Portfolio insurance

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Aim of portfolio insurance

- Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- This can be driven by regulatory requirements or the need to meet given obligations, for example pension payments
- In principle derivatives can be used to achieve this aim as they allow to hedge these risks
- In many cases derivatives are not readily available or the number of instruments that need to be used is prohibitively high

Buy-and-hold strategy

- An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
- $V_T \geq \alpha V_0$
- The investor invests into the risk-free asset so this final value is ensured is ensured: $B_t = \alpha V_0 e^{-r(T-t)}$

$\Rightarrow$ $B_T = \alpha V_0$

- The remainder is invested into the risky asset: $C_t = V_t - B_t = V_t - \alpha V_0 e^{-r(T-t)}$
- This implies usually a low investment into risky assets or even a short position

Maximum loss of the risky asset

- Assume the risky asset cannot make losses exceeding a certain threshold before it can be liquidated
- This risk of such a loss may evaluated using Value-at-Risk or other risk measures
- With Value-at-Risk as a risk measure this threshold would be the reasonable loss the investor could make, the Value-at-Risk itself

An alternative investment strategy

- The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- The investor will always be guaranteed to obtain the floor, plus any accumulated interest
- The investment strategy is to invest mC_t into the risky asset and $B_t = V_t - mC_t$ into the risk-free asset

Worst case scenario

- The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain
- $V_{t+1} \geq V_t - \gamma m C_t + r\left(V_t - mC_t\right)$

$$\Rightarrow V_{t+1} = C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r\left(V_t - mC_t\right)$$
$$= C_t + B_t - \gamma m C_t + r\left(C_t + B_t - mC_t\right)$$
$$= C_t\left(1 + r - m\left(\gamma + r\right)\right) + \left(1 + r\right) B_t$$

- If we keep the investments constant, then $B_{t+1} = \left(1 + r\right) B_t$

$$\Rightarrow C_{t+1} \geq C_t\left(1 + r - m\left(\gamma + r\right)\right)$$

Optimal investment into the risky asset

- If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = \left(1 - \alpha e^{-rT}\right) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$

$\Rightarrow$ $1 + r - m(\gamma + r) > 0$

$\Rightarrow$ $m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$

- As assets can at most lose all their value, $\gamma < 1$ and $m > 1$
- The investor can invest more into the risk-free asset than in a buy-and-hold strategy

Constant proportion portfolio insurance

- As the investment consists of a fixed multiple m of the cushion invested into the risky asset, it is called the Constant Proportion Portfolio Insurance (CPPI)
- As the cushion changes every time period due to the value of the risky asset changing, the amounts invested into risky assets changes constantly
- This requires a continuous adjustment of the wights between risky and risk-free assets

CPPI as a hedging tool

- As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0
- As not the entire investment is in the risky asset, the payoff at maturity will be different to that of a hedge with put options, but no premium is payable either
- Transaction costs of adjusting the portfolio constantly can be prohibitive
- CPPI can be used as an alternative to the use of derivatives or where derivatives are not available

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