Credit default swaps

Andreas Krause

Outline

- Insuring default risk
- Default rates
- Valuing credit default swaps
- The relationship to discount rates for risky bonds
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