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Credit default swaps
NEW YORK STOCK EXCHANGE
LIVERMORE

Outline

- Insuring default risk
- Default rates
- Valuing credit default swaps
- The relationship to discount rates for risky bonds
- Summary

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Payment on default

- Credit default swaps make a payment to the buyer if the underlying entity defaults on its obligation
- The underlying entity can be companies or governments and the obligation is usually a bond they have issued
- The payment are determined as the losses arising from the default
- The loss is the amount that the entity does not pay to bondholders

Credit default rate spread

- The buyer has to pay the seller of the credit default swap a premium, which is known as the spread
- The spread is paid in regular intervals until the default occurs
- The total amount the buyer pays will depend on the timing of the default

Hazard rate

- The probability of a company defaulting in a time interval is assumed to be linear in this time interval: Prob(default in $[t, t + \Delta t]) = h\Delta t$
- The probability of default is characterised by the hazard rate
- The hazard gives the probability of default in a single time period

Probability of not defaulting

- The probability of not defaulting is the complement of defaulting:
  Prob (no default in $[t, t + \Delta t]$) $= 1 - h\Delta t$
- The time to maturity of the credit default swap is the limit of interest for defaults and we divide this into a number of time periods: $\tau = N\Delta t$
- Defaults are independent over time:
  Prob (no default until maturity) $= (1 - h\Delta t)^N$
- We now increase the number of time periods, $N \to \infty$, requiting that the length of a time period reduces, $\Delta t \to 0$

$\Rightarrow$ Prob (no default until maturity) $= e^{-h\tau}$

Value of fee payments

- The spread is paid continuously until the maturity of the credit default swap, as long as the entity has not defaulted
- The spread payment needs to be discounted to its present value
- $$\begin{aligned} V_{\text{Fee}} &= \int_0^\tau s e^{-r(t-\tau)} e^{-h(t-\tau)} dt \\ &= s \int_0^\tau e^{-(r+h)(t-\tau)} dt \\ &= \frac{s}{r+h} \left(1 - e^{-(r+h)\tau}\right) \end{aligned}$$

Contingent payment

- The seller pays the amount not recovered from the obligation of the entity, if the entity defaults
- The entity can default at a specific point of time, provided it has not defaulted before
- The payment is then discounted to its present value
- $$\begin{aligned} V_{\mathsf{Pay}} &= (1-R)\int_0^\tau e^{-r(t-\tau)} h e^{-h(t-\tau)} dt \\ &= (1-R)\, h \int_0^\tau e^{-(r+h)(t-\tau)} dt \\ &= (1-R)\, h \frac{1-e^{-(r+h)\tau}}{r+h} \\ &= (1-R)\, \frac{h}{s} V_{\mathsf{Fee}} \end{aligned}$$

The CDS spread

- The credit default swap is priced fairly if the payments made by the buyer equal the payments they receive from the seller
- $V_{\text{Fee}} = V_{\text{Pay}}$

$\Rightarrow$ $s = (1 - R)\, h$

- The spread reflects the probability of default, adjusted for any repayments the entity might make

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The value of a risky zero bond

- A zero bond does not make coupon payments, but only repays its face value at maturity
- Its value is the present value of this future repayment
- If the entity does not default before maturity, it will make a full repayment
- If the entity does default before maturity, it will only make a partial repayment
- $B = \left(e^{-h\tau} + \left(1 - e^{-h\tau}\right) R\right) e^{-r\tau}$
  $= e^{-(r+h)\tau} (1 - R) + e^{-r\tau} R$

CDS spreads as measuring bond risks

$$
\begin{aligned}
&= e^{-(r+h)\tau}\left(1-R\right) + e^{-(r+h)\tau}e^{h\tau}R \\
&\approx e^{-(r+h)\tau}\left(1-R\right) + e^{-(r+h)\tau}(1+h\tau)\,R \\
&= e^{-(r+h)\tau}\left(1+hR\tau\right) \\
&\approx e^{-(r+h)\tau}e^{hR\tau} \\
&= e^{-(r+h(1-R))\tau} \\
&= e^{-(r+s)\tau}
\end{aligned}
$$

$\Rightarrow$ The discount for a risky bond is the risk-free rate adjusted by the spread

- The spread of credit default swaps represents the risk of the bond

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Fair credit default swap spreads

- Payments on credit default swaps can be determined by comparing the payments a buyer makes to the seller and payments received from the seller
- The spread reflects the default risk, adjusted for any partial repayment if the entity defaults
- The spread is independent of the time to maturity as regular payments are made until maturity or default
- The spread is not affected by the risk-free rate as spread payments are discounted as is the bond repayment

Relationship to bond pricing

- The spread of credit default swaps reflects the default risk of the bond of the entity
- The discount rate applied to risky bonds is adjusted by the CDS spread to capture the default risk
- As corporate bonds and non-investment grade government bonds are not very liquid, using this relationship for arbitrage to exploit mispricings is difficult

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