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Chapter 16.1

Optimal remuneration

Outline

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- Investment banks are known to offer amongst the highest remuneration to their employees, along with law firms, management consultancies, and accounting firms.
- Pay at investment banks is also considerably higher than in commercial banks or insurance companies.
- we will here explore they reason why investment banks offer such generous remuneration, which is exceeds the marginal productivity of their employees.

Outline

- We will here compare the remuneration of commercial banker employees and investment bankers.
- To make this comparison we will determine the benefits employees generate to commercial and investment banks respectively, and based on this assessment determine the size of the remuneration package that should be offered to employees.

- We will compare the profits generated by an employee employed by a commercial bank ('banker') and that of an investment banker ('trader').
- We make this distinction as the banker originates the loan and the investment banker (trader) will only sell loan on to other investors, thus trade it without adding value to the loan itself.

Bankers and traders

- Bankers assess securities and loans and monitor them, thereby adding value to the through increased value or higher likelihood of being repaid
- Traders assess the value of the securities and loans provided by other banks with an aim to benefit from trading these
- Remuneration of bankers and traders should be based on the value they add to the bank employing them
- Bankers add social value, while traders do not add social value, they only re-distribute value
- How is remuneration determined for these two types of employees?

Bankers and traders

→ We distinguish between bankers providing a loan and traders, who merely buy and sell an existing loan.

▶
- The role of bankers is to assess the value of companies and the likelihood of any loans being repaid and subsequently monitor the companies to ensure the proceeds from the loan are used effectively.
- This activity adds value in that loans to companies who do not have good investment opportunities are not provided with loans and monitoring ensures the loans given are used effectively, in both cases increasing the value of the loan and the company.
- Banks also ensure through their monitoring that they detect early signs of likely default such that they can advise the company to take measures at an early stage, reducing the likelihood of default.

▶
- Traders, investment bankers, assess the value of loans that have already been granted, but do not add additional value to the process.
- The aim of traders is to benefit from buying and selling such loans, they are not interested in the quality of the loans itself.

▶ The remuneration employees receive for their roles should be based on the marginal value they add to their employers.

▶
- Through their activity, bankers add social value as they improve the investments that are made in an economy; they ensure that inefficient loans are not given and through monitoring ensure that loans remains valuable.
- In contrast, traders only re-distribute any value between the buyer and seller, no new social value of produced.

▶ We will not look at how these different roles affect the remuneration of bankers and traders.

→ With this distinction we will be able to determine the value these two types of employees add to their bank.

Loan repayments

- Banks have given a loan L with interest r_L
- Probability that the loan is repaid is either π_H^i or $\pi_L^i = \delta\pi_H^i$
- State H occurs with probability p, but this probability for an individual loan is not known to other banks, only thr bank originating the loan
- The other banks receive a signal about the state and this is correct with probability ρ_i

Loan repayments

→ We will focus now on the loans that a bank provides; of particular concerns are the repayments of the loan.

- Commercial banks provide a loan at a given loan rate, this decision s seen as exogenously given.
- This loan can be repaid with either a high or a low probability, which the bank providing the loan knows.
-
  - The fraction of loans with a high probability of repayment (low default rate) in the economy is known.
  - Other banks do not hold information on the probability of a loan being repaid, on the bank providing the loan has this information.
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  - Other banks will obtain information about the state of the loan, which is commonly referred to as a signal.
  - This signal is not perfect and will occasionally give the wrong information. It is common knowledge how likely the signal is to be correct.

→ We can use this modelling set-up now to determine the value of such loans from the perspective of another bank.

- We will first determine the value of the loan from the point of view of another bank.

Loans with low and high signals

- A bank does not know the probability with which the loans of another bank is repaid, but from expectations $E_i\left[\pi_s^j\right]$
- Bank i will assess a loan given by bank j if given a signal L as
- $P_L^{ij} = \rho_i E_i\left[\pi_L^j\right](1+r_L)\,L + (1-\rho_i)\,E_i\left[\pi_H^j\right](1+r_L)\,L$
- The signal L can be correct or incorrect, and gives the inference of the high or low probability of default
- For the high signal we get similarly
- $P_H^{ij} = \rho_i E_i\left[\pi_H^j\right](1+r_L)\,L + (1-\rho_i)\,E_i\left[\pi_L^j\right](1+r_L)\,L$

Loans with low and high signals

→ We distinguish between loans for which other banks receive a high signal and those for which other banks receive a low signal.

- The other bank doe snot know the probability of the loan being repaid as this information is not given.
  - It can only form expectations about the loan repayments in the two possible states.
- We can now determine how another bank assess the value of the loan, given that the bank has received the low signal for a loan.
- *Formula*
- We need to distinguish whether the signal received is correct or incorrect.
  - If the signal is correct, the loan is repaid with the low probability
  - If the signal is incorrect, the loan is repaid with the high probability, as the signal implied a low probability of repayment.
- The same arguments can now be used for the high signal being received; a correct signal implies the high repayment rate and a wrong signal the low repayment rate.
- *Formula*

→ Having determined the values of the loan for different signals, we can now determine whether the originating bank would be willing to sell the loan.

Seller accepting low offers

- If $\rho_i > \frac{1}{2}$, then $P_L^{ij} < P_H^{ij}$
- A bank can always offer to sell at P_L^{ij} and the loan will be purchased, if the banks wants to sell at P_H^{ij}, then the buyer needs to have the high signal
- We need $P_L^{ij} > pP_H^{ij}$ for the seller to accept the low offer
- This gives $\rho_i \leq \frac{1-p\delta}{(1-\delta)(1-p)}$
- Adverse selection must not be too high for the seller willing to accept low offers

Seller accepting low offers

→ We will look at the conditions under which a bank selling the loan is willing to accept a low offer, that is the offer made by another bank that has received the low signal.

- Let us assume that the signal is more likely to be correct than incorrect. In this case we can show that the value of the loan when receiving a low signal is lower than when receiving a high signal.
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  - A bank could always sell any loan at the low value and the other bank would be willing to buy this loan. If the loan is of low value it would break at least even and if it is of high value, it would make a profits.
  - In order to sell at the high price, the buying bank needs to have received the high signal. Only if the high signal is received, will be the buying bank break even.
- When setting its price, the seller doe snot know the signal of the buying bank. It will be able to sell the loan with certainty if asking for the low price. The high signal is received with probability p, the likelihood of the high state occurring, hence if setting a high price, it will only sell the loan if the high signal is received. Thus for a seller to quite a low price we need the certainty of making the sale at the low price, to exceed the expected proceeds from selling at the high price, taking into account that they might not sell.
- Solving this equation requires the precision of the signal to be not too large.
- It is thus that the information the buyer has is not too precise and thus the informational asymmetry must not be too high.

→ Knowing when banks are able to sell a loan to another bank and at which price, we can now determine the bank profits resulting from such sales.

- We will look at banks facing a liquidity shock that requires them to sell loans. It will be the implications of such a forced sale of loans that we are concerned with.

Liquidity shock

- A bank faces a liquidity shortage with probability λ and has to sell loans
- A bank not facing a liquidity shortage has excess liquidity and would buy these loans
- Banks employ bankers, who can affect the probability of loans being repaid, π_j^i
- Banks employ traders, who obtain signals with precision ρ_i

Liquidity shock

→ banks may be forced to sell loans due to an exogenous shock, called a 'liquidity shock'.

- We assume that a bank faces a liquidity shock that forces it to sell its loans. Such a liquidity shock might occur if a bank faces the withdrawal of deposits.
- A bank that does not face such a liquidity shortage will have excess cash that allows it to buy the loans the other bank is selling. This bank might face a lower than expected withdrawal of deposits, or even an inflow of deposits that have been withdrawn from the other bank.
- We now assume that banks employ 'bankers', who have the ability to affect the probability of loans being repaid. They have this ability only on loans they have originally given.
- In addition, banks employ 'traders' who are able to obtain a signal on the quality of the loans made by another bank. They do not affect the repayment rate of loans, but only the precision of information on loans they are seeking to purchase.

→ We can now determine the profits of a bank in this context.

Bank profits

- Banks facing a liquidity shortage, sell the loan for what the other bank thinks it is worth, P_L^{ji}
- Banks not facing a liquidity shortage, retain their loan
- and purchase at a price P_L^{ij} the loan of the other bank
- They pay depositors and their bankers and traders
- $$\Pi_B^i = \lambda P_L^{ji} + (1-\lambda)\left(p\pi_H^i\left(1+r_L\right)L + (1-p)\,\pi_L^i\left(1+r_L\right)L \right.$$
$$+ \left.\left(pE_i\left[\pi_H^j\right]\left(1+r_L\right)L + (1-p)\,E_i\left[\pi_L^j\right]\left(1+r_L\right)L - P_L^{ij}\right)\right)$$
$$- \left(1+r_D\right)D - w_T N_T^i - w_B N_B^i$$

Bank profits

→ We can now determine the profits of banks facing a potential liquidity shock and selling loans, or when facing no liquidity shock buying additional loans.

- We assume that the bank facing a liquidity shortage sells the loan at the lower price to ensure it is sold as it needs to raised the required funds. Adverse selection is assumed to be sufficiently low for this strategy to be optimal.
- A bank who doe snot face a liquidity shortage, will retain the loan and receive the possible payoffs, with either a high or a low repayment rate.
- In addition, they buy the loans of the other bank at the low price. They will then receive the loan repayment from the purchased loans, making inferences about their probabilities of repayment.
- Banks then repay their deposits which are used to fund the original loan and they pay the wages for each of their bankers and each of their traders.
- *Formula*

→ We now have to make an inference about the price the other bank is willing to pay for the loans of a bank.

Price of the loan the other bank pays

- The price paid will be determined by the inference the bank has on the quality of the signal by the other bank
- $P_L^{ji} = E_i\left[\rho_j\right]\pi_L^i\left(1+r_L\right)L + \left(1-E_i\left[\rho_j\right]\right)\pi_H^i\left(1+r_L\right)L$
- Probability of loans being repaid and the quality of the signal are not given but banks will optimize them

Price of the loan the other bank pays

→ The other bank has to make inferences about the quality of the loan they are purchasing and based on that, they will determine the price they are offering.

- They bank knows the probabilities of repayment for their own loans, but has to make an inference about the quality of the signal the other bank has.
- We focus on the low price, this price will be determined by the low repayment rate, if the signal the other bank receives is correctly low, or the high repayment rate if the signal received is incorrectly high.
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  - As we will now argue, the repayment rate and the quality of the signal are not exogenously given,
  - but instead they are the result of banks making optimal decisions on the amount invested into each activity.

→ We will therefore now assess how bankers and traders affect these variables.

- We will be looking at the impact bankers and traders have on the repayment probabilities and the signal quality.

Hiring more bankers and traders

- Hiring more bankers increases the likelihood of loans being repaid: $\frac{\partial \pi_s^i}{\partial N_B^i} > 0$
- Hiring more traders increases the precision of the signal: $\frac{\partial \rho_i}{\partial N_T^i} > 0$
- The total number of bankers and traders is limited to N_k each
- If a bank hires N_k^i bankers or traders, the remaining banks share $N_k^j = \frac{N_k - N_k^i}{N-1}$

Hiring more bankers and traders

→ We will firstly seek to argue that hiring more employees increases the quality of the work they provide.

- If more bankers are employed, they each have more time to assess companies seeking loans and monitoring the use of the loan proceeds; this should increase the quality of the work they provide and hence reduce the default rate or increase the repayment rate.
- If more traders are employed, they each have more time to assess loans; this should increase the quality of the work they provide and hence increase the precision of the signal they receive.
- However, in the market, the number of bankers and traders that are available is limited. Thus banks compete with each other for bankers and traders. We also assume that bankers and traders are distinct types and a banker cannot become a trader and the same holds vice versa.
- Thus if a bank hires a number of bankers (traders), this reduces the number of bankers (traders) that are available to the remaining banks. In total we have N banks.

→ As a first step, we can now determine the effect of bankers and traders on the outcome of other banks.

Influence of bankers and traders

- Influence of bankers on loan repayments of other banks:
  $$\frac{\partial \pi_H^j}{\partial N_B^i} = \frac{\partial \pi_H^j}{\partial N_B^j} \frac{\partial N_B^j}{\partial N_B^i} = -\frac{1}{N-1} \frac{\partial \pi_H^j}{\partial N_B^j} < 0$$
- Influence of traders on signal precision of other banks:
  $$\frac{\partial \rho_j}{\partial N_T^i} = \frac{\partial \rho_j}{\partial N_T^j} \frac{\partial N_T^j}{\partial N_T^i} = -\frac{1}{N-1} \frac{\partial \rho_j}{\partial N_T^j} < 0$$
- As the number of bankers and traders is limited, hiring more will reduce the number available to other banks and thus reduce their probability of loan repayment or signal precision

Influence of bankers and traders

→ With the number of bankers (traders) given, a bank employing an additional banker (trader) will reduce the number of bankers (traders) available to other banks, which in turn will affect their ability to increase the repayment rate (quality of the signal).

- We can now determine the effect the hiring of a banker has on the repayment rate of the loan of another bank. Using the relationships above, we easily get this *formula*.
- We can now determine the effect the hiring of a trader has on the signal quality of another bank. Using the relationships above, we easily get this *formula*.
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  - With only a limited number of bankers (traders) available, hiring an additional banker (trader) will reduce the number of bankers (traders) the other banks can employ.
  - This will reduce their repayment rates and the quality of their signal.

→ We thus see that the hiring decision of one bank, affects other banks and banks will compete for bankers and traders.

- We can now determine the optimal wages for bankers and traders, initially assuming that we ignore the effects that competition has on other banks. This result can than be used as a benchmark when considering the case where the hiring of bankers and traders affects other banks.

Number of bankers and traders

- Ignoring the effect hiring a banker has on the ability of other banks, the optimal number of bankers and traders to hire is given from $\frac{\partial \Pi_B^i}{\partial N_T^i} = \frac{\partial \Pi_B^i}{\partial N_B^i} = 0$
- All banks are alike, hence banks will infer that they behave like them: $E_i\left[\pi_H^j\right] = \pi_H^i$ and $E_i\left[\rho_j\right] = \rho_i$
- Traders: $w_T^* = (1-\lambda)(1-\delta)\pi_H^i(1+r_L)L\frac{\partial \rho_i}{\partial N_T^i}$
- Bankers: $w_B^* = (V + (1-\delta)\lambda(1-\rho_i-p)(1+r_L)L)\frac{\partial \pi_H^i}{\partial N_B^i}$

Number of bankers and traders

→ We now seek to determine the optimal number of bankers and traders a bank will employ.

- We initially ignore the effect that hiring has on the number of employees other banks can hire themselves. The optimal number of bankers and traders are then given at the point where the bank profits are maximal.
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  - In order to solve these two equations, we need to make an additional assumption; we will assume that all banks a identical, thus lend to the type of companies, apply the same loan rates, deposit rates, but also have the same probability of facing a liquidity shock.
  - This implies that banks will infer that the other banks will make the exact same decisions as they do. Hence the expected repayment rate of loans of the other bank will be identical to its own repayment rate.
  - he expected signal quality of other banks will also be identical to its own signal quality.
- Inserting this, we can solve the first order conditions for the optimal wages that are provided. For traders the wage will be as in this *formula*.
- For bankers the wage will be as given in this *formula*.

→ The first order condition represents the marginal revenue these employees generate to the bank, less their marginal costs, which is trivially the wage. Thus the wage represents the marginal profits of the bank from hitring an additional employee.

Equilibrium wages

- A Pareto optimal allocation of resources within banks requires that the marginal products of bankers and traders are identical
- The marginal product of a banker or trader is its wage
- This implies that $w_T^* = w_B^* = w^*$
- The wages are adjusted by hiring the requisite number of bankers and traders to adjust $\frac{\partial \rho_i}{\partial N_T^i}$ and $\frac{\partial \pi_H^i}{\partial N_B^i}$ accordingly

Equilibrium wages

→ Having determined the optimal wages of bankers and traders as a function of the marginal repayment rate and marginal signal quality, we can now implicitly determine these values.

- Hiring bankers and traders can be seen as an allocation of resources between these two 'products'. We know that in order to achieve Pareto optimality, the marginal products of the two products have to be identical. Hence the marginal product the bankers and traders contribute to the profits of the bank must be identical.
- In this context the marginal product of the banker and marginal product of the trader will be their respective wages. This is the 'price' of the product.
- Hence the wages of bankers and traders must be the same.
- To achieve the same wages, the bank will adjust the number of banker and traders accordingly such that the marginal repayment rate and signal quality, respectively, achieve this. From this we can then determine the optimal repayment rate and signal quality, which then translate into the optimal number of bankers and traders that need to be hired. We cannot determine both variables simultaneously as we only have a single equation, the equality of wages, for these two variables. However, we might impose an additional restriction, such as that competition erodes any bank profits to add a constraint that allows us to determine both values.

→ This result serves as a benchmark to investigate the case where banks compete for bankers and traders.

- We will now take into account that when maximizing their profits, banks hiring bankers and traders affect the ability of hiring for other banks.
- We then can compare these results with that of the wages when ignoring the effect.
- When ignoring these effects, we have seen that the wages reflect the marginal product of the banker and trader, which is generally regarded as the efficient wage to be paid.

Optimal wages with impact on other banks

- The bank will now take into account the effect its hiring of a banker or trader has on the ability of the other bank to do likewise
- The optimal number of bankers and traders to hire is given from $\frac{\partial \Pi_B^i}{\partial N_T^i} = \frac{\partial \Pi_B^i}{\partial N_B^i} = 0$
- Traders: $w_T^{**} = (1-\delta)\left((1-\lambda) + \frac{\lambda}{N-1}\right)(1+r_L)\,L\,\frac{\partial \rho_i}{\partial N_T^i}$
- Bankers: $w_B^{**} = \frac{\partial \pi_H^i}{\partial N_B^i}\left(V + \left(\lambda + \frac{1-\lambda}{N-1}\right)(1-\delta)(1-\rho_i - p)(1+r_L)\,L\right)$

Optimal wages with impact on other banks

→ We will now look at the optimal wages in this case where banks are competing for bankers and traders.

- When optimising their profits, the bank will take into account that their hiring of employees will affect the availability of employees to competitors. Competitors will have less bankers available, making their loans more risky, and the signal precision lower.
- The optimal number of bankers and traders are then given at the point where the bank profits are maximal. In contrast to the case above, we now explicitly take into account the effect on other banks when deriving the first order conditions..
- Solving these two first for the optimal wages are obtained. For traders the wage will be as in this *formula*.
- For bankers the wage will be as given in this *formula*.

→ We can now compare these wages with the efficient wages we have derived previously.

Traders are paid more than bankers

- Comparing with the efficient wage, we get $w_T^{**} > w^* > w_B^{**}$
- Traders are paid more than bankers
- Traders are paid more than their marginal product, they are overpaid
- Bankers are paid less than their marginal product, they are underpaid

Traders are paid more than bankers

→ We can now establish the relationship between these wages and the efficient wage determined before.

- Comparing the expressions we see that the wages of traders are higher than the efficient wage, while the wages of bankers are lower.
- A direct implication is that while it was efficient to pay both, bankers and traders, the same wages, here the traders are paid more than the bankers.
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  - As the efficient wage, w^*, represented the marginal product of the employee, traders are paid more than their marginal product.
  - Traders are therefore overpaid, relative to the efficient wage.
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  - As the efficient wage, w^*, represented the marginal product of the employee, bankers are paid less than their marginal product.
  - Bankers are therefore underpaid, relative to the efficient wage.

→ We have thus established that traders are paid more than bankers and that their wages are noth not efficient.

Overpaid traders

- Traders contribute to bank profits by evaluating loans the bank buys
- Hiring traders contributes also to the bank achieving a higher sales price for their loans

$\Rightarrow$ Denying other banks a trader, reduces the precision of their signal

$\Rightarrow$ As $\frac{\partial P_L^{ij}}{\partial \rho_i} < 0$, the sale price of the loan increases

- Traders indirectly contribute more than their marginal product from signal precision to the profits of the bank

Overpaid traders

→ We will first look at the reasons why traders are overpaid, that is paid more than their marginal product.

- The contribution traders make to their bank's profits arises from the precision of their signals to determine the loan a bank buys, provided it does not face a liquidity shock.
- Hiring traders also increases the price the bank gets for the loan they sell if faced with a liquidity shock. This arises from the fact that hiring an additional employee would reduce the ability of the other banks to assess the quality of the loan due to this bank being able to hire less traders.
- [⇒] Thus, hiring more traders increases the precision of their own signals, but also decreases the quality of the signal of other banks.
- [⇒] As the price will increase with less precise signals, the bank will receive a higher price for their loans.
- Traders therefore contribute not only through the higher quality of their signal to reduce the price the bank pays for loans, their marginal product considered in the efficient case, but they also reduce the ability of other banks to obtain information and this also benefits the bank. Thus their contribution to the profits of banks is larger than implied by the efficient case and it is therefore that they are paid a higher wage.

→ Thus traders are still paid their marginal product, but we have to take into account that they contribute also indirectly through the negative impact their hiring has on other banks.

Underpaid bankers

- Bankers increase the value of the loan the bank holds through higher probabilities of repayment
- This also increases the value of the loan to any purchaser as loans are sold at a discount, increasing the loss to the selling bank
- This causes an externality and the banker contributes less than its marginal product from increasing the probability of repayment

Underpaid bankers

→ We can now look at the reasons why bankers are underpaid, relative to the efficient case.

- Bankers increase profits to the bank by reducing the default rate of loans (increasing the repayment rate). This directly increases the profits of the bank and constitutes the marginal profits to the bank this employee produces in the efficient case above.
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  - This reduced default rate also increases the value of the loan to a purchaser of the loan as we assumed that the loans are sold at the low price. Hence, if the value is high as the repayment rate is high, the loan the purchaser obtains will be worth even more than what they paid.
  - This implies on the other hand that the loan is sold at an even larger discount, increasing the loss of the bank from selling the loan.
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  - This loss can be seen as a negative externality affecting the profits of the bank that hired the banker.
  - The banker overall contributes less than their marginal product to the bank's profits, which was the impact of the reduced default rate; the effect of the externality has to be deducted.

→ Again, the banker is paid its marginal product, once their negative externality is taken into account.

- So we have seen that traders produce a positive externality (other banks are less able to assess the loans they are purchasing), while bankers produce a negative externality (other banks are obtaining a more valuable loan). We can now distinguish between private benefits of bankers and traders as opposed to their social benefits.

Private benefits

- Traders create profits by buying loans at a higher discount and preventing other banks to purchasing loans at a high discount
- They benefit banks in two ways, making their remuneration high
- Bankers create value to the bank by reducing the default rate of loans, this also benefits the purchaser of a loan as it will be paid at a higher discount
- They create an externality that reduces bank profits, making their remuneration low

Private benefits

→ The private benefits accrue to the bank and they will take into account the external effect their employees have and how they impact their profits.

- Traders generate profits obtaining a higher discount when purchasing loans as they have better information; this increases the profits of the bank.
- At the same time by taking away resources from other banks, they have less information and will therefore not able to purchase the loan at a high discount, reducing the loss to the bank.

- Traders benefit banks in both ways,
- which will increase their wages.

- Bankers reduce the default rate of loans, increasing the value of loans and hence the profits to the bank.
- This also benefits the purchaser of a loan as the discount they can obtain will be higher; the increase in the loan value is not fully incorporated into the loans price.

- Bankers generate an effect that reduces the profits of the bank.
- As they reduce the bank profits through this effect, their wages will be lower.

→ We can now contrast this result with that of the social benefits these two types of employees generate.

Social benefits

- Bankers produce social value by reducing defaults
- Traders produce no social surplus as they only redistribute value between banks
- The activity increasing welfare is paid less than the activity adding no welfare
- It is privately rational to reward traders more highly

Social benefits

→ The private benefits are very different from the social welfare the respective activities generate.
- The social value of bankers consists of their ability tow work with lenders to reduce the default rate of loans and make company investments more successful.
- Traders only redistribute wealth from the sellers of loans to the purchasers of loans and as such they do not add social value.
- We thus see that the activity creating welfare is paid less than the activity creating no welfare.
- This discrepancy arises from the private benefits each of these types of employees generate.

→ We thus have a scenario in which an activity that has social benefits (bankers) is paid less than an activity that does not create any social benefits (traders). As remuneration is paid based on banks competing for employees they will reward them up to the marginal profits they can generate and this will lead to a situation in which traders (investment bankers who distribute wealth between buyers and sellers of securities) are better paid than those generating the value (bankers, but also employees in companies that work for investment to succeed). It also provides an explanation for the high salaries and boni paid by investment banks if compared to other employers.

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