



Credit default swaps

Outline

- Insuring default risk
- Default rates
- Valuing credit default swaps
- The relationship to discount rates for risky bonds
- Summary

- For a long time it was not possible to hedge effectively against credit risk. This was changed in the 1990s with the introduction of Credit Default Swaps.
- Credit Default Swaps make a payment to the purchaser that offsets any losses arising from the default of on a specific bond.
- We will seek to determine the value of this credit derivative.

- We will first explain how Credit Default Swaps work to insure the purchaser against credit risk, then discuss some properties of the default rate.
- Using this information, we then determine the value of Credit Default Swaps and how their value relates to the value of risky bonds.

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- We will first establish how Credit Default Swaps make payments in the case a bond defaults and how the purchasers pay for this protection.

Payment on default

- ▶ Credit default swaps make a payment to the buyer if the underlying entity defaults on its obligation
- ▶ The underlying entity can be companies or governments and the obligation is usually a bond they have issued
- ▶ The payment are determined as the losses arising from the default
- ▶ The loss is the amount that the entity does not pay to bondholders

- We first consider what payments the seller of the Credit Default Swap has to make in case of the underlying bond defaulting.
- ▶ The seller of a Credit Default Swap has to make a payment to the buyer if the entity having issued the bond defaults. What constitutes a default will be defined in the contract that is entered and may be case if any payment, including coupon payments, is not made at the designated time, or it might be if there is a minimum delay in payments, or the arrears exceed a certain threshold.
- ▶
 - The underlying entity is often a large listed company,
 - but in many cases also a government, developed countries as well as developing countries.
 - The Credit Default Swap is often tied to a specific bond, but in principle can be issued such that payment is triggered if the entity defaults on any of its obligations. Issuing Credit Default Swaps can be done without the consent of the entity.
- ▶ The payments the seller of the derivative has to make are the losses the holder of the bond has from this default.
- ▶ Losses are the value of the missed payments, less any partial payments the entity might have made.
- These payments are due as the entity defaults. It can be that the buyer obtain the full payment of all payments defaulted on and the seller then recovers any payments from the entity itself.

Credit default rate spread

- ▶ The buyer has to pay the seller of the credit default swap a premium, which is known as the spread
- ▶ The spread is paid in regular intervals until the default occurs
- ▶ The total amount the buyer pays will depend on the timing of the default

- Now that we have established what the payments from the credit default swap are, we can determine how the purchase price is set.
 - ▶
 - When purchasing a credit default swap, the buyer has to pay a premium to the seller for this protection,
 - this premium is called a 'spread'
 - ▶ Unlike in options, the spread is paid in regular intervals, such as annually or quarterly, and the spread is paid only until the default occurs and not for the entire length of the contract. It is thus more akin to an insurance contract which is also terminated once a claim has been accepted, in this case here the default has occurred.
 - ▶ It is therefore the total amount that the buyer has to pay will depend on when the default is, the later the default, the higher the payments overall. This is also different to options, where the premium is fixed even if an American option is exercised early.
- Knowing this different way of buyers paying for the credit default swap, we can now determine the spread.

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- Before being able to determine the spread that is required for a credit default swap, we consider some properties of the rate of default that will be used to determine the spread.

Hazard rate

- ▶ The probability of a company defaulting in a time interval is assumed to be linear in this time interval: $\text{Prob}(\text{default in } [t, t + \Delta t]) = h\Delta t$
- ▶ The probability of default is characterised by the hazard rate
- ▶ The hazard gives the probability of default in a single time period

- Closely related to the concept of default is the hazard rate, which represents the probability of default in a single time period, while probability of default covers the entire time until the end of the time horizon.
 - ▶ We assume that the probability of default grows linearly with the time in which default can happen; thus the probability of a default to happen in a given time interval is proportional to the length of the interval.
 - ▶ Apart from the time interval considered, the determinant of the probability of default is the hazard rate h .
 - ▶ The probability of default for a single time period, $\Delta t = 1$, is called the 'hazard rate'.
-

Probability of not defaulting

- ▶ The probability of not defaulting is the complement of defaulting:
 $\text{Prob}(\text{no default in } [t, t + \Delta t]) = 1 - h\Delta t$
 - ▶ The time to maturity of the credit default swap is the limit of interest for defaults and we divide this into a number of time periods: $\tau = N\Delta t$
 - ▶ Defaults are independent over time:
 $\text{Prob}(\text{no default until maturity}) = (1 - h\Delta t)^N$
 - ▶ We now increase the number of time periods, $N \rightarrow \infty$, requiring that the length of a time period reduces, $\Delta t \rightarrow 0$
- $\Rightarrow \text{Prob}(\text{no default until maturity}) = e^{-h\tau}$

Probability of not defaulting

- We will now determine the probability of the entity not defaulting during a number of time periods.
- ▶ As we had defined the probability of default in a given time interval, the probability of not defaulting is simply the complement of this expression.
- ▶
 - Credit Default Swaps provide protection only for a given period of time, until their maturity, which may or may not coincide with the maturity of the bond it seeks to protect. Any default occurring after the maturity of the credit default swap is not relevant for the value of the derivative.
 - Let us now assume that this time to maturity, τ , is divided into N time periods of length Δt .
- ▶ We assume that default are independent over time, this implies that we will not observe a default by the entity if it does not default in any of the N time periods.
- ▶
 - We now increase the number of time periods we consider by increasing N .
 - As the time to maturity is constant, this necessitates that the length of each time period Δt is reduced.
- ▶ [⇒] Taking this limit, we can show that the probability of observing no default until maturity is given as in the *formula*.
- Having established these properties of the defaults, we can now continue to determine the value of the credit default swap.

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- We can now determine the value of the credit default swap, which requires us to determine the payments the buyer needs to make, the spread.

Value of fee payments

- ▶ The spread is paid continuously until the maturity of the credit default swap, as long as the entity has not defaulted
- ▶ The spread payment needs to be discounted to its present value
- ▶
$$\begin{aligned}V_{\text{Fee}} &= \int_0^{\tau} s e^{-r(t-\tau)} e^{-h(t-\tau)} dt \\ &= s \int_0^{\tau} e^{-(r+h)(t-\tau)} dt \\ &= \frac{s}{r+h} (1 - e^{-(r+h)\tau})\end{aligned}$$

Value of fee payments

- We will first seek to determine the payments the buyer of the derivative has to make.
 - ▶
 - We had established that the spread is paid until the derivative matures,
 - unless a default occurs before that. We assume here that the spread is paid continuously
 - ▶ Any future payments need to be discounted to their present value to account for the different timings of payments.
 - ▶ *Formula*
 - ▶ We can take the spread s out of the integral and collect the exponents.
 - ▶ We can now conduct the integration and obtain this *formula*.
- This expression gives us the value of payments the buyer of a credit default swaps makes. This is also referred to as the 'fee leg'.

Contingent payment

- ▶ The seller pays the amount not recovered from the obligation of the entity, if the entity defaults
- ▶ The entity can default at a specific point of time, provided it has not defaulted before
- ▶ The payment is then discounted to its present value

$$\begin{aligned} V_{\text{Pay}} &= (1 - R) \int_0^{\tau} e^{-r(t-\tau)} h e^{-h(t-\tau)} dt \\ &= (1 - R) h \int_0^{\tau} e^{-(r+h)(t-\tau)} dt \\ &= (1 - R) h \frac{1 - e^{-(r+h)\tau}}{r+h} \\ &= (1 - R) \frac{h}{s} V_{\text{Fee}} \end{aligned}$$

- We can now turn to the payments the seller of the derivative makes. This payment occurs if the entity defaults.
- ▶ The seller has to make a payment if the entity defaults. The net payment the seller makes is what he cannot recover from the entity after this default. The fraction of the obligation that is recovered is denoted by R .
- ▶
 - The default happens at some point of time,
 - but only if there has been no default before. Naturally we need to exclude the possibility of having multiple defaults of the same entity.
- ▶ These future payments need to be discounted to their present value to account for the different timings of payments made, depending on the time of default.
- ▶ *Formula*
- ▶ We can take the hazard rate h out of the integral and collect the exponents.
- ▶ We can now conduct the integration and obtain this *formula*.
- ▶ Comparing with the fee leg previously, we can rewrite this so-called 'contingent leg' as shown in the *formula*.
- Now that we have determined the amounts paid by buyers and sellers, we can determine the spread that buyers need to be charged.

The CDS spread

- ▶ The credit default swap is priced fairly if the payments made by the buyer equal the payments they receive from the seller
- ▶ $V_{\text{Fee}} = V_{\text{Pay}}$
- ⇒ $s = (1 - R) h$
- ▶ The spread reflects the probability of default, adjusted for any repayments the entity might make

- The price of the credit default spread is the amount the buyer needs to pay until the entity defaults, the spread.
- ▶ A fair value for both sides is achieved, if the payments made by the buyer and the seller are equal; in this case neither side makes a gain from entering the agreement.
- ▶ *Formula*
- ▶ [⇒] We can solve this equality for the spread as shown in the *formula*.
- ▶
 - The spread consists of the hazard rate, that is the probability of default. Thus the payment the seller makes offsets exactly the defaults that might occur.
 - This is adjusted only for the recovery of payments from the entity.
- The spread of a credit default swap reflects the defaults risk of the entity. We can now look at the relationship of this spread with the pricing of a corporate bond.

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- We will now consider how a risky bond, such as a corporate bond is priced and use the spread of a credit default swap to adjust the discount rate of this bond.

The value of a risky zero bond

- ▶ A zero bond does not make coupon payments, but only repays its face value at maturity
- ▶ Its value is the present value of this future repayment
- ▶ If the entity does not default before maturity, it will make a full repayment
- ▶ If the entity does default before maturity, it will only make a partial repayment
- ▶
$$B = (e^{-h\tau} + (1 - e^{-h\tau}) R) e^{-r\tau}$$
$$= e^{-(r+h)\tau} (1 - R) + e^{-r\tau} R$$

The value of a risky zero bond

- We will first determine the value of a risky bond before relating introducing the spread.
- ▶
 - For simplicity we consider a zero bond, that is a bond which makes no coupon payment,
 - but is only repaid at maturity at face value.
- ▶ The value of this bond is then the present value of this future repayment.
- ▶ The bond is risky in that it might not be repaid, thus the entity issuing the bond might default. If there is no default the bond is repaid in full.
- ▶ However, if the entity defaults, then the bond will be repaid only partially.
- ▶ *Formula*
- ▶ We can consolidate terms and rewrite the value of the bond as given in the *formula*.
- We can now make more transformations and relate this value to the spread of a credit default swap.

CDS spreads as measuring bond risks

$$\begin{aligned} &= e^{-(r+h)\tau} (1 - R) + e^{-(r+h)\tau} e^{h\tau} R \\ &\approx e^{-(r+h)\tau} (1 - R) + e^{-(r+h)\tau} (1 + h\tau) R \\ &= e^{-(r+h)\tau} (1 + hR\tau) \\ &\approx e^{-(r+h)\tau} e^{hR\tau} \\ &= e^{-(r+h(1-R))\tau} \\ &= e^{-(r+s)\tau} \end{aligned}$$

- ⇒ The discount for a risky bond is the risk-free rate adjusted by the spread
- ▶ The spread of credit default swaps represents the risk of the bond

- We now conduct a number of transformations to introduce the spread.
 - ▶ We firstly can expand the second term as shown here.
 - ▶ The exponential in the second term can be approximated as shown here as long as the hazard rate h and time to maturity τ are not too large.
 - ▶ We now collect terms
 - ▶ and then reverse the approximation made previously.
 - ▶ We now can collect these terms.
 - ▶ We can now use that $s = h(1 - R)$ and introduce the spread into the equation.
 - ▶ [⇒] This final expression represents the present value of the future payment if the discount rate is $r + s$, thus the risk-free rate adjusted by the spread.
 - ▶ The spread represents the risk of the bond and hence the discount rate needs to be adjusted from the risk-free rate to account for this risk.
- It is therefore that the value of corporate bonds is given by discounting at the risk-free rate plus the CDS spread. Transferring this to a bond with coupon payments, the coupons should have a rate of $r + s$ if the bond is to be issued at face value.

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Fair credit default swap spreads

- ▶ Payments on credit default swaps can be determined by comparing the payments a buyer makes to the seller and payments received from the seller
- ▶ The spread reflects the default risk, adjusted for any partial repayment if the entity defaults
- ▶ The spread is independent of the time to maturity as regular payments are made until maturity or default
- ▶ The spread is not affected by the risk-free rate as spread payments are discounted as is the bond repayment

- We can now summarize the main results we have obtained about credit default swaps.
- ▶ We have seen that credit default swaps are priced such that the fees paid by the buyers equals the payments they expect to receive from the seller.
 - ▶
 - The resulting spread reflects the default risk of the entity
 - and is only adjusted for any partial payment made in the event of default.
 - ▶ The time to maturity of the credit default swap is irrelevant for the spread, unlike the option premium. This is because option premia are paid for the entire time to maturity, but the spread is paid continuously either until maturity or a default happening, thus if the maturity is longer, the payment would be made over a longer period of time; this allows the spread to be independent of the time to maturity.
 - ▶ There is also no inclusion of the risk-free rate and that is because spread payments made by buyers as well as those payments made by sellers, are discounted; these effects cancel each other out.
- We have thus established that the CDS spread is compensation for the risk of default and the associated costs of this default.

Relationship to bond pricing

- ▶ The spread of credit default swaps reflects the default risk of the bond of the entity
- ▶ The discount rate applied to risky bonds is adjusted by the CDS spread to capture the default risk
- ▶ As corporate bonds and non-investment grade government bonds are not very liquid, using this relationship for arbitrage to exploit mispricings is difficult

- We can also use CDS spreads, where available, to determine the appropriate discount rate of risky bonds.
- ▶ The CDS spread captures the costs of default, the probability of default and the likely costs of such a default.
- ▶ We have seen that the discount rate of a corporate bond consists of the risk-free rate and the CDS spread to account for the risk of the bond.
- ▶
 - In principle we could use credit derivatives and the relevant bond to develop an arbitrage strategy and exploit any mispricings between the two, but corporate bonds are usually not very liquid and hence prices react sensitively to any demand for such bonds.
 - Due to such price adjustments due to the demand arising from arbitrage, any trading strategy might not be profitable and mispricings can persist.
- Thus CDS spreads can be used not only to hedge default risk, but can also be used as an indicator for the default risk of the underlying entity.



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