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Collaterised debt obligations

- We now consider the valuation of another credit derivative that was widely used until the Great Financial crisis of 2007/8, the Collaterised Debt Obligation.
- These CDOs were not developed as a tool to hedge credit risk directly, but allowed banks to sell risky loans to investors not seeking a large exposure to credit risk.
- To do this, only some part of the loan was sold, the remainder retained by the bank. The derivative was constructed such that those parts sold off were supposed to be nearly free of credit risk and most credit risk retained by the bank.

Tranching losses

- A number of fixed income assets can be combined and securities be sold based on this portfolio
- Fixed income assets can include mortgages, bonds, car loans, student loans, credit card receipts, ...
- Securities are designed such that they bear losses from defaults in order of priority
- The first tranche will bear the losses until it receives no more repayments
- Only once the first tranche has been eliminated, will the second tranche bear any losses, and so on from junior tranches to senior tranches
- The higher the tranche, the more losses are required before the tranche is not repaid in full

Tranching losses

→ We will now explore the idea behind Collaterised Debt Obligations and how they are able to generate elements that are nearly free of credit risk.

- We will combine a number of fixed income assets into a single new security, most commonly loans, but it could also be corporate bonds. This new security (the portfolio of loans) is then sold to investors.
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    - The loans we can include into such new security can be varied. Most prominent were the use of mortgages (Collaterised Mortgage Obligations, CMOs),
    - or corporate bonds, mostly junk bonds,
    - but we also frequently found car loans
    - and student loans (in the USA).
    - We have also seen the use of credit card receipts,
    - and many many more types of loans.
- While thus far it is only that a number of such loans are combined into a portfolio and sold off to investors, it is the way that losses are allocated that makes CDOs different. Losses are allocated to different parts of the newly issued securities by priority. these different parts are called 'tranches'.
- The first tranche (equity tranche), bears all initial losses from defaults on the original loans until these losses have completely eliminated the tranche.
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    - One the equity tranche (the first tranche) has been eliminated by losses, any further losses are then allocated to the second tranche (mezzanine tranche).
    - This tranche is then allocated all losses until they eliminate this tranche as well and then further losses are allocated to the next tranche. This process caries on through potentially various tranches all the way to the most senior tranche, which is the last tranche.
- It is therefore that losses are only allocated to a more senior tranche once all junior tranches have been eliminated; thus the higher the tranche, the less likely losses are and the tranche should be safer.

→ It were the most senior tranches, those that were to be affected by losses from defaulting loans last, that banks sold to investors. These tranches were regarded as being very safe and thus received AAA ratings.

Pricing collaterised debt obligations

- Collaterised debt obligations are structured like ordinary bonds with a coupon payment and uncertain repayment
- The pricing of CDOs consists of finding an appropriate coupon payment
- The expected repayment, including coupon payment and the repayment of the principal, has to equal the repayment from a risk-free bond of the same maturity
- Solving this equality, will result in a spread over the risk-free rate

Pricing collaterised debt obligations

- → We will not oitline the principles on which such CDOs can be valued.
- ▶
  - In principle, CDOs are bonds and they bear a coupon payment, similar to ordinary bonds.
  - The repayment of the bond is uncertain, however, as it will depend on the losses from defaults of the original loas that have accumulated.
- ▶ It is this coupon payment, which if the CDO is sold at face value determines the yield that constitutes the 'price' of the CDO, it represents the yield of the bond (CDO).
- ▶ The pricing principle is that the expected repayment of the CDOs, consisting of the coupon payments and the final repayment of the bond, has to be the same as that of a risk-free bond with the same maturity.
- ▶ While making such a calculation is not trivial, it can be conducted using numerical methods, and we obtain a yield of the CDO, which will consist of the risk-free rate and a spread to account for the risk of the CDO.
- → Conducting this pricing itself is more difficult in practice as the following considerations show.

Additional considerations in pricing CDOs

- The pricing is conducted similarly to credit default swaps, but additional factors have to be taken into account
- The fixed income instruments have different default rates, but it is commonly assumed default rates are identical
- The defaults of the fixed income instruments will be correlated, it is commonly assumed correlations are identical
- A tranche may be not repaid at all, fully repaid, or partially repaid, depending on the number of defaults of the entities included
- Using Monte-Carlo simulations, the spread of CDOs can be determined

Additional considerations in pricing CDOs

→ The pricing of CDOs is much more complex than for CDSs as the payments ro investors are affected by a number of other concerns.

- - The idea behind the pricing of CDOs is similar to that of a credit default swap; we need to determine the spread based on the probability of default.
  - However, there are a number of factors that complicate the derivation of the spread.
- - First each of the loans that is included into the CDO will have a different probability of default, which should be considered in the derivation of the spread.
  - As a simplification, it is assumed that all default rates are identical, such as the average default rate.
- - We also need to consider the correlation of defaults across the different loans as they will have an impact on the distribution of the number of defaults we have in a CDO.
  - Correlations will be different between loans, but we make the simplification that all correlations are identical, using the average correlation.
- - We have to consider not only the case that a tranche is not repaid fully with a fixed recovery rate as in the case of CDSs,
  - or it being repaid in full,
  - But a tranche might be partially repaid if the losses accumulated have exceeded the amount of more junior tranches, but there are not sufficient defaults to eliminate the tranche considered fully.
  - We will therefore have to consider the number of defaults in a tranche; this is where the correlation of defaults becomes relevant.
- Such pricing is complex to model and the spread of a CDO will in general be determined using computer simulations, also called Monte-Carlo simulations. What we effectively do is we use the probabilities of default and correlations across the assets and then through the use of random losses from the resulting distribution generate the payments an investor in the CDO would get and set the spread such that the average payment matches that from a risk-free bond.

→ We will not look at the details of this technique to determine the CDO spread, but will instead look

Key drivers of CDO spreads

- Higher default rates increase the spread due to the increased risk
- Higher correlations have an ambiguous effect on the spread
- For senior tranches the spread is increasing in the correlation
- For the equity tranche it is decreasing in the correlation
- Mezzanine tranches have an initial decrease and then increase in the correlation

Key drivers of CDO spreads

→ We will not look at the way such Mone-Carlo simulations can be conducted, but instead will immediately focus on the key results that emerge out of such an analysis of the spread.

- It is not surprising that a higher default rate will increase the spread incerasing. This is due to the higher risk of an investor making losses from such more frequent defaults. This applies to tranches of all seniority.
- We find that the effect of the correlation of defaults on the spread of CDOs depends on the seniority of the tranche we consider.
- We observe that the spread of the most senior tranches are increasing in the correlation of defaults.
- We observe that the spread of very junior and equity tranches are decreasing in the correlation of defaults.
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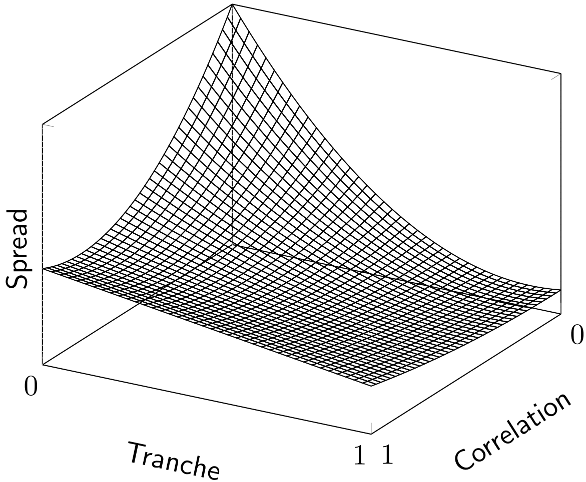
    - The spread of intermediate tranches are initially decreasing in the correlation of defaults,
    - but then once the correlation of defaults is sufficiently high, increase again.

→ We can now illustrate the size of the spread graphically.

Spread of collaterised debt obligations



Spread of collaterised debt obligations

- → We consider here the spread of a CDO as an example to illustrate the results introduced before.
  - The consider the spread of the CDO as determined by different correlations of defaults (we only consider positive correlations) and the seniority of tranches, where 0 denotes the equity tranche and 1 the most senior tranche.
  - We see here that the spread reduces for junior tranches as the correlation increases, while the spread increases for senior tranches. Of course, the spread os more senior tranches is lower than for more junior tranches as the junior tranches bear any losses from default first.
- → We will now seek to provide a rational for this result.

Defaults and correlation

- As the correlation increases, the likelihood of no losses at all increases, reducing the spread of the equity tranche
- At the same time the probability of high losses increases ("all-or-nothing"), causing the spread of senior tranches to increase
- For intermediate correlations, these two effects combine and as the correlation increases, the spread initially decreases and then increases again

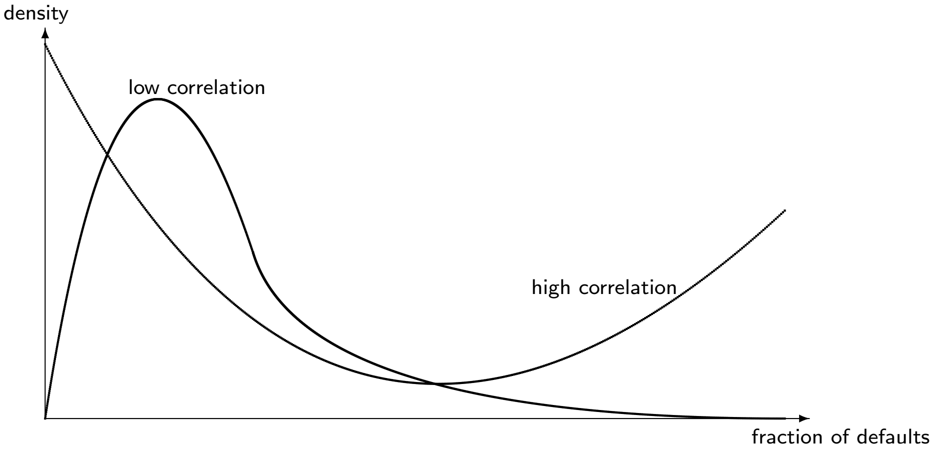
Defaults and correlation

→ We will now look at the impact different correlations have on the spread of different CDO tranches.

- ▶
  - If the correlation increases, then the likelihood of having no default at all is increasing. Suppose the correlation is 1; in this case either all loans default or all loans do not default, there will be no case of some loans defaulting. This now means that there is a reasonable chance of the equity tranche not suffering any losses; this change is the probability of the loans not defaulting. If ont he other hand, the correlation is lower, there will most likely be some losses as we can have situations where a small number of loans default while other do not default; it is very unlikely that no loan will default if the correlation is low. It is therefore very likely that the equity tranche will face a loss.
  - Thus a lower correlation implies a higher loss to the equity tranche and this will result in a higher spread as the correlation increases.
- ▶
  - The same mechanism works in the opposite direction for senior tranches. With a correlation of 1, the loans will either all be repaid or none is repaid.
  - This is a situation of either all loans default or no loan defaults.
  - With lower correlations, the chances of all loans defaulting is very low, less than with a correlation of 1. This implies that the spread with a high correlation should be higher as the losses are higher.
- ▶
  - For intermediate tranches, mezzanine tranches, both effects are relevant and we see that the initially the effect reducing the spread is stronger. An increase in the correlation makes it less likely that they are affected by losses to their tranches.
  - Once the correlation is sufficiently high, the effect increasing the spread dominates.

→ We can now illustrate the effect the correlation of defaults has on the fraction of defaults and hence the spread of the CDO tranches.

Fraction of defaults



Fraction of defaults

→ Let us now consider the fraction of defaulting loans in a CDO for a low and a high correlation.

- We look at the distribution of the fraction of defaults (density).
- Let us first consider the case of a low correlation. The underlying variable has Bernoulli distribution in that it is either default or not default. If these defaults are independent of each other, thus have a correlation of zero, combining a large number of such loans will result in a binomial distribution with a mean at the probability of default, which we had assumed to be identical for all loans.
- If we increase the correlation, the outcomes (default and no default, respectively), become more and more alike. With a high correlation it will be the case that either most loans are not defaulting or most loans are defaulting, it will be quite unlikely to be that some loan default, while others do not default; this gives rise to the shape of the distribution is indicated here.

→ We can now make use of this result in explaining better the results of the CDO spread we obtained above.

Higher correlations

- As correlations increase, it becomes more and more a situation where either no entity defaults or all entities default
- With low correlations a large number of defaults is unlikely and senior tranches are unlikely to face losses, while lower tranches are likely to face large losses
- High correlations reduce the risks of junior tranches as small number of defaults that only affect them become less likely
- Higher correlations make a large number of losses also more likely, increasing the risks to senior tranches

- → We can now look at the impact correlations of defaults have on the spread in more detail.
- ▶
  - As in the case of high correlations of default shown in the previous figure, a high correlation gives a situation in which either most loans are defaulting
  - or very few loans are defaulting. Intermediate cases will be rare.
- ▶
  - In contrast, with a low correlation of defaults, it is unlikely that we observe a large fraction of default (assuming the probability of default to be low).
  - In this case senior tranches are unlikely to face losses and will demand a low spread for such low risk.
  - More junior tranches are facing larger losses as shown in the distribution and will demand higher spreads to compensate for this higher risk.
- ▶ High correlations reduces the risk to junior tranches as a small number of losses that affects them only become less likely, apart from the very first part, the graph above shows the density high correlations to be below that of low correlations for the first part.
- ▶
  - Higher default correlations make it more likely that many loans are defaulting, some that with low correlations was unlikely to occur.
  - This increases the risk of senior tranches and hence increases the spread.
- → We thus see that the correlation of defaults is an important determinant of the spread of CDOs, in addition to the probability of default. By merely increasing this correlation, while keeping the probability of default constant, spreads can change.

The importance of correlations

- A higher default rate will lead to a higher spread
- Correlations between the default of entities are an important determinant of the spread of CDOs
- The impact differs depending on the seniority of the tranche
- The risks when holding collateralised debt obligations are difficult to evaluate

The importance of correlations

- → Default correlations are an important factor affecting the spread of CDOs.
- ▶ It is intuitu=ive clear that a higher default rate will increase the spread; or for a given spread, an increase in the default rate will reduce the value of all tranches.
- ▶ We have seen that the correlation of defaults is another key component in determining the spread, or for a given spread, the value of a tranche.
- ▶ What the impact is depends on the seniority of the tranche, a more senior tranche is likely to require a higher spread if the correlation increases, while a junior trance is likely to require a lower spread. Thus if the correlation increases, for a given spread, the value of a senior CDO tranche reduces while that of a junior tranche increases.
- ▶ Overall, the risks to the value of a CDO are complex and not easy to determine as the effects also depend on the seniority of the tranches, as well as the interaction between default probabilities and the correlation of default.
- → The absence of analytical solutions to the value of CDOs makes it difficult for investors to evaluate risks and the complexity of the interactions between default rates and correlations, makes a proper risk assessment difficult. With both parameters changing and often in the same direction, it the magnitude of the effect any changes have are difficult to predict and hedging against adverse movements become difficult.

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