



Parities

# Outline

- Arbitrage with goods and investment
- Purchasing power parity
- Interest rate parity
- Summary

- One way to determine exchange rates is by means of arbitrage when investing abroad or purchasing good abroad.
- If it is more profitable to invest in one country than another, this country should see an inflow in investments and thus demand for its currency increases and the currency appreciates; this would continue until the investments in both countries are equal.
- Similarly, if goods are cheaper in one country than another, purchases in that country would increase, leading to an increase demand for that country's currency and the currency appreciates; this would continue until the goods in both countries cost the same.

- We will look at how this arbitrage would work and then consider the implications for arbitrage in goods markets and in investments, respectively.

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- We can now consider how cross-border investments and goods purchases can be used to determine exchange rates.

# Trade and investment

- ▶ Currencies are exchanged if goods are traded across borders
- ⇒ Such trade can be used to determine appropriate exchange rates
- ▶ Currencies are exchanged if investments are made abroad
- ⇒ Such investments can be used to determine appropriate exchange rates

- International trade and investments involve the use of different currencies and we propose that this can be used to determine exchange rates that would provide an equilibrium.
- ▶ If goods are traded between different countries, the payment for such goods involves the exchange of the two currencies of the countries involved.
- ▶ [⇒] We can thus use the trade flow between countries to determine the exchange rate that would lead to an equilibrium.
- ▶ If investments are made overseas, the investment requires the exchange of the two currencies of the countries involved.
- ▶ [⇒] We can thus use the cross-border investments to determine the exchange rate that would lead to an equilibrium.
- We will therefore use these flows of funds between countries to determine an exchange rate that gives an equilibrium between countries, thus it would make investments in both countries equally profitable and goods have the same price.



## Limits to the theories

- ▶ International trade and investment faces substantial transaction costs
- ▶ These transaction costs make foreign goods and investments not directly comparable
- ▶ This will limit the applicability of theories based on such approaches

- While the idea to use international trade and investments seems an obvious way to determine the exchange rate as an equilibrium, there are limits to its applicability in practice.
  - ▶ Despite progress in international transactions through online payment platforms, transaction costs remain substantial. This includes actual costs of transferring funds internationally, but also time delays and various import and export restrictions for goods, services and investments.
  - ▶ Having such substantial transaction costs makes goods and investments in other countries not directly comparable as the costs of foreign goods and investments are substantially higher, which can lead to substantial distortions in the exchange rate, which can nevertheless not be exploited.
  - ▶ Hence transaction costs limit severely the ability of such equilibria to explain actual exchange rates.
- Although the equilibrium exchange rates might differ from actual exchange rates substantially due to transaction costs, will obtain these equilibria now as a useful benchmark to assess exchange rates against.

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- We will first determine the exchange rate when based on the purchase of goods from overseas.

## Equal goods prices

- ▶ If goods are produced in two countries and they can be traded, the costs of purchasing them should be the same in both countries
- ▶ If prices were different, arbitrage could occur with a trader buying the goods in the country with the lower price and selling it in the country with the higher price
- ▶ Of course, the price of the goods will be priced in different currencies and the exchange rate needs to be applied to transform the prices of the domestic country into that of the foreign country
- ▶  $P_t^* = e_t P_t$
- ▶ This is known as the absolute purchasing power parity

- We will argue that internationally, goods prices should be equal.
- ▶ If you have a goods, produced in two countries and this good can be traded between these countries, the goods should have the same price in both countries.
  - ▶
    - If the good was cheaper in one country than the other, traders could make profits from arbitrage.
    - They would purchase the good in the country where it is cheaper
    - and then sell it in the country in which it is more expensive. This would generate them a profits from the differences in the price.
  - ▶
    - As we are considering tow countries, the prices will be denominated in different currencies, the local currencies, respectively.
    - We therefore need the exchange rate to to adjust the prices in the different currencies.
- ▶ The price of the good abroad will be the price of the good at home, adjusted by the exchange rate.
- ▶ This equation is known as purchasing power parity and the exchange rate should reflect the ration of the prices of the good in the respective currencies.
- This absolute purchasing parity is difficult to assess as multiple goods can be traded and the exchange should reflect this variety of goods. However, using price indices of goods are problematic as they often contain goods in different weights, as well as different goods, making the absolute purchasing power parity difficult to determine.

## Relative purchasing power parity

- ▶ Price levels across an economy are often difficult to compare due to the variety of goods produced
  - ▶ We will instead compare inflation rates between countries
  - ▶ Absolute purchasing power parity holds:  $P_t^* = eP_t$
- ⇒  $\ln P_t^* = \ln e_t + \ln P_t$
- ▶ The same holds for the previous time period:  $\ln P_{t-1}^* = \ln e_{t-1} + \ln P_{t-1}$
- ⇒  $\ln e_t - \ln e_{t-1} = (\ln P_t^* - \ln P_{t-1}^*) - (\ln P_t - \ln P_{t-1})$
- ⇒  $\Delta e_t = \pi_t^* - \pi_t$

- We will seek to overcome the problems of determine the price of a basket of goods (an index) by looking at how prices evolve over time. This is known as the relative purchasing power parity.
- ▶ Comparing price levels across countries is difficult, as different goods are included in indices and even where the same goods are included the relative weights might well be different.
- ▶ Instead, we will compare the price changes, thus inflation. While inflation, based on the index, suffers in principle from the same problem, prices of goods are changing more similarly over time, even if index is calculated differently.
- ▶ Our starting point is the absolute purchasing power parity.
- ▶ [⇒] We take the logarithm of this expression.
- ▶ We now shift the equation by one time period.
- ▶ [⇒] Solving both equations for the exchange rate and taking the difference gives us this *formula*.
- ▶ [⇒] The difference in log-exchange rates is the relative change of the exchange rate and the first term on the right-hand side reflects the relative change of the price level abroad; that is the inflation abroad. The second term on the right-hand side reflects the relative change of the price level at home; that is the domestic inflation.
- We thus see that the change in the exchange rate should be equal to the difference in the inflation rate.



## Exchange rates adjusting to inflation

- ▶ The change in the exchange rate should reflect the difference in inflation
- ▶ A country experiencing higher inflation would see its currency devalue over time
- ▶ Not all goods are traded, many services need to be provided locally, if these are included in inflation, the purchasing power parity might not hold

# Exchange rates adjusting to inflation

- We have established a clear link between the change in the exchange rate and the difference in inflation between two countries.
  - ▶ The exchange rate changes such that it reflects the difference between the foreign and domestic inflation rates.
  - ▶ If a country has higher inflation than another country, its currency depreciates. To see this consider that the foreign inflation is higher, thus the right-hand side term is positive and the exchange increases; this means more of the foreign currency is needed to purchase the domestic currency; the foreign currency as depreciated.
  - ▶ A limitation of this approach is also that not all goods are traded; local services cannot be provided internationally, such as haircuts. Usually such non-tradeable goods are included in the price index and hence in the inflation figure. As arbitrage can only work with traded goods, this might distort the results. This is especially the case if the lack of international competition allows non-tradeable goods and services to have very different price changes and thus affect the overall inflation rate.
- While purchasing power parity should ensure that goods can be purchased at the same prices and exchange rates adjust to this effect, there are significant obstacles in terms of international trade to achieve this. We also have implicitly assumed that prices or inflation in both countries are given and the adjustment to purchasing power parity is made through exchange rates. However, we can easily include an effect on inflation from such international competition, which will affect the inflation rate and hence the change in the exchange rate.

- Arbitrage with goods and investment
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- rather than focussing on trading goods, we could similarly focus on international investment and determine the equilibrium exchange rate from this perspective. This is known as the interest rate parity.

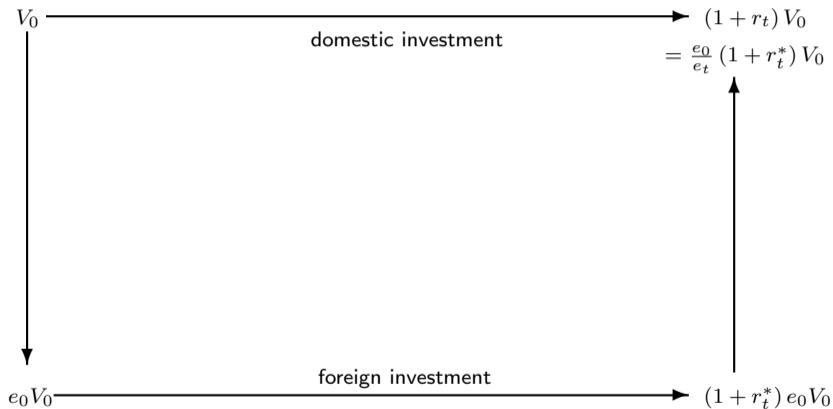
## Investing at home and abroad

- ▶ Investors can invest into bonds in their home country or into bonds denominated in a different currency
- ▶ Investing abroad involves exchanging their home currency for the foreign currency
- ▶ At the end of the investment, the investment is exchanged back into the domestic currency
- ▶ Investing abroad and overseas should yield the same outcome

# Investing at home and abroad

- We will now consider the impact that the possibility to invest either domestically or abroad has on the on the determination of exchange rates.
- ▶ We only consider investments into bond markets, both assumed to be risk free. This is done to avoid any discrepancies arising from the relative risk of assets.
- ▶ If investing into a bond abroad, the investor will have to exchange their domestic currency for the foreign currency before making the investment.
- ▶ Once the investment has been concluded, the investors reverses this transaction and exchanges the foreign currency for the domestic currency.
- ▶ If investing overseas provides a higher return, no domestic investment would be found and if investing domestically is preferred no investment in the foreign country would occur. Both cannot be an equilibrium, hence we need that domestic and overseas investment provide the same return.
- We will now explicitly compare the investment strategies at home and abroad.

# Investment strategies



- We will now see how the return of the domestic and foreign investment is determined.
  - ▶ The initial investment is made.
  - ▶ If the investment is made domestically, it will yield a given return at the end of the time period based on the domestic interest rate.
  - ▶ If the investment is made abroad, the investment is first converted into the foreign currency.
  - ▶ This amount is then invested and yield the return from the interest as applicable in the foreign country.
  - ▶ The amount obtained from investment is then converted back into the domestic currency,
  - ▶ These two investment strategies should now be yielding the same outcome.
- We can now analyse this equality further to obtain the interest rate parity.



# Exchange rates and interest rate differentials

- ▶ Domestic and foreign investments should have equal outcomes:

$$(1 + r_t) V_0 = \frac{e_0}{e_t} (1 + r_t^*) V_0$$

$$\Rightarrow \ln(1 + r_t) = \ln e_0 - \ln e_t + \ln(1 + r_t^*)$$

$$\Rightarrow r_t = -\Delta e_t + r_t^*$$

$$\Rightarrow \Delta e_t = r_t^* - r_t$$

# Exchange rates and interest rate differentials

- We can now determine the interest rate parity using the equality of investment outcomes.
- ▶ We had established that the outcome of the domestic and foreign investment should give the same outcome.
- ▶ [⇒] We can now eliminate  $V_0$  and take the logarithm.
- ▶ [⇒] Using the approximation that  $\ln(1+x) \approx x$ , we can rewrite this equation in terms of the risk-free rates in the two countries.
- ▶ [⇒] We can now solve this expression for the change in the exchange rate and see that it is determined by the interest rate differential between the two countries.
- Thus we see that the risk-free rates of the two countries determine how the exchange rate changes.

## Exchange rates adjusting to interest rate differentials

- ▶ Exchange rates reflect the difference in interest rates of the countries
- ▶ The exchange rate of the country with the higher interest rate should depreciate over time
- ▶ This result holds for the length to maturity of the bonds affected and could be used to forecast future exchange rate movements over long periods of times
- ▶ The interest rate parity implies that both investment have the same risk, but this can be adjusted to account for any differences in risk

# Exchange rates adjusting to interest rate differentials

- We can now intuitively interpret the interest rate parity.
- ▶ The interpretation of interest rate parity is that the exchange rate reflects the difference in the interest rates between two countries.
- ▶ If the interest rate of the foreign country is higher, the right-hand side expression is positive and hence the value of that country depreciates. This is because the return on investment overseas is higher than domestically and this needs to be compensated for by the lower exchange rate. Here a higher exchange rate implies that more foreign currency is obtained for a given amount of domestic currency.
- ▶
  - We can observe the yields to maturity of government bonds, the risk-free rate, for various maturities and we did not specify the lengths of investment in our interest rate parity.
  - Thus we could use the interest rate differential to predict the future exchange rates for  $t$  time periods ahead.
- ▶
  - Our model assumed both investments, domestic and abroad, had the same risk (they were risk-free),
  - but we can include risky assets as well and adjust for the different risks these pose and will obtain the same result.
- Thus exchange rate differentials drive any changes in the exchange rate.

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- We can now summarize our results on these two parities that determine exchange rates, the purchasing power parity and the interest rate parity.

# International Fisher effect

- ▶ Exchange rates will take into account price changes in tradeable goods and different yields to investments in countries
- ▶ Purchasing power parity and interest rate parity should both hold
- ▶  $\Delta e_t = \pi_t^* - \pi_t = r_t^* - r_t$
- ⇒  $r_t - \pi_t = r_t^* - \pi_t^*$
- ▶ The real interest rates across countries have to equal

# International Fisher effect

- We will firstly combine the two parities as they should both hold to prevent arbitrage in investments but also in goods.
- ▶ We have seen that exchange rates are affected by the price changes in the two countries, their inflation, and the interest rates in those countries.
- ▶ Both parities should hold and the exchange rate should change such that it is consistent with the difference in inflation as well as interest rates.
- ▶ We now set both parities equal.
- ▶ [⇒] We can now solve for this *formula*. The term  $r_t - \pi_i$  ( $r_t^* - \pi_i^*$ ) gives the real interest rate of the domestic (foreign) country; the real interest rate is the nominal interest rate less inflation.
- ▶ Thus if both parities were to hold, the real interest rates of both countries would have to be equal. This is known as the 'International Fisher effect'.
- It thus possible for both, purchasing power parity and interest parity, to hold, provided the real interest rates in both countries are equal.



## Limits in the use of parities

- ▶ Information on inflation is only available with substantial delays, making the forming of expectations necessary
- ▶ Interest rates are affected by monetary policy decisions and these are linked to macroeconomic factors, including inflation
- ▶ Such interdependencies, lack of information and monetary policy interventions limits the applicability of parities

- While parities are an easy tool to determine exchange rate changes based on the avoidance of arbitrage in goods markets and international investments, respectively, there are significant limitations to the practical applicability of these theories.
  - ▶
    - In order to make use of purchasing power parity, we need access to information about the inflation of countries. Actual inflation data are not instantly available are published with substantial delay and particularly in developing countries are not always a reliable indicator of the actual inflation due to limited resources in collecting data.
    - Consequently, we have to rely in expectations about inflation, which may prove to be wrong.
  - ▶
    - When looking at interest rate parities, the interest rates are available, but they are affected by many other factors, one such factor is monetary policy.
    - Monetary policy is linked to wider macroeconomic factors, such as economic growth and therefore expectations about future changes will complicate the assessment of future exchange rate changes. It will also affect the assessment of past exchange rate changes as these effects need to be considered, given they will have affected investors' decisions.
    - Inflation will also influence monetary policy, making a connection between interest parities and purchasing power parities, further complicating the picture.
  - ▶ Taking into account these complications arising when using parities to assess exchange rate changes, the direct use of them as implied by ex-post data, will not allow us to assess the validity of these theories adequately.
- We thus see that while parities are an easy and intuitive way to understand the changes in exchange rates, they are affected by wider macroeconomic factors and monetary policies, which should also be taken into account.



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