indications of interest) possible. Under auctions or fixed-price public offers, underwriters are free to do road shows and to ask for indications of interest. However, without the ability to make allocations dependent on the information reported, underwriters have no way to give investors the incentive to accurately report their information, as was first discussed in Benveniste and Spindt (1989) and Benveniste and Wilhelm (1990).

The ability to control allocations through book building includes the ability to pre-commit to specific allocation and pricing rules, such as those of a particular auction. One example of this is the U.S. investment bank W. R. Hambrecht, which began offering sealed-bid, dirty Dutch auctions in 1999 without any changes in legislation. The question for U.S. regulators, then, is not whether to allow

---

1 A dirty auction or “leaving something on the table,” is one in which the issuer is allowed to choose an offering price strictly below the market-clearing price. In other words, the price could be set below the maximum level that would allow all shares to be sold (which means that shares must be rationed). Dirty auctions have been used in Australia, Belgium, Finland, France, Hungary, New Zealand, the United Kingdom, and the United States.

In finance and on the Internet, uniform price (nondiscriminatory) auctions are sometimes mistakenly called Dutch auctions. A Dutch auction is an open (not sealed-bid) descending price auction. A Dutch auction for multiple units involves selling the same item for many different prices and is, in this sense, closer to a discriminatory than to a uniform price auction. Uniform price auctions are also sometimes mistakenly called Vickrey auctions. Ausubel and Cramton (2002) discuss the “uniform-price auction fallacy.”
sealed-bid auctions, but whether to require them. This paper points out two drawbacks to mandating rigid IPO share allocation rules.

I model both book building and two types of sealed-bid auctions for the same environment, one in which the number of investors and the accuracy of investors’ information are endogenous. The two auction types are discriminatory (pay what you bid) and uniform price, both of which have been used for IPOs in many countries. This paper contributes to the auction literature by modeling these multi-unit auctions in a setting that captures crucial aspects of new equity issues.

A key characteristic of IPOs is that the shares are difficult (and therefore costly) to evaluate. There are no past analyst reports to read and no market prices to observe. Corporate insiders have a clear, absolute advantage in terms of their knowledge of current assets and past performance, but valuation requires more than just this. Valuation also involves forecasting the future of the company, its competitors, and the industry as a whole and evaluating the quality of management and its strategic vision. Because the effort of evaluating an IPO is largely unobservable, there is a free rider problem (Sherman and Titman, 2002).

A second key aspect of IPOs is that the number of potential entrants to the auction is extremely large, relative to the number of bidders that the auction can profitably accommodate. For a U.S. IPO auction, the number of potential bidders easily exceeds 100 million. Each potential entrant has many investment alternatives and is not compelled to participate in, or to evaluate, any one particular offering.

Past IPO and auction research has tended to focus on an endowed information, fixed entry environment. For example, Biais and Faugeron-Crouzet (2002) show that the outcome of a dirty Dutch auction is equivalent to the Benveniste and Spindt (1989) book-building solution in this setting. For either book building or auctions in this environment, underpricing goes to zero as the number of informed investors increases. Thus, uniform price IPO auctions should result in zero underpricing and zero aftermarket volatility, if a sufficiently large number of exogenously informed investors is sure to bid.

However, if it takes time and effort for an outside investor to produce a superior independent estimate of the offering’s value, and if investors decide for themselves whether or not to participate in each IPO, then the no underpricing, no volatility auction result does not hold. Jagannathan and Sherman (2004) show that the empirical evidence on IPO auctions cannot be explained by endowed information, fixed entry models.

In the information production, endogenous entry environment of this paper, two key differences exist between book building and sealed-bid auctions. First, the issuer-underwriter has substantial control over information acquisition through book building, but little or no control in the auctions. This control can be used either to maximize expected proceeds from the current offering or to induce investors to more
carefully evaluate the issue, resulting in a more accurate aftermarket price. The disadvantage of the two auctions is not that they always lead to either too much or too little evaluation (or underpricing), it is that they seldom, except by chance, lead to the optimal level. Sealed-bid auctions are a one-size-fits-all approach, whereas controlling access to shares allows more customized solutions.

The second advantage of coordinating entry to the IPO process is that there is less uncertainty about the number of bidders. The expected number of shares sold is higher because undersubscription is less likely when the number of participants is coordinated. With book building, the underwriter recruits investors. It cannot force investors to like the issue, but it can promise them a reasonable allocation at a sufficiently low price to cover their time and effort, guaranteeing that a number of investors will at least consider the offering.

With sealed-bid auctions, the lack of investor coordination leads to increased risk for both issuers and investors, because both sides must make decisions without knowing how many bidders will participate. Ex post, there could be too few entrants and the offering could fail, or there could be too many entrants who bid away all of the potential profits, preventing investors from recovering their information costs (see Levin and Smith, 1994). This risk lowers the entry incentives of all investors, making them less willing to participate.

The two advantages of controlling entry help to explain why auctions have not been as useful for IPOs as for other securities, most notably government bonds. The first advantage, the ability to control information expenditures, is minor if the cost of evaluation is sufficiently small. Or, if information is preexisting, then controlling information production costs is not an issue at all. For the weekly auctions of 13-week U.S. Treasury bills, the cost of evaluation is negligible because so many close substitutes already are trading, including the off-the-run bills and the new bills trading in the when issued market.

The second advantage to controlling access, the ability to monitor and influence the number of bidders, is less important when the number of potential bidders is small and stable relative to the number that can profitably be accommodated. In U.S. Treasury auctions, 22 primary dealers participate regularly. Even so, the U.S. Treasury has a procedure (noncompetitive bids) for small orders ($1 million) to be filled separately in a way that does not disrupt the price-setting process.

The advantages of preferential IPO allocations apply to all issuers, regardless of issuer characteristics such as quality, risk level, or preference for price accuracy. They must be weighed against the disadvantages of book building, particularly the inherent conflicts of interest between issuers and underwriters (and between investors and underwriters). However, if the advantages identified here are sufficiently important, then the result will be a corner solution, with all issuers choosing book building. In other words, if the benefits of controlling access are sufficiently large, issuers will not voluntarily surrender that control.

While a formal test of this model would be difficult, because of the scarcity of IPO auction data and the fact that a corner solution is predicted, my results offer an explanation for the empirical regularity shown by Jagannathan and Sherman (2004). Jagannathan and Sherman examine the IPO methods used in 47 countries and find
that book building, which was rare outside North America in the early 1990s, is now common around the world.

Uniform price and discriminatory IPO auctions have been tried in many countries, but virtually all have abandoned them. IPO auctions were tried in Italy, the Netherlands, Portugal, Sweden, Switzerland, and the U.K. in the 1980s and in Argentina, Malaysia, Singapore, Taiwan, and Turkey in the 1990s, but they were abandoned years before book building became popular. IPO auctions were most robust in France, being used alongside both fixed-price public offers and a restricted form of book building for many years. Even in France, however, IPO auctions were abandoned once standard book building-public offer simultaneous hybrids were allowed.3

The abandonment of the IPO auction method has occurred across a variety of cultures and market conditions, across variations in the regulatory and procedural details, and regardless of whether the final decision was made by issuers or by regulators (often in response to complaints from investors). The consistency of this pattern indicates that allocation flexibility has significant advantages. This paper suggests two such advantages.

This is a positive, not a normative, analysis, comparing three existing methods in a particular environment instead of attempting to derive the optimal mechanism. I analyze the two standard IPO auction methods, even though auction theorists have long recognized problems with these particular methods.4 If the term “auction” is interpreted in a broad sense, it is almost a tautology that an appropriate auction could be designed for IPOs. However, the optimal IPO auction most likely would allow the auctioneer to favor reliable regular investors and those who provide information and would give the auctioneer discretion in setting the offer price after aggregating information from all bids. For example, the optimal auction in Spatt and Srivastava (1991) incorporates both pre-play communication and participation restrictions. I leave the design of the ideal auction to future work and concentrate here on demonstrating some of the benefits of preferential allocations.

The rest of the paper is organized as follows. Section 2 reviews some of the most relevant literature. Section 3 contains models of the book-building, discriminatory auction, and uniform price auction methods. Section 4 compares the methods and offers empirical implications regarding underpricing, aftermarket volatility, and partial adjustment of offering prices to publicly available information. Section 5 is the conclusion.

3Derrien (2005) describes the simultaneous hybrid book-building method introduced to France in 1999 as “unique,” although it had been in use for at least half a decade and was already common throughout Europe and Asia. Derrien and Womack (2003) found that, by requiring the offer price to be set too far in advance, France’s earlier sequential hybrid book-building method led to higher initial returns than either auctions or pure public offers.

4Ausubel (2002) argues that “both formats inevitably yield inefficient outcomes.” More generally, the Klemperer (2002) list of “notable fiascoes” shows that auction design matters and thus that it should not automatically be assumed that one form of auction will fit all situations.
2. Review of related literature

This paper builds on previous models of the book-building process by Benveniste and Spindt (1989), Benveniste and Wilhelm (1990), and Maksimovic and Pichler (2001), who argue that underpricing is needed to induce investors to report information once they have it, and Sherman (2000) and Sherman and Titman (2002), who argue that underpricing is needed to compensate investors for the cost of acquiring information. I extend past book-building models by allowing the information’s accuracy to be a continuous decision variable. In a similar environment, Sherman and Titman explore the effects of the one price rule and of various participation restrictions, characterizing when such restrictions lead to informed investors receiving economic rents. Yung (2005) models both investor evaluation and banker screening of IPOs, explaining why the size of an IPO investor pool could be restricted.

In the auction literature, relatively little work has been done on endogenous entry and information production in a common value setting. Notable exceptions include Hausch and Li (1993) and Harstad (1990), both of which consider only the single unit case. Levin and Smith (1994) and Bajari and Hortacsu (2003) model endogenous entry in a single-unit, endowed information setting. Matthews (1987) considers information production in single-unit auctions with risk-averse buyers. Habib and Ziegler (2003) show that posted-price selling of corporate debt could be superior to an auction, if there is a cost to evaluation.

Chemmanur and Liu (2003) analyze information acquisition in uniform price auctions and fixed-price public offers. Public offers allow issuers to control price but not allocations, book building allows issuers to control both, and standard auctions do not allow issuers to control either. Chemmanur and Liu demonstrate how fixed-price offers allow the issuer to induce a higher level of information acquisition but do not allow the offering price to reflect the information acquired.

This paper shows that controlling both price and allocations, through book building, brings two advantages. First, it allows the issuer to induce the optimal amount of information acquisition (whether that optimum is high or low). Second, it allows the issuer to control the number of participants, thus reducing risk for issuers and investors. Thus, this paper combined with Chemmanur and Liu can explain the patterns of issuer choice shown in Jagannathan and Sherman (2004): that issuers prefer fixed price to auctions and book building to fixed price, making auctions a distant third among IPO methods.

Ausubel (2002) suggests that IPOs should be sold through an ascending clock auction, which he shows to be efficient in an independent private values model with an exogenous number of bidders. Klemperer (2002) points out that a multi-unit uniform price auction “is analogous to an ascending auction (in which every winner pays the runner-up’s willingness-to-pay),” because the lowest winning bid “is typically not importantly different from the highest losing bid.” With IPO auctions, hundreds of bids tend to be at the market-clearing price. Ausubel recommends ascending auctions to reduce the winner’s curse, although reducing the winner’s curse increases the free rider problem if information must be produced.
Biais and Faugeron-Crouzet (2002) model IPO auctions, showing that a dirty Dutch auction can prevent tacit collusion among bidders and can truthfully elicit preexisting information from investors in much the same way as book building. Parlour and Rajan (2002) also identify similarities between dirty auctions and book building, showing that “leaving something on the table” can reduce the winner’s curse, thus eliciting more aggressive bids, under a variety of allocation rules, including some that allow the underwriter to discriminate between bidders. Bulow and Klemperer (2002) show that it could be optimal to select an auction price at which there is excess demand, as in a dirty auction. None of these papers models information production, however, and thus they do not examine the free rider problem.

Loughran et al. (1994) were the first to examine global patterns for IPOs, showing that underpricing of IPOs exists to some extent in virtually all countries and for all issue methods. They were also among the first to point out the importance of considering the offering method when evaluating IPOs. For a survey of U.S. IPO papers, see Ritter and Welch (2002). For a more extensive list of empirical papers on non-U.S. IPOs, see Jagannathan and Sherman (2004).

Chowdhry and Sherman (1996b) explain one global pattern: the tendency for large orders to be favored in book building and for small orders to be favored in fixed-price public offers. They show that, among risk-averse investors who are identical ex ante (same wealth, preferences, opportunities, etc.), those who become informed will optimally place larger orders than those who remain uninformed. Thus, favoring small over large orders reduces the winner’s curse of the uninformed, while favoring large over small orders discourages free riding. In a fixed-price offering, information collected during the subscription period arrives too late to be used in pricing the offering, so it is optimal to favor small orders to reduce the risk that the offering will fail. For book building, information that is revealed during the marketing stage can later be used to set the price.

However, even if the underwriter-issuer values the information that is revealed during the marketing stage, small orders should be favored if the marginal cost of evaluation is zero. Favoring small over large orders reduces the winner’s curse of the uninformed without driving the return to information acquisition below the marginal cost, if information is endowed. But if there is a cost to information production, and if underwriters value pricing accuracy (for whatever reason), then large orders should be favored over small orders, because favoring large orders increases the returns to information production.

Cornelli and Goldreich (2001) and Jenkinson and Jones (2004) use unique data sets to examine the orders and allocations for investors in book building IPOs (Cornelli and Goldreich, 2003, examine the determinants of offer prices). Both find that large orders are favored over small orders. Jenkinson and Jones (JJ) find that early bids receive favorable allocations. This is consistent with underwriters trying to discourage free riders, because those that place firm, early bids are clearly acting on their own opinions about the issuer. However, Cornelli and Goldreich (CG) do not find that early orders are favored in terms of allocation size. CG find that rationing varies between bidders and that about 30% of bidders do not get shares at all,
showing that underwriters are actively managing allocations (for better or for worse), instead of passively rationing everyone when demand is high. Both papers also find a core of frequent investors who tend to be favored in terms of allocations.

The strongest difference between the two studies is over whether price-limited bids are favored. This effect is strong in the CG sample, indicating that underwriters are rewarding bidders who provide information, but JJ find little evidence of this. One difference between the two samples is that the upper end of the price range seems to be binding for the JJ sample, making it difficult to distinguish uniformed bids from informed bids signaling a high valuation. An informed investor that valued the shares at, say, $26 but was faced with a price range of $20 to $22 would optimally place a strike price bid, if the upper end of the range is binding. Consistent with this, JJ (and CG) find far fewer limit bids for hot offerings.

JJ is also able to use a ranking of the quality of institutional investors to investigate how perceived investor quality, particularly the probability that an investor will hold the stock long term, affects allocations. They find that high-quality investors, those judged least likely to flip or stag the shares, are favored. The more effectively an underwriter weeds out excessive free riders whose only goal is to flip for a fast profit, the more easily it can encourage serious long-term investors to evaluate the stock.5

Other papers also provide indications that book building involves costly information production, a key claim of this paper. Ljungqvist and Wilhelm (2002) find evidence that underpricing is “directly related to information production” and that discretionary allocations promote information acquisition. Benveniste et al. (2003) find evidence that underwriters bundle offerings to spread out information production costs, thus preventing coordination failures. Ljungqvist et al. (2003) compare book building and public offer IPOs for many countries. They find that book building leads to lower underpricing when conducted by U.S. banks or targeted at U.S. investors.

More attention has recently been given to cross-country variations in the book-building method, an important area of research that complements the results of this paper: Book building has some strong advantages that make it worth further study. Jenkinson et al. (2005) explore the greater ability of underwriters to collect pricing information from investors before setting the initial pricing range in European book-building IPOs, offering an explanation for the fact that European underwriters seem to voluntarily commit to not price above the range.

Pichler and Stomper (2003) explore the roles of book building and when issued or grey market trading, showing that acquiring private information (through book building) could be necessary to reduce the Glosten and Milgrom (1985) problem. Glosten and Milgrom show that a market could fail to open if there are extreme

---

5This implies that underwriters will try to prevent unauthorized flipping for hot as well as cold issues. Flipping is sometimes seen as a problem only for cold issues that are receiving price support. However, if the underwriter is trying to discourage investors from free riding off the information production of others, it will also be concerned about flipping of hot issues. Ljungqvist et al. (2006) get a similar result in a model through irrationally exuberant sentiment investors. See Aggarwal (2000), Boehmer and Fishe (2001), and Fishe (2002) for more information on flipping in IPOs.
information asymmetries. Pichler and Stomper demonstrate that book building could reduce the information asymmetry regarding the value of an IPO, thus allowing the when issued market to open.

Derrien (2005) models IPO underpricing as being driven by suboptimal regulation that mandates aftermarket price support for all IPOs, in an environment in which price support would not occur voluntarily. The paper does not give information on which countries, if any, mandate aftermarket price support. The model is tested on French data, although French IPOs are not price-supported in the aftermarket, as evidenced by the first day initial returns reported in the paper.

3. The model

This model is based on the idea that investors might be able to produce information on the value of IPO shares that was not previously known by the issuer. This idea is controversial, given that corporate insiders clearly have more information, in an absolute sense. Yet outsiders can contribute to the valuation process, because valuation incorporates not just hard information on, say, physical assets but also soft information on management quality, strategy, and ability to outperform competitors, as well as forecasts of future developments in the company, industry, and economy.

An experienced investment professional is a less biased judge of managers’ abilities than the managers themselves. Plus, professional investors have better access to the company’s main competitors and spend their careers weighing the conflicting claims and forecasts of various companies. A common situation faced by analysts and fund managers is the following. Suppose 20 companies are in an industry dominated by five main companies. The fund manager or analyst visits four of the five main companies, and each of the four forecasts that it will have at least a 30% market share within the industry over the next year or two. Or, all four companies predict that total sales for the industry will grow by only 4% over the next year, but that their own sales will expand by 12–15%.

Clearly, the forecasts cannot all be correct. The four companies combined cannot capture a 120% market share, and the rest of the industry would have to experience huge drops in sales (drops not directly predicted by any of the companies) for all four of the five main companies to drastically outgrow the industry average. Nevertheless, each management team could believe its own forecasts. The fund manager-analyst regularly faces conflicting information from company insiders and has to decide which forecast is most likely. The consensus judgment of these experienced investment professionals helps to determine the market value of the stock.

Why would the issuer want this information to be acquired? Sherman (2000) shows that an issuer preference for price accuracy arises endogenously if uninformed investors are more risk-averse than informed investors. Chemmanur and Liu (2003) endogenize this preference through assuming that issuers will want to do follow-on offerings later (thus, good companies want an accurate aftermarket price), while
Busaba and Chang (2002) endogenize a concern for price accuracy through aftermarket trading.6

Other reasons that the issuer and underwriter might prefer a more accurate aftermarket price include better investment choices (Sherman, 1992), underwriter and issuer reputation, aftermarket liquidity, and lawsuit avoidance. Last, for agency reasons, managers could feel that their future job performance will be judged more fairly if the initial price is more accurate. Companies that see a large price drop during aftermarket trading (the result of an inaccurately high assessment during the IPO) could be more likely to become a takeover target or to face pressure to replace the current management team.

Whether or not one believes that issuers want this feedback, book building clearly is an information-gathering process (information flows from investors to issuers and underwriters, as well as vice versa) because the final price and quantity of IPO shares depend on responses from investors during the book-building process. As for auctions, potential bidders sometimes pay to collect information. For example, test drilling can be expensive but is often done before auctions of offshore oil tracts.

The environment is the same for all models. The issuer requires a fixed amount of capital and plans to sell a fixed number of shares, \( X \). If it raises more than is needed, any excess will be paid to the original shareholders. The issuer is risk-neutral and maximizes the expected proceeds minus \( f(P(\cdot, 0)) \), a term that reflects the possibility of price inaccuracy. In other words, the issuer prefers higher to lower expected proceeds but also prefers a more accurate valuation of the issue. \( P(\cdot, 0) \) is the probability that the true state is not discovered. I assume that \( f(P(\cdot, 0)) \) is sufficiently large, relative to the cost of the information \( [C(\alpha)] \), that the underwriter will always choose to induce investors to purchase at least some information.

There are three dates. The market price at date two, the initial trading date, reflects the information acquired during the IPO.7 The true state, high or low, is revealed and incorporated in market prices in period three. The market price at date three, given state \( j \), \( j \in \{ h, \ell \} \), is \( s' \). For simplicity and without loss of generality,

---

6The issuer might know all relevant information already but want it to be independently certified by investors, presumably because the issuer and underwriter cannot credibly convey the information directly. While that is a different approach from the one in this paper, the two are not mutually exclusive.

7I assume that, if informed investors trade in the aftermarket, their information would be revealed, thus giving the informed zero ability to take advantage of their information in the early aftermarket. If I assume the opposite extreme—that the informed can trade in the early aftermarket with zero effect on the price, so that there is no Glosten and Milgrom (1985) market maker that adjusts prices even slightly (but with some limit on the number of shares, so that they cannot make infinite profits)—then underpricing could have to be higher than it otherwise would be, to induce information revelation during the offering, and the optimal \( K \) could be somewhat larger, because the reasoning of Sherman and Titman (2002) holds even when the amount of information collection by each investor is endogenous. The underwriter will always select a \( K \) large enough that the information acquisition constraint binds (although the information reporting constraints must also be satisfied, and some could be satisfied with equality). The way to think of this is that, if the underwriter will have to increase underpricing to satisfy another constraint, then additional information acquisition could be induced at zero cost.
$s^h = 1$ and $s^f = 0$. State $h$ occurs with probability $\theta$. Thus, the expected value per share is $\theta s^h + (1 - \theta) s^f = \theta$.

There are $N$ investors with access to both capital and information (informed investors). They must purchase their information, paying $C(z)$ dollars for a signal that has probability $z$ of revealing the true state ($H$ if $h$ or $L$ if $\ell$). With probability $1 - z$, the investor receives a neutral signal ($M$). There is zero probability that the investor gets a false high or low signal ($H$ if $\ell$ or $L$ if $h$).$^8$ $C(z)$ is twice differentiable, strictly increasing and strictly convex for $z \in (0, 1)$, and $C(0) = 0$. Assume $N > X$, so informed investors can purchase the entire issue. Also, uninformed investors (investors unable to acquire information at a competitive price) are in plentiful supply.

All investors have an alternative investment with an expected net rate of return of $r$, which to simplify notation is set equal to 0. Investors that wish to participate in the IPO must pay a fixed entry cost $e > 0$, which can be thought of as a dead weight “bid preparation” cost or as a fixed cost of evaluation.

The unconditional probabilities of state $h$ occurring and of none of the $K$ informed investors receiving signals $H$ or $L$ are $P(h, 0) = \theta (1 - z)^K$ and $P(\ell, 0) = (1 - \theta)(1 - z)^K$. The unconditional probability that none of the $K$ investors receives informative signals is $P(\cdot, 0) = P(h, 0) + P(\ell, 0) = (1 - z)^K$. The conditional probability that 0 of the $K - 1$ other investors will receive an $H$ or $L$ signal is $P'(h, 0) = P'(\ell, 0) = P'(\cdot, 0) = (1 - z)^{K-1}$. The unconditional probability of state $h$ occurring and of $k$ of the $K$ informed investors receiving signal $H$ is

$$P(h, k) = \theta \binom{K}{k} z^k (1 - z)^{K-k}. \quad (1)$$

Similarly, $P(\text{entry} = i)$ is the probability that $i$ of $N$ potential auction bidders will choose to enter the auction (given that the probability of entry by each individual is $p$):

$$P(\text{entry} = i) = \binom{N}{i} p^i (1 - p)^{N - i}. \quad (2)$$

And the probability of undersubscription in the auction is

$$P(\text{undersubscription}) = \sum_{i=0}^{X} \binom{N - 1}{i} (p^i (1 - p)^{N - 1 - i}). \quad (3)$$

### 3.1. The book building solution

I assume that no conflicts of interest exist between underwriters and issuers; both prefer higher issue prices but also value price accuracy. Biais et al. (2002) examine the other extreme, assuming that the underwriter colludes with all informed investors.  

---

$^8$High or low signals could be interpreted as the detection of favorable or unfavorable information on the market value of a firm that was not discovered by the underwriter. I assume that information known by the firm or the underwriter or both has already been signaled to investors.
against the issuer, so that the issuer is effectively auctioning the shares to only one buyer. In practice, underwriters act in their own interests. But, as middlemen, their survival depends on offering acceptable terms to both issuers and investors.

The underwriter selects $K$ risk-neutral potentially informed investors to attend the road show and evaluate the offering. The underwriter chooses the allocation and price for each investor, based on both the signal reported by that investor and the signals reported by other investors (i.e., on $k$, the number of investors out of $K$ that report either $H$ or $L$). Each investor is limited to at most one share, for the sake of consistency with the auction models.

The underwriter is allowed to charge different prices to different investors in this model. Benveniste and Wilhelm (1990), Sherman and Titman (2002), and others analyze the effects of a one-price rule for book building. I do not incorporate such a restriction into this model because it complicates the formulas without changing the results. For an environment similar to this one, Sherman and Titman show that a one-price rule does not lead to investors collecting economic rents unless the issuer has an unusually strong desire for price accuracy (in which case auctions would not allow the issuer to collect a sufficient amount of information), or binding participation restrictions are placed on investors (which, if also applied to auction bidders, would limit the efficiency of auctions as well as book building).

Informed investors could report $H$, $L$, or $M$ (neutral), while the uninformed report only $U$. The following notation is for all $i \in \{H, L, M, U\}$, $j \in \{H, M\}$, and $t \in \{M, U\}$.

- $s_{i,j,k}$ = issue price to an investor who reports $i$ when $j$ is reported by $k$ of $K$ informed investors;
- $q_{i,j,k}$ = allocation to an investor who reports $i$ when $j$ is reported by $k$ of $K$ informed investors;
- $s_{t,M}$ = issue price to an investor who reports $t$ when all informed investors report $M$; and
- $q_{t,M}$ = allocation to an investor who reports $t$ when all informed investors report $M$.

Below is a summary of the timing for book-building offerings.

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Date 2</th>
<th>Date 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underwriter selects $K$, announces price and allocation schedule</td>
<td>Underwriter sets prices, allocates shares (based on announced schedule)</td>
<td>Shares start to trade; price reflects IPO shares information revealed</td>
</tr>
<tr>
<td>Announces price and allocation schedule</td>
<td>Informed investors select $z$, pay $C(z)$</td>
<td>True value</td>
</tr>
<tr>
<td>Informed investors report signal, pay $e$</td>
<td>Informed investors</td>
<td></td>
</tr>
</tbody>
</table>

### 3.1.1. Information reporting constraints

As part of its book-building strategy, the underwriter must design an allocation and pricing schedule that elicits accurate information from investors. Because the investment bank will use the reported information to price the issue, the pricing and
allocation strategy must counteract investor incentives to withhold favorable information that will lead to a higher issue price. I consider Nash equilibria in which, conditioned on the underwriter’s strategy, investors have an incentive to truthfully reveal their information, given their expectation that other investors will also report information accurately.

The underwriter’s problem has multiple solutions, with many sets of allocations and prices that elicit truthful revelation and give the issuer and investors the same expected utility. To eliminate these extraneous equilibria, I assume without loss of generality that all investors who report the same signal receive the same allocation. I also assume that the underwriter allocates zero shares to all investors who reveal conflicting signals. Although such conflicting reports will not exist within the equilibrium, this out-of-equilibrium assumption is needed to fully specify the equilibrium.

Let $R_B(j,i)$ be the expected profit to an informed investor who receives signal $j$ and reports signal $i$ (under the book-building system). In equilibrium, informed investors are induced to report their information truthfully, which implies that the following truth-telling constraints (described in more detail in Appendix A) must be satisfied.

$$R_B(j,j) \geq R_B(j,i) \text{ for all } j,i \in \{H, M, L\}. \quad (4)$$

3.1.2. Participation and information collection constraints

Constraints are also needed to guarantee that informed investors choose to participate and to acquire information. The binding constraint is that buying and reporting a signal offers a higher expected profit than not purchasing a signal and falsely reporting $M$.

$$\alpha \theta R_B(H, H) + \alpha(1 - \theta)R_B(L, L) + (1 - \alpha)R_B(M, M) - C(\alpha) - e \geq R_B(M, M). \quad (5)$$

This participation constraint (PC) for book building can be rewritten as

$$PC_B = \theta \sum_{k=1}^{K-1} P'(h, k)((s^h - s_{H,H,k+1})q_{H,H,k+1} - (s^h - s_{H,H,k})q_{M,H,k})$$

$$+ \theta P'(h, 0)[s^h - s_{H,H,1}]q_{H,H,1} - (s^m - s_{M,M})q_{M,M}]$$

$$+ (1 - \theta)P'(L, 0)\{[s^L - s_{L,L,1}]q_{L,L,1} - (s^m - s_{M,M})q_{M,M}]$$

$$+ (1 - \theta)\sum_{k=1}^{K-1} P'(L, k)[(s^L - s_{L,L,k+1})q_{L,L,k+1} - (s^L - s_{L,L,k})q_{M,L,k}]$$

$$- (1/\alpha)(C(\alpha) + e) \geq 0. \quad (6)$$

Buying a signal and reporting it must also be at least as good as saving the costs $e$ and $C(\alpha)$, and either falsely reporting $H$ or $L$, or not participating at all. If the truth-telling constraints are satisfied, the return to falsely reporting $H$ or $L$ will never be higher than the return to falsely reporting $M$. This solution is based on the interpretation of $e$ as a fixed information cost, instead of an entry fee. The issuer would have even more flexibility if $e$ is interpreted as an entry fee, allowing the issuer to use the entry fee to prevent free riders.
In addition, to guarantee that the choice of alpha is optimal for the informed investor, the derivative of the participation constraint with respect to \( \alpha \) must equal zero. In other words, the following first-order condition (FOC) must be satisfied with equality for \( 0 < \alpha < 1 \).

\[
\text{FOC}_B = \theta \sum_{k=1}^{K-1} P'(h, k)[(s^h - s_{H,H,k+1})q_{H,H,k+1} - (s^h - s_{M,H,k})q_{M,H,k}]
\]

\[
+ \theta P'(h, 0)[(s^h - s_{H,H,1})q_{H,H,1} - (s^m - s_{M,M})q_{M,M}]
\]

\[
+ (1 - \theta) P'(\ell, 0)[(s^f - s_{L,L,1})q_{L,L,1} - (s^m - s_{M,M})q_{M,M}]
\]

\[
+ (1 - \theta) \sum_{k=1}^{K-1} P'(\ell, k)[(s^f - s_{L,L,k+1})q_{L,L,k+1} - (s^f - s_{M,L,k})q_{M,L,k}]
\]

\[
- C'(\alpha) \geq 0.
\]  

The second-order condition is satisfied, given that \( C(\alpha) \) is convex.

Next, uninformed investors must have an incentive to participate. Uninformed investors face no participation costs, so they will be willing to purchase shares unless the shares are overpriced, which leads to the following set of constraints.

\[
s_{U,j,k} \leq s^d \quad \text{and} \quad s_{U,M} \leq s^m \quad \text{for all} \quad j \in \{H, L\}, k \in \{1, 2, \ldots, K\}.
\]  

Last, informed investors will not participate, once the state is revealed, if the shares are overpriced. This provides a final set of investor constraints.

\[
s_{i,j,k} \leq s^d \quad \text{and} \quad s_{M,M} \leq s^m \quad \text{for all} \quad j \in \{H, L\}, i \in \{H, M, L\}, k \in \{1, 2, \ldots, K\}.
\]  

3.1.3. The underwriter’s objective

The issuer-underwriter prefers higher to lower expected proceeds but also places some value on the accuracy of the issue price. The underwriter has a large number of choice variables in this model, some of which can be determined by substituting from the constraints already presented, simplifying the exposition. Proposition 1 describes basic features of the equilibrium solution.

**Proposition 1.** The book-building equilibrium will be such that

1. \( s_{U,j,k} = s^d \) and \( s_{U,M} = s^m \) for all \( j \in \{H, L\}, k \in \{1, 2, \ldots, K\} \);
2. \( s_{j,L,k} = s^d \) for all \( j \in \{L, M\}, k \in \{1, 2, \ldots, K\} \); and
3. \( s_{M,H,k} = s^h \) and \( s_{M,M} = s^m \) for all \( k \in \{1, 2, \ldots, K\} \).

In other words, the underwriter will not underprice shares to uninformed investors; when state \( L \) is revealed; or to informed investors who report a neutral signal (\( M \)). Shares will be underpriced only to informed investors who report signal \( H \).

**Proof.** See Appendix B.

Thus, the underwriter chooses \( K, \alpha \), and the prices and allocations for informed investors who report a good signal, to maximize a utility function that is a separable
function of the accuracy of the initial aftermarket price and the expected proceeds of
the issue. In particular, the underwriter chooses $K$, the number of investors invited to
participate in the offering, and $\alpha$, the accuracy of each informed investor’s signal, by
trading off the increase in accuracy associated with a larger number of more accurate
signals against a corresponding increase in the required underpricing to compensate
investors for their evaluation costs. The underwriter’s maximization problem is

$$\text{Max}_{K, \alpha, \theta, H, k, \text{for } k \in \{1, 2, \ldots, K\}} \sum_{k=1}^{N} \left( \theta - K \right) P(h, k) (s^h - s_{H, H, k}) q_{H, H, k} - f(P(\cdot, 0))$$

subject to Eqs. (4), (6)–(9), and the restrictions that prices and allocations cannot be
negative, that no investor is allowed to purchase more than one share, and that
exactly $X$ shares are sold.

3.2. The auction models

In this section, I determine the bidding strategies and entry probability in two
multi-unit, sealed-bid auctions: a uniform price and a discriminatory auction. I
assume that all valid bids are filled even if the number of bids is less than $X$, the
number of shares for sale. The lowest acceptable bid price is the lowest possible
value, the value of type $L$ shares. This can be thought of as the reservation price for
the auctions. The uniform price auction is an $X + 1$st auction, where all $X$ winning
bidders pay the $X + 1$st bid. If there are less than $X + 1$ bids, all bidders pay the
reservation price, the value of type $L$ shares.

I allow each bidder to bid for only one share. Ausubel and Cramton (2002) have
shown that, if multiple bids are allowed in a uniform price auction, bidders can shave
their bids on subsequent units, after the first, in the hopes of lowering the market-
clearing price. In IPO auctions, the large number of bidders makes this effect
negligible, because dozens or hundreds of bids likely are at the market clearing price.
Similarly, the likelihood of collusion could change when multiple bids are allowed
(Back and Zender, 1993), but this risk is also reduced with thousands of expected
bidders and millions of potential bidders.

I do not specify the issuer’s maximization problem because, although the issuer
has the same preferences as with book building, it has no choice variables. The rules
of these standard auctions do not allow the issuer to choose either price or
allocations. In practice, issuers in some countries could control some aspects of their
auctions. In its 2004 U.S. IPO auction, Google essentially used a secret reserve price,
reserved the right to price below market-clearing and to throw out excessively high
bids that it considered speculative (as in French IPO auctions), and required all
bidders to obtain a unique bidder identification number before bidding began, thus

---

10This is not necessarily the optimal reservation price. It is the lowest reservation price that the issuer
might choose, because all shares can always be sold at this value. I leave it to future research to examine
the optimal reservation price in multi-unit sealed-bid auctions with endogenous entry and information
acquisition.
giving Google an estimate of the maximum possible number of bidders while
blocking additional free riders from deciding to enter after bidding started.11

Each potential bidder chooses \( p \), the probability that he will enter the auction, and 
\( x \), the accuracy of the information purchased. There are \( N \) potential bidders for the \( X \) available shares, \( N > X \). The timing is as follows.

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Date 2</th>
<th>Date 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N ) potential bidders</td>
<td>Shares start to trade;</td>
<td>True</td>
</tr>
<tr>
<td>Choose ( p ), probability of entering</td>
<td>Price of</td>
<td>Value</td>
</tr>
<tr>
<td>( p )</td>
<td>( x ), accuracy</td>
<td>( X )</td>
</tr>
<tr>
<td>( e ), place bids</td>
<td>highest</td>
<td>Revealed</td>
</tr>
<tr>
<td>( C(x) )</td>
<td>information</td>
<td></td>
</tr>
</tbody>
</table>

Before the auction begins, bidders choose the symmetric equilibrium pair \( (p, x) \) (for sufficiently large \( N \)) to satisfy the following participation constraint.

\[
PC_A = x\theta A(b|H) + (1 - x)R_A(b|M) - C(x) - e \geq 0, \quad \text{for } A \in (D, U), \quad (11)
\]

where \( PC_A \) is the expected profit of a bidder who enters the auction with probability \( p \) and, on entering, acquires information of accuracy \( x \); and \( R_A(b|S) \) is the expected profit given that the bidder enters, purchases a signal, and observes \( S, S \in \{H, M\} \). This general equation is the equivalent of Eq. (6) for the book-building method.

For \( p < 1 \), the participation constraint must be satisfied with equality, so that potential bidders are indifferent, ex ante, about whether or not they enter. If \( N \) is small enough, the participation constraint could be strictly positive and \( p = 1 \). In other words, for a sufficiently small \( N \), the auction can profitably accommodate all bidders, leading potential bidders to enter with probability one. Because the goal of most IPO auctions is to open up bidding to everyone, I focus on the case where \( N \) is large enough that the optimal \( p \) is less than one.

The following first-order condition for \( x \) must also be satisfied.

\[
FOC_A = \theta A(b|H) - R_A(b|M) - C'(x) = 0, \quad \text{for } A \in (D, U), \quad (12)
\]

---

11As of January 2005, Google had not publicly announced whether or not its auction was dirty. The initial filing range was $108 to $135 per share, but the range was later revised to $85 to $95. The IPO was priced at $85, with winning bidders getting around 75% of their orders. The rationing rate indicates that either the auction’s clearing price was above the $85 offering price or a substantial cluster of bids was at $85, the bottom of the range. At least some rationing at the clearing price is virtually inevitable in IPO auctions. The stock closed at $100.34 on its first day of trading and $108.31 on its second day. The 18% initial return for the Google IPO auction was below the mean initial return of 30% for the ten U.S. IPO auctions that preceded it.
where $F_{CA}$ is the derivative of the bidder’s participation constraint with respect to that bidder’s choice of $a$ (the accuracy of the information purchased), given the choices of other bidders. The second-order condition is satisfied, given that $C(a)$ is convex.

The FOCs for the discriminatory ($D$) and uniform price ($U$) auctions can be rewritten

$$FOCD = \theta(1 - p\alpha)N^{-X} \frac{1 - \theta + \theta(1 - p)^{N-X}}{1 - \theta + \theta(1 - p)^{N-X}} - \theta(1 - p)^{N-X} - C'(a) = 0$$

and

$$FOCU = \theta \int_{m}^{\bar{m}} (1 - z)f^H(z)dz - C'(a) = 0. \tag{14}$$

Last is the constraint guaranteeing that it is not strictly profitable for uninformed to free ride by bidding in the same range as informed that receive an $M$ signal:

$$R_A(b|M) \leq e \quad \text{for } A \in (D, U). \tag{15}$$

The solution to the auction is offered in Proposition 2.

**Proposition 2.** In the first stage of the auction, potential bidders decide whether or not to enter, and they choose their information accuracy. They select $a$ and $p$ to uniquely solve $PC_A = 0$ and $F_{CA} = 0$.

In the second stage of the auction, the bidding, bidders who receive signal $L$ bid the full value of the shares, those with signal $M$ draw a bid from the cumulative bidding distribution $M_A(b; a, p)$, for $b \in (m_A, \bar{m}_A)$, and those who observe signal $H$ draw a bid from $H_A(b; a, p)$, for $b \in (h_A, \bar{h}_A)$.

**Proof.** See Appendix C.

The cumulative bidding distributions are given in Appendix C, along with the limits to the bidding ranges for each auction type. Bidders bid in distinct intervals in both auctions. Those who receive a low signal bid the expected value of the shares (zero), while those who receive a neutral signal randomize over a range from $m_A$ to $\bar{m}_A$. Those who receive a high signal randomize over the interval from $h_A$ to $\bar{h}_A$.

For the discriminatory auction, the upper limits of the optimal bidding ranges for investors who receive either an $M$ or $H$ signal are always strictly less than the expected values given those signals, and thus the auction leads to underpricing. For the uniform price auction, bidders who observe $H$ bid the full value, and thus there is zero underpricing of $H$ shares if at least $X + 1$ bidders observe a high signal.

The bidders’ ex ante expected profits are zero in both auctions. Although information expenditures are a sunk cost by the time bids are submitted, the entry probability chosen in stage 1 depends on information costs. Expected entry to the auction will be sufficiently low for the bidders to expect to recover their cost of evaluation. The seller ends up indirectly paying for the evaluations of the investors through a lower expected selling price.
4. Comparison of the offering methods

Both the book-building and the auction equilibria involve underpricing. Proposition 3 lays out the expected proceeds under each method.

**Proposition 3.** The book-building and auction solutions will result in the following expected proceeds or seller’s expected revenue, \( \text{SER}_B \) for book building and \( \text{SER}_A \) for either type of auction.\(^{12}\)

\[
\text{SER}_B = \theta X - K[C(x) + e]
\]

\[
\text{SER}_A = \theta X \left( 1 - \sum_{i=0}^{X-1} P(\text{entry} = i) \left( 1 - \frac{i}{X} \right) \right) - pN[C(x) + e].
\]

**Proof.** See Appendix D.

A comparison of Eqs. (16) and (17) demonstrates the similarities and differences of the methods. For both, the expected value of the shares for sale is \( \theta X \) (\( X \) shares worth, on average, \( \theta \) each). Both SERs are reduced by the last term, the expected information expenditures of \( C(x) + e \) per investor. The information costs vary through both \( x \) and the expected number of investors. For book building, the underwriter selects \( K \) investors. For the auctions, the expected number of bidders is \( N \), the number of potential investors, times \( p \), the entry probability of each bidder. The first term of \( \text{SER}_A \) is more complicated than the \( \theta X \) from \( \text{SER}_B \), because fewer than \( X \) shares could be sold.

In other words, for all of the methods the expected proceeds are equal to the expected value of the shares sold minus the expected information costs of the investors. In this sense, the book-building and auction solutions at first seem to be equivalent, but there are two key differences: (1) the greater ability to control information expenditures, and thus control expected proceeds or aftermarket volatility, with book building; and (2) the possibility of undersubscription in the auction, which strictly lowers expected proceeds and increases risk.

4.1. Example of differences in the methods

I illustrate differences in the methods through a simple numerical example. Consider two otherwise identical issuers who differ only in their relative preferences for greater accuracy versus a higher expected issue price. In other words, the two issuers have two different functions for \( f(P(\cdot,0)) \), the value of information. Readers skeptical of the idea that issuers might value anything other than the highest possible expected proceeds could choose to focus only on the solution for the issuer that puts little weight on price accuracy. Let \( \theta = 0.5 \), \( C(x) = x^4 \), \( e = 0.01 \), \( X = 5 \), and \( N = 15 \). The preferences for price accuracy are \( f(P(\cdot,0)) = 0.02 (P(\cdot,0))^{0.5} \) for Issuer 1, who

---

\(^{12}\)I will sometimes use the term “seller’s expected revenue” or SER because it is standard in the auction literature, although the funds received from a stock issue are proceeds, not revenue.
has only a slight preference for more information, and \( f(P(\cdot, 0)) = 2.0 (P(\cdot, 0))^{0.5} \) for Issuer 2, who has a stronger preference for price accuracy in the aftermarket.

The information level in Table 1 is the probability that the firm’s value is revealed during the IPO, meaning that the initial (Date 2) trading price is accurate. My model assumes that the value of the company is always revealed eventually (by Date 3) but does not assume that it is costlessly revealed the instant the shares start trading. The Date 2 trading price reflects information produced during the IPO. If more information is purchased during the offering, then the trading price at Date 2 is more likely to accurately reflect the firm’s value.

Table 1 shows that book building can offer a better combination of underpricing and information for each issuer. The main point of Table 1 is that the auction solutions are the same for both issuers, regardless of each issuer’s preferences, because the issuer-underwriter cannot select the outcome. The auctions are efficient in the sense that investors do not receive excess returns. They could, in some cases, give an issuer a near-optimal solution (ignoring risk), but this occurs only by chance. Under book building, the underwriter is able to offer less underpricing for Issuer 1 and greater pricing accuracy for Issuer 2, while selecting participants in a way that minimizes the costs of achieving a certain level of accuracy.

The second key difference between the methods is that auctions are riskier. With book building, there is no risk that the issue will fail as a result of a lack of scrutiny, because the underwriter will recruit a sufficient number of investors who each have an incentive to seriously consider the offering. Book building can be seen in part as a coordination mechanism to ensure entry of the optimal number of participants. With auctions, the expected number of bidders could be optimal, but ex post a chance still exists that too few bidders will show up and the auction will be undersubscribed. This possibility of undersubscription lowers the issuer’s expected proceeds, holding information expenditures constant, as shown in Proposition 4.

**Proposition 4.** Holding expected evaluation costs constant, the seller’s expected proceeds are strictly higher for book building than for either auction.
Proof. See Appendix E.

The added uncertainty of auctions leads to lower expected utility even for a risk-neutral issuer. If issuers or investors are risk-averse, auctions are at an even greater disadvantage. The risk to investors in an auction comes from the fact that they must decide whether or not to enter and how much information to purchase without knowing whether or not they will receive shares. Each potential bidder thus factors in the probability that, ex post, either too few bidders will enter and the auction will fail, or too many bidders will enter and bid away all potential profits (see Levin and Smith, 1994). In either of these cases, the bidder could end up paying to evaluate the issue, only to receive nothing.

Many countries require all IPOs to be underwritten (meaning that the underwriter guarantees the proceeds), regardless of the issue method: book building, auction, or public offer. For auctions, this usually means guaranteeing the purchase of all shares at the minimum or reservation price, because the guarantee must be at a prespecified price. In this model, the reservation price is the value of type $L$ shares (which, for simplicity, is zero). If the issuer wanted ex ante insurance from the underwriter at a price above the value of type $L$ shares, the fee would have to cover the expected cost of having to buy type $L$ shares at above their value. Being able to insure against losses stemming from undersubscription does not eliminate the cost to the issuer, it only evens out the distribution of that cost. Such insurance might nevertheless be valuable if issuers face large dead-weight costs from a failed offering.

4.2. Opening up an auction to more potential bidders

My example involves a relatively small number of potential bidders, raising the question of whether the problems with IPO auctions, particularly the risks, disappear when the process is opened up to a sufficiently large number of potential investors. In endowed information, fixed entry models, more informed investors mean more competition and a more accurate offering price. If bidders decide for themselves whether or not to enter, however, an increase in the number of potential bidders adversely affects the entry probability of each bidder, meaning that the expected number could increase only slightly or even decrease. In addition, the variance in the number of bidders could increase, adding risk for both investors and issuers. These effects are summarized in Proposition 5.

Proposition 5. In the uniform price and discriminatory auctions, when $N$ (the number of potential entrants) increases, either the variance in the number of entrants increases or the expected number of entrants decreases, or both.

Proof. See Appendix E.

To understand the result that opening up an auction to a large number of potential bidders could increase, instead of decrease, risk, it helps to consider how small the entry probability must be in practice. For Taiwan’s discriminatory IPO auctions, the average number of bidders was around 1,150 (Liu et al., 2001). More than 16 million adults were eligible to bid in each auction, giving an average participation rate of
roughly 72 per million. If the participation rate of the eligible population shifted by 7/1,000 of 1% in either direction, bids would either almost double or almost vanish. (In addition, large average numbers of bidders will not eliminate the risk of undersubscription if there is some coordination, or leakage of information, as discussed in Chowdhry and Sherman, 1996a.)

In the numerical example from Section 4.1, with five shares being auctioned, if the number of potential bidders is doubled (from 15 to 30), the standard deviation in the number of bidders increases from 1.9 to 2.6 for both auctions, while the mean increases from 8.9 to 9.8 bidders for the uniform price auction and from 9.1 to 10.0 for the discriminatory auction. The $N$ that maximizes SER in this example is 8 for the uniform price auction and 20 for the discriminatory auction. For both auction types, as $N$ increases, underpricing decreases up to a point and then increases, while $p$ consistently decreases.

Table 2 shows that doubling the number of potential bidders in my example increases both underpricing and the probability of undersubscription for both types of auctions. Total information production increases for the uniform price auction but decreases for the discriminatory auction. One must be cautious in interpreting examples. However, this example shows that opening an auction up to more potential bidders does not automatically reduce either underpricing or the risk of undersubscription, as one might suspect. More potential entrants leads to increased risk for potential bidders, which affects their endogenous entry probability.

These results can also explain the frequent use of private auctions, in which entry is restricted to “invitation only.” Restricting access to an auction could increase the number of bidders because people who receive an invitation will give the auction more serious consideration (i.e., choose a higher entry probability, $p$), because an entry limit implies that competition will not be excessive, and thus that entrants will have a reasonable chance of being satisfied with the auction results.

Last, I do not analyze the possibility that some uninformed investors could bid a very high price, as opposed to bidding in the neutral range, in the uniform price auction. This possibility is left for future research, given that it is a subject in itself. Even in this model in which all entrants collect the same amount of information, however, uniform price auction bidders are partial free riders, because the price paid by each winning bidder depends on the information (if any) in the bids of the other $X$.

Table 2
The effect of increasing the number of potential bidders in previous example

<table>
<thead>
<tr>
<th>Potential bidders</th>
<th>Under pricing</th>
<th>Information level</th>
<th>Probability of undersubscription</th>
<th>Entry probability $p$</th>
<th>Accuracy level $\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform price auction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N = 15$</td>
<td>3.8%</td>
<td>10.3%</td>
<td>3.8%</td>
<td>0.593</td>
<td>0.012</td>
</tr>
<tr>
<td>$N = 30$</td>
<td>4.3</td>
<td>11.6</td>
<td>4.3</td>
<td>0.325</td>
<td>0.013</td>
</tr>
<tr>
<td>Discriminatory auction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N = 15$</td>
<td>5.6</td>
<td>93.9</td>
<td>0.9</td>
<td>0.604</td>
<td>0.265</td>
</tr>
<tr>
<td>$N = 30$</td>
<td>5.7</td>
<td>93.6</td>
<td>1.2</td>
<td>0.333</td>
<td>0.240</td>
</tr>
</tbody>
</table>
highest bidders. For discriminatory auction bidders, each bidder’s price depends only on that bidder’s own information. In all of the examples that I have calculated (see Table 2, for instance), the optimal information production per investor, \( a \), was lower for the uniform price auction than for the discriminatory auction, consistent with the idea that uniform price bidders are partial free riders.

4.3. Empirical implications regarding underpricing and aftermarket trading

This model yields several empirical implications. First, because book building offers lower risk for both issuers and investors, it should lead to less underpricing (holding information costs constant), as shown in Proposition 4. The long-term relationships modeled in Sherman (2000) also imply that book building will lead to less underpricing, relative to auctions. But book building’s flexibility in controlling information expenditures could lead to either more or less underpricing, depending on the preferences of the issuer.

One policy implication is that issue methods should not be judged based solely on underpricing levels. A major advantage of controlling allocations is that underpricing can be tailored to the preferences of issuers and can adapt to the circumstances of various countries or time periods. If issuers use their control to induce higher levels of information collection, and thus higher underpricing, this by itself is not evidence that issuers are worse off.

Another implication of this paper is that lower average underpricing should be correlated with greater aftermarket volatility. My model assumes that the true value of the firm is revealed eventually but that it will not be revealed during initial trading unless the information was produced during the IPO. More information production leads to more underpricing, a higher probability that the firm’s true value is reflected in the initial trading price, and a lower probability of aftermarket price shifts (because the initial price was more accurate). Proposition 6 thus compares expected underpricing to aftermarket volatility.

**Proposition 6.** For each of the three offering methods, offerings with lower expected underpricing have higher expected variance in returns from Date 2 (the first date of trading) to Date 3 (the date that the true value is revealed).

**Proof.** Follows directly from Proposition 3.

This model, combined with previous work, offers several reasons that book building might lead to lower underpricing but only one reason (greater information acquisition) that the control of allocations might lead to greater underpricing (and less aftermarket volatility). This yields a testable empirical implication, presented in Proposition 7.

**Proposition 7.** If book building leads to greater expected underpricing relative to uniform price or discriminatory auctions, then it should also lead to less volatility in aftermarket trading from Date 2 (the first date of trading) to Date 3 (the date that the true value is later revealed).
Proof. Follows directly from Proposition 3.

Returning once more to the numerical example, the book-building solution for Issuer 2 offers an expected initial return of 7.5%, which is greater than the expected underpricing for either auction (see Table 1). The probability that the true value is revealed during the offering process is 97.9% for book building, meaning that there is a 2.1% chance that the Date 2 price will be inaccurate and thus will change by Date 3. The probabilities of a change in price from Dates 2 to 3 are 6.1% for the discriminatory auction and 89.7% for the uniform price auction.

This model also predicts partial adjustment of IPO prices to both private and public information, for both book building and auctions. Partial adjustment to private information – the fact that underpricing tends to be greater for offerings that are priced at a higher than expected level, based on demand from investors—was first explained for book building IPOs by Benveniste and Spindt (1989) and was first established empirically by Hanley (1993).

This is, to my knowledge, the first model that predicts the same partial adjustment to private information for sealed bid IPO auctions. For both the uniform price and discriminatory auctions in my model, a higher market clearing or weighted average winning bid price from the auction means that the value of the firm is more likely to be high and thus that the IPO will lead to a positive initial return. Similarly, a lower than expected market clearing or weighted average winning bid price implies a higher probability that the offering has been overpriced and will fall on the aftermarket.

Partial adjustment to public information is evidenced by a positive correlation between initial returns and publicly available information such as market returns before the IPO.13 Both Loughran and Ritter (2002) and Edelen and Kadlec (2005) offer explanations for this, and both correctly point out that book-building models with endowed information cannot explain it. However, models of information production predict a positive correlation between underpricing and recent market returns. Underpricing in these models compensates investors for their evaluation costs, the biggest of which is the opportunity cost of devoting time to one particular IPO instead of to the many stocks already trading, or to other IPOs.

This opportunity cost explanation is consistent with underwriter claims that it is hard to convince investors to attend road shows during hot markets but easy to fill up road shows during slow times, when few other IPOs or SEOs (seasoned equity offerings) are being priced. This leads to Proposition 8, that underpricing will be correlated to publicly observable variables, for any of the issue methods.

**Proposition 8.** If the cost of evaluation, \( C(x) \), is in part an opportunity cost that is correlated to publicly observable variables, then book building, uniform price auctions, and discriminatory auctions will all exhibit partial adjustment to both private and public information.

---

13Lowry and Schwert (2004) find that this relationship is statistically significant but argue that it is too small, after adjusting for other factors, to be economically significant.
Proof. Follows directly from Proposition 3.

Thus, to test for the connection between underpricing and aftermarket volatility predicted in Proposition 6, one must first adjust for variations in both the need for information and the cost of that information. The best proxies for the cost of information would be recent market conditions such as stock price returns and the number of IPOs and SEOs pricing at about the same time.

In addition to the potential for high aftermarket volatility, this model predicts that sealed-bid auctions could lead to wide variations in the number of bidders per auction. Amihud et al. (2003) and Kandel et al. (1999) report large variations in the number of bidders for Israeli IPO auctions. Jagannathan and Sherman (2004) give some examples of fluctuations in bidder levels in various countries. Bajari and Hortacsu (2003) find “significant variation in the number of bidders” in eBay coin auctions, a common value (but single-unit) setting with large numbers of potential entrants.

5. Conclusion

This paper contributes to the auction literature by modeling the two types of auctions most used for IPOs (discriminatory and uniform price auctions) in a setting that captures crucial features of new equity issues. I model book building and the two sealed-bid auctions for the same environment, endogenizing the number of investors and the accuracy of their information. The ability to control allocations (via book building) yields two advantages: less risk for both issuers and investors, leading to less underpricing even under risk neutrality; and more control over information expenditures, which means more control over either underpricing or aftermarket volatility. These advantages should appeal to all issuers, offering an explanation for the global trends in IPO methods over the last decade.

The overall conclusion is that book building is less risky than a sealed-bid auction because someone is managing the process, making sure that at least a certain minimum number of investors carefully evaluate the offering. If too many investors want to participate, the underwriter can guarantee that serious regular investors receive enough shares so that their time evaluating the offering is not wasted. Standard sealed-bid auctions cannot ensure that a reasonable number of investors will ever take the time to seriously evaluate an IPO.

Surprisingly, increasing the number of potential bidders in an auction could make the issuer worse off, in terms of both underpricing and the risk of failure. More potential bidders leads to more risk and to potentially lower returns for each bidder, thus discouraging both entry and evaluation. Auctions open to large numbers of potential investors could lead to inaccurate pricing, high aftermarket volatility, and wide variation in the number of bidders. Jagannathan and Sherman (2004) offer anecdotal evidence that all of these problems have occurred with IPO auctions in practice.

These results are relevant for many multi-unit online auctions, as long as entry is endogenous; the number of potential bidders is large, relative to the number of
participants who can comfortably be accommodated; and the objects have a common value, but substantial uncertainty regarding that value exists. Examples might include limited editions of art or collectibles or the new release (most recent vintage) of a fine wine. This model could explain why some auctions are “by invitation only,” because limiting potential entrants can reduce risk, encourage evaluation, and perhaps even increase the average number of bidders.

My models predict that, for a country that switches from IPO auctions to book building, any increase in average underpricing should be accompanied by a decrease in aftermarket volatility. The new book-building regime should lead to a wider range of intentional underpricing levels and to a wider range of companies going public. My models do not support the popular belief that auctions should always lead to lower expected underpricing, and they predict that issuers will prefer book building to auctions regardless of the direction of the change in underpricing levels.

Last, my model predicts a correlation between underpricing and publicly available information for both book-built IPOs and sealed-bid IPO auctions. Partial adjustment to public information occurs because expected underpricing compensates investors for their time spent evaluating an offering, and the opportunity cost of that time depends on the returns to other current IPOs and SEOs as well as to traded stocks.

The advantages of allowing underwriters to control share allocations must be weighed against disadvantages such as the inherent conflict of interest between issuers and underwriters, possible cost differences, and the actual or perceived fairness of discriminatory allocations. The theoretical justification of book building relies heavily on competition among underwriters, yet Chen and Ritter (2000) offer evidence that underwriting fees in the United States do not respond to competition. This by itself is inconclusive, because underwriters might be competing on other dimensions, but it raises questions that should be addressed.

Similarly, evidence is available that underwriters allocate shares in hot issues to regular investors based on their general relationship, not based exclusively on the current issue, or even based exclusively on repeated IPO participation. This also is not, by itself, a problem for issuers because any side benefits that an underwriter expects to receive from handling an IPO should be factored into the general package of fees and services that it offers to an issuer, if underwriters compete for investment banking business. If underwriters can win investment banking business by spinning (allocating hot IPO shares to the personal accounts of executives at potential client firms), then the book-building system is seriously flawed.

This paper analyses only two of the three main global IPO methods: auctions, which do not allow the underwriter to control either price or allocations, and book building, which allows the underwriter to control both. The third method, fixed-price public offer, allows the underwriter to control the price but not allocations. By controlling only the price through a public offer, the underwriter can still increase information production (Chemmanur and Liu, 2003) and ensure against failure of the offering (Chowdhry and Sherman, 1996a). Jagannathan and Sherman (2004)
show that the fixed-price method has been consistently chosen over IPO auctions in nearly all countries that allow both methods.

Thus, if the United States or other countries decide that the problems with underwriter control of allocations outweigh the advantages of risk reduction, etc., they should consider fixed-price public offers as a way to partially disintermediate the process without surrendering all control. Another alternative would be for underwriters to make the book-building allocation process more transparent by announcing the general criteria for evaluating various orders, as proposed in Jagannathan and Sherman (2005).

A frequently stated reason for reforming the U.S. IPO process is to allow everyone to participate. Two main motivations have been cited for wanting to open up the IPO process to all investors: to price IPOs more efficiently, by first getting feedback from a larger number of potential investors; and to have a “fair” system in which all investors have a chance at participating. My research indicates that the first goal, more accurate pricing, could be better achieved by controlling access. If estimating the value of a company requires effort, then allowing too many investors to participate could decrease the accuracy of the pricing process.

The second goal, giving small investors a chance to participate in the process, is a legitimate one for policy makers. Nevertheless, broadening access does not automatically mean allowing small investors to set the offering price. Just as the U.S. Treasury uses noncompetitive bids to allow small investors to participate in Treasury auctions without affecting the price-setting process, IPOs could be sold in a way that gives all investors a chance to purchase shares but still allows issuers to obtain the benefits discussed in this paper.

Appendix A. Information-reporting or truth-telling constraints

The full set of six truth-telling or information-reporting constraints are given, expanding Eq. (4).

1. $R_B(H,H) \geq R_B(H,M)$

$$
\sum_{K=1}^{K-1} P'(h,k) \left[ (s^h - s_{H,H,k+1}) q_{H,H,k+1} - (s^h - s_{M,H,k}) q_{M,H,k} \right] 
+ P'(h,0) \left[ (s^h - s_{H,H,1}) q_{H,H,1} - (s^h - s_{M,M}) q_{M,M} \right] \geq 0. \tag{18}
$$

2. $R_B(M,M) \geq R_B(M,L)$

$$
(1 - \theta) \sum_{K=1}^{K-1} P'(l,k) \left[ (s^l - s_{M,L,k}) q_{M,L,k} - (s^l - s_{L,L,k+1}) q_{L,L,k+1} \right] 
+ \theta \sum_{K=1}^{K-1} P'(h,k) \left[ (s^h - s_{M,H,k}) q_{M,H,k} - 0 \right] 
+ P'(h,0) \left\{ (s^m - s_{M,M}) q_{M,M} - (s^m - s_{L,L,1}) q_{L,L,1} \right\} \geq 0. \tag{19}
$$
3. $R_B(H,H) \geq R_B(H,L)$

\[
\sum_{k=1}^{K-1} P'(h,k) [(s^h - s_{H,H,k})q_{H,H,k} - 0] + P'(h,0) \{ (s^h - s_{H,H,0})q_{H,H,0} - (s^h - s_{L,L,1})q_{L,L,1} \} \geq 0. \tag{20}
\]

4. $R_B(M,M) \geq R_B(M,H)$

\[
\theta \sum_{k=1}^{K-1} P'(h,k) [(s^h - s_{M,H,k})q_{M,H,k} - (s^h - s_{H,H,k+1})q_{H,H,k+1}] + (1 - \theta) \sum_{k=1}^{K-1} P'(\ell,k) [(s^\ell - s_{M,L,k})q_{M,L,k} - 0] + P'(\ell,0) \{ (s^\ell - s_{M,M})q_{M,M} - (s^\ell - s_{H,H,1})q_{H,H,1} \} \geq 0. \tag{21}
\]

5. $R_B(L,L) \geq R_B(L,M)$

\[
\sum_{k=1}^{K-1} P'(\ell,k) [(s^\ell - s_{L,L,k+1})q_{L,L,k+1} - (s^\ell - s_{M,L,k})q_{M,L,k}] + P'(\ell,0) \{ (s^\ell - s_{L,L,1})q_{L,L,1} - (s^\ell - s_{M,M})q_{M,M} \} \geq 0. \tag{22}
\]

6. $R_B(L,L) \geq R_B(L,H)$

\[
\sum_{k=1}^{K-1} P'(h,k) [(s^\ell - s_{L,L,k})q_{L,L,k} - 0] + P'(h,0) \{ (s^\ell - s_{L,L,1})q_{L,L,1} - (s^\ell - s_{H,H,1})q_{H,H,1} \} \geq 0. \tag{23}
\]

**Appendix B**

**Proof of Proposition 1. Proof of (1).** The underwriter-issuer prefers the highest issue prices possible. The only constraints on prices to the uninformed are from the participation constraints of the uninformed, Eq. (8), which bind.

**Proof of (2).** Because $s^\ell = 0$, $\ell$ shares cannot be underpriced in this model (assuming non-negativity of prices). More generally, however, the truth-telling restrictions for $H$ bind before the truth-telling restrictions for $L$, because $s^\ell < s_{M,M}$ and $s^\ell < s_{H,H,k}$ except under extreme underpricing, whereas $s^h > s_{M,M}$ and $s^h < s_{L,L,k}$ unless there is overpricing. This makes it more efficient to compensate investors through underpricing of $H$ instead of $L$ shares.

**Proof of (3).** As long as prices are non-negative and no shares are strictly overpriced, Eqs. (22) and (23) will not bind. Similarly, (19) and (20) will not bind as
long as \( q_{L,L,1} = 0 \). Thus, of the truth-telling constraints, only Eqs. (18) and (21) affect the equilibrium.

Underpricing to those that report \( M \) lowers the expected proceeds of the issuer. It will also make it easier to satisfy Eq. (21), but in constraints Eqs. (6), (18), and (21), only the differences between the returns to those that report \( H \) and the returns to those that report \( M \) show up. Thus, to satisfy these constraints, a decrease in the expected return to those that report \( H \) has the same effect as an increase in the expected return to those that report \( M \). Because a decrease in expected return to those that report \( H \) is always at least as good in terms of satisfying the constraints, and it strictly dominates in terms of maximizing the underwriter’s expected proceeds, it will always be strictly preferred (and therefore chosen) by the underwriter.

Appendix C

Proof of Proposition 2. The two cumulative bidding distributions for the discriminatory auction are as follows. \( M_D(b; x, p) \) is the unique positive real root of the polynomial

\[
\theta(1 - b)[1 - p + p(1 - z)M_D(b; x, p)]^{N-X} \\
- b(1 - \theta)[1 - p(1 - z) + p(1 - z)M_D(b; x, p)]^{N-X} - \theta(1 - p)^{N-X}
\]

for \( b \in m_D, \bar{m}_D \) and

\[
H_D(b; x, p) = \frac{1 - xp}{xp} \left( -1 + \left( \frac{1 - \theta + \theta(1 - p)^{N-X}}{(1 - b)[1 - \theta + \theta(1 - px)^{N-X}]} \right)^{1/(N-X)} \right),
\]

for \( b \in (h_D, \bar{h}_D) \), where \( M_A(b; x, p) \) is the cumulative distribution function (cdf) for bidders that observe signal \( M \) and \( H_A(b; x, p) \) is the cdf for bidders that observe signal \( H \). A \( \in \{D, U\} \). The limits to the bidding ranges are: \( m_D = 0, \bar{m}_D \) and

\[
\bar{m}_D = \bar{h}_D = \frac{\theta[(1 - px)^{N-X} - (1 - p)^{N-X}]}{1 - \theta + \theta(1 - px)^{N-X}},
\]

and

\[
\bar{h}_D = 1 - (1 - px)^{N-X} \frac{1 - \theta + \theta(1 - p)^{N-X}}{1 - \theta + \theta(1 - px)^{N-X}}.
\]

To demonstrate that this is the solution to the auction, I need to show that the bidders are acting optimally through their choice of the equilibrium \( p, x \) pair in stage 1 and through the bidding strategies given for stage 2. The way \( p \) and \( x \) are chosen [i.e., such that \( PC_D(x, p) \) and \( FOC_D(x, p) \) are zero] guarantees that they are optimal for stage 1. For stage 2, the expected profit to bidding \( b \in (m_D, \bar{m}_D) \) with signal \( M \)}
The first set of terms is the probability that the issue has a high value, \( \theta \), times the probability of winning and thus paying \( b \) for shares worth one \( (1 - b) \), times the probability of winning with bid \( b \), given that the firm is type \( H \). This last probability is the sum over all combinations of bids that could lead to \( b \) being a winning bid, given that the firm is type \( H \). Of the other \( N - 1 \) bidders, \( j \) (between 0 and \( X - 1 \)) either receive signal \( H \) and bid above (with probability \( xp \)) or receive signal \( M \) and bid above [with probability \( (1 - x)p(1 - M_D(b; z, p)) \); \( i \) other bidders (between 0 and \( N - 1 - j \)) receive signal \( M \) and bid below [with probability \( p(1 - z)M_D(b; z, p) \); and the remaining \( N - 1 - j - i \) other bidders do not enter the auction (with probability \( 1 - p \)). If more than \( X - 1 \) other bidders bid above you, you do not have a winning bid and thus get zero.

The second set of terms is the expected profit to paying \( b \) for shares worth zero, \(-b\), times the probability that the shares are type \( L \) and thus worth nothing, \( 1 - \theta \), times the probability of winning with bid \( b \), given that the firm is type \( L \). Again, the probability is the sum over all combinations of bids for the \( N - 1 \) other bidders that lead to \( b \) being a winning bid, given the firm’s type. The key difference in the second probability of winning is that, if the firm is type \( L \), bidders that receive an informative signal will bid below, not above, bidders that observe \( M \). We can rewrite the expected profit for investors with neutral signals as

\[
R_D(b|M) = \theta(1 - b)(1 - p + p(1 - z)M_D(b; z, p))^N - X - X - b(1 - \theta)(1 - p + p(1 - z)M_D(b; z, p))^N - X.
\]  

(29)

An investor that gets a neutral signal has a riskless alternative to bidding in the range from \( m_D \) to \( m_D \), and that is to bid the lowest possible share value, which in this case is zero. Because entry is endogenous, a chance exists that not enough other bidders will enter the auction, and therefore such a bid will win. The bid is riskless, because even low-value shares are worth zero, while high-value shares are worth strictly more. Because bids are accepted at this minimum value, the expected return to bidding \( b \in (m_D, m_D) \) must be at least as great as the expected return to bidding zero, given that a bidder receives a neutral signal. The expected profit to bidding zero in this auction is \( \theta(1 - p)^N - X \), because a bid of zero is successful only if less than \( X \) other bidders choose to enter the auction. Thus, \( R_D(b|M) = \theta(1 - p)^N - X \). Setting this version equal to the earlier formula for \( R_D(b|M) \) gives Eq. (24).

From Eq. (24), I can also find \( m_D \) and \( m_D \). For bidders to be willing to bid anywhere in the range \( m_D \) to \( m_D \), the expected profits must be the same for all bids.
Because $M_D(b; z, p)$ is the cumulative density function for the bids, I know that $M_D(\hat{m}_D; z, p) = 1$ and $M_D(\tilde{m}_D; z, p) = 0$. Plugging these into Eq. (24) reveals that the lower end of the distribution is zero, and the upper end is given by Eq. (26).

The expected profit to bidding $b \in (\hat{h}_D, \tilde{h}_D)$ with signal $H$ in the discriminatory auction is

$$R_D(b|H) = (1 - b) \sum_{j=0}^{X-1} \sum_{i=0}^{N-1-j} \binom{N-1-j}{i} \binom{X-1}{j} (1-p)^{N-j-1-i} (zp(1-H_D(b; z, p)) prop\{p(1-z(1-H_D(b; z, p)))\}^j$$

$$= (1 - b)[1 - zp(1 - H_D(b; z, p))]^{N-X}. \quad (30)$$

The expected profit must be the same for all bids in the range $\hat{h}_D$ to $\tilde{h}_D$ (because investors otherwise would not be willing to bid anywhere within the range). An investor with signal $H$ that bids at the top of the range, $\tilde{h}_D$, wins with probability one and gets a payoff of $1 - \tilde{h}_D$. An investor with signal $H$ that bids at the bottom of the range will win only if less than $X$ other bidders enter and receive $H$ (because all other bidders with $H$ will bid above $\hat{h}_D$). The payoff if the bid wins is $1 - \hat{h}_D$, and the probability of winning is $1 - zp)^{N-X}$. Thus

$$1 - \tilde{h}_D = (1 - h_D)(1 - zp)^{N-X}. \quad (31)$$

It must also be true that $\hat{h}_D = \hat{m}_D$ (if not, bidders with $H$ could lower their bids, increasing their payoff if they win without changing their chance of winning, which would make the higher bids suboptimal). This gives the $H_D(b; z, p)$ and $\tilde{h}_D$ formulas given earlier.

Last, it is straightforward to show that bidding $b \in (\hat{h}_D, \tilde{h}_D)$ with signal $M$ or bidding $b \in (\hat{m}_D, \tilde{m}_D)$ with signal $H$ would lead to a lower expected return, meaning that the bidding strategy given in Proposition 2 is optimal.

In the uniform price auction, $\hat{h}_u = \tilde{h}_u = 1$. Bidders who receive an $H$ signal will bid the full value of the shares but will pay the $X + 1$st price. Let $z$ be the expected $X + 1$st or clearing price. The expected profit to a bid of one, given signal $H$, is

$$R_U(b|H) = \sum_{i=0}^{X} \binom{N-1}{i} (p)^i (1-p)^{N-1-i}$$

$$+ \int_{\hat{m}_U}^{\tilde{m}_U} (1-z)p(1-z)m_U(z) \sum_{i=0}^{X-1} \binom{X-1}{i} (zp)^i [p(1-z(1-M_U(z; z, p))]^{X-1-j}$$

$$\times \sum_{i=0}^{N-X-1} \binom{N-X-1}{i} (1-p)^i [p(1-z)M_U(z; z, p)]^{N-X-1-i} dz, \quad (32)$$

where $m_U(z)$ is the probability density function (pdf) of a bid $z \in [\hat{m}_U, \tilde{m}_U]$, and $M_U(z; z, p)$ is the cdf, as defined earlier.

The first set of terms in $R_U(b|H)$ represent the probability that the number of entrants to the auction is less than or equal to $X$, meaning that the price paid by all bidders will be the reservation price (because there is no $X + 1$st price). The reservation price in this model is the lowest possible value of the stock, zero.
Second, we have the integral over possible reservation prices, $z \in [\mu_U, \bar{m}_U]$, of the expected profit to buying shares at the $X+1$st price, $(1-z)$, times the probability of price $z$ being the $X+1$st price. For price $z$ to be the $X+1$st bid [i.e., the $X^{th}$ of the $N-1$ other potential bids, given that this bidder receives signal $H$ and bids 1.0], there must be one bid at $z$ (with probability $p(1-z)\mu_U(z)$); $j$ (between 0 and $X-1$) that enter, receive signal $H$ and bid above $z$ (with probability $zp$); $X-1-j$ that enter, receive signal $M$ and bid above $z$ [with probability $p(1-z)(1-M_U(z;\alpha,p))$]; $i$ (between 0 and $N-X-1$) bidders that do not enter (with probability $1-p$); and $N-1-X-i$ bidders that enter, receive signal $M$ and bid below $z$ [with probability $p(1-z)M_U(z;\alpha,p)$]. If $X$ or more other bidders enter and get signal $H$, then the $X+1$st price will be 1.0, and there will be no profit.

The pdf of the $X+1$st price, given signal $H$, is

$$f^H(z) = p(1-z)\mu_U(z) \sum_{j=0}^{X-1} \binom{X-1}{j} (zp)^j [p(1-z)(1-M_U(z;\alpha,p))]^{X-1-j} \times \sum_{i=0}^{N-X-1} \binom{N-X-1}{i} (1-p)^i [p(1-z)M_U(z;\alpha,p)]^{N-1-X-i} = p(1-z)\mu_U(z) [p - p(1-z)M_U(z;\alpha,p)]^{X-1} [1-p + p(1-z)M_U(z;\alpha,p)]^{N-1-X}. \tag{33}$$

These definitions allow the expected profit, given a signal of $H$, to be rewritten as

$$R_U(b|H) = P(\text{undersubscription}) + \int_{\bar{m}}^{\bar{m}} (1-z)f^H(z)\,dz. \tag{34}$$

Similarly, the expected profit to bidding $b \in (\mu_D, \bar{m}_D)$ in a uniform price auction, given that signal $M$ has been observed, is

$$R_L(b|H) = \theta P(\text{undersubscription}) + \theta \int_{\bar{m}}^{\bar{m}} (1-z)f^L(z)\,dz - (1-\theta) \int_{\bar{m}}^{b} zf^L(z)\,dz, \tag{35}$$

where $f^L(z)$ is the pdf of the $X+1$st price, $z$, given that the firm is type L, which is

$$f^L(z) = p(1-z)\mu_U(z) [p(1-z)(1-M_U(z;\alpha,p))]^{X-1} [1-p(1-z)(1-M_U(z;\alpha,p))]^{N-1-X}. \tag{36}$$

In other words, the expected return to a bidder with an $M$ signal equals the probability that the issue is type $H$ times the expected profit to winning the bid, given that the issue is type $H$ (this includes both the probability that the issue is undersubscribed and thus that the share is purchased for zero, and the probability that the $X+1$st price, $z$, is less than $b$), plus the probability that the issue is type $L$ times the expected loss $(-z)$ to winning the bid and buying shares, given that the issue is type $L$, integrated over all $z < b$.

Besides the fact that bidders with a neutral signal do not know the value of the shares and thus might lose money by bidding too much, this expected profit term
differs from that for bidders with $H$ signals because the integration goes only up to that bidder’s bid, $b$, not to the upper limit of the bidding range, $\bar{m}_U$. This is because the bidder wins a share only if his bid, $b$, is greater than the reservation price, $z$. Bidders with an $H$ signal bid the maximum amount, 1.0, and thus they could fail to win shares only if at least $X$ other bidders also bid 1.0, in which case the shares are fairly priced, so the bidder is indifferent to winning.

For bidders with signal $M$ to be willing to bid any price within the range $m_U$ to $\bar{m}_U$, the expected profit must be the same for all bids within the range. In other words, the derivative of $R_U(b|M)$ with respect to $b$ must equal zero. This gives the restriction that

$$\theta (1 - b) f^H(b) = (1 - \theta) z f^L(z) \text{ for all } b \in (m_U, \bar{m}_U).$$  

Substituting this into $R_U(b|M)$ and simplifying yields

$$R_U(b|M) = \theta P(\text{undersubscription}).$$

Holding $p$ constant, $R_U(b|M)$ [which equals $\theta P(\text{undersubscription})$] is strictly greater than $R_D(b|M)$ [which equals $\theta (1 - p)^{N-X}$]. In a discriminatory auction, only those that bid the reservation price pay it, whereas in a uniform price auction other bidders also pay the reservation price if the auction is undersubscribed (i.e., if less than $X+1$ enter).

Substituting into these equations reveals that $\bar{m}_U = 1.0$ and

$$m_U = \frac{\theta p^{N-1}(1-p)^{N-X-1}}{\theta p^{N-1}(1-p)^{N-X-1} + (1-\theta)(p(1-z))^{X-1}(1-p(1-z))^{N-X-1}}.$$  

Again, it is straightforward to show that bidding $b \in (m_U, \bar{m}_U)$ with signal $H$ would lead to a lower expected return, meaning that the bidding strategy given in Proposition 2 is optimal.

**Appendix D**

**Proof of Proposition 3.** I will derive $\text{SER}_B$. Given Proposition 1, I can rewrite Eqs. (6), (18) and (21) (the only potentially binding constraints) as

$$\sum_{k=1}^{K} \theta P(h, k - 1)(s^h - s_{H,H,k}) q_{H,H,k} \geq (1/z)(C(z) + e),$$  

$$\sum_{k=1}^{K} P(h, k - 1)(s^h - s_{H,H,k}) q_{H,H,k} \geq 0,$$  

and

$$-\theta \sum_{k=1}^{K} P(h, k - 1)(s^h - s_{H,H,k}) q_{H,H,k} + P(\cdot, 0)(1 - \theta) s_{H,H,1} q_{H,H,1} \geq 0.$$  

(42)
Eq. (41) will not bind as long as Eq. (40) is satisfied, given that $C(x) + e > 0$. Eq. (21) will limit the degree of overpricing, meaning that there is a feasibility limit to how much information the issuer can purchase or induce. Eq. (26) gives a minimum amount of underpricing for a given level of information collection. It will always bind, so I can substitute the amount of underpricing from Eq. (6) into SER_B, giving the equation shown in Proposition 3.

For SER_A: this formula follows directly from the fact that bidders choose $a$ and $p$ such that both PCA and FOCA equal zero (i.e., such that their ex ante expected profit equals zero).

Appendix E

Proof of Proposition 4. Because expected evaluation costs are the same for the two methods, it is sufficient to show that

$$\left(1 - \sum_{i=0}^{X-1} P(\text{entry} = i)\left(1 - \frac{i}{X}\right)\right) < 1,$$

which follows from $i/X < 1$, $N \geq X$ and $\sum_{i=0}^{N} P(\text{entry} = i) = 1$.

Proof of Proposition 5. The mean number of entrants is $pN$ and the variance is $p(1 - p)N$ (the standard formulas for binomial probabilities). Thus, it is sufficient to show that $\partial p/\partial N$ is nonpositive, i.e., that the optimal entry probability $p$ does not strictly increase as $N$ increases. This is straightforward, because the increased competition from more potential bidders makes entry less, not more, profitable, thus decreasing the optimal entry probability.

References


