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Portfolio insurance

# Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by regulatory requirements or the need to meet given obligations, for example pension payments
- ▶ In principle derivatives can be used to achieve this aim as they allow to hedge these risks
- ▶ In many cases derivatives are not readily available or the number of instruments that need to be used is prohibitively high

## Buy-and-hold strategy

- ▶ An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
- ▶  $V_T \geq \alpha V_0$
- ▶ The investor invests into the risk-free asset so this final value is ensured is ensured:  $B_t = \alpha V_0 e^{-r(T-t)}$
- ⇒  $B_T = \alpha V_0$
- ▶ The remainder is invested into the risky asset:  $C_t = V_t - B_t = V_t - \alpha V_0 e^{-r(T-t)}$
- ▶ This implies usually a low investment into risky assets or even a short position

## Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses exceeding a certain threshold before it can be liquidated
- ▶ This risk of such a loss may evaluated using Value-at-Risk or other risk measures
- ▶ With Value-at-Risk as a risk measure this threshold would be the reasonable loss the investor could make, the Value-at-Risk itself

## An alternative investment strategy

- ▶ The amount not invested into the risk-free asset,  $C_t = V_t - B_t$ , is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be guaranteed to obtain the floor, plus any accumulated interest
- ▶ The investment strategy is to invest  $mC_t$  into the risky asset and  $B_t = V_t - mC_t$  into the risk-free asset

## Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain
- ▶  $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$
- ⇒ 
$$\begin{aligned} V_{t+1} &= C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t) \\ &= C_t + B_t - \gamma m C_t + r (C_t + B_t - m C_t) \\ &= C_t (1 + r - m (\gamma + r)) + (1 + r) B_t \end{aligned}$$
- ▶ If we keep the investments constant, then  $B_{t+1} = (1 + r) B_t$
- ⇒  $C_{t+1} \geq C_t (1 + r - m (\gamma + r))$

# Optimal investment into the risky asset

- ▶ If the investor starts with a cushion  $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$ , a positive cushion ensures that the total investments meet the condition that  $V_T \geq \alpha V_0$ 
  - ⇒  $1 + r - m(\gamma + r) > 0$
  - ⇒  $m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$
- ▶ As assets can at most lose all their value,  $\gamma < 1$  and  $m > 1$
- ▶ The investor can invest more into the risk-free asset than in a buy-and-hold strategy

## Constant proportion portfolio insurance

- ▶ As the investment consists of a fixed multiple  $m$  of the cushion invested into the risky asset, it is called the Constant Proportion Portfolio Insurance (CPPI)
- ▶ As the cushion changes every time period due to the value of the risky asset changing, the amounts invested into risky assets changes constantly
- ▶ This requires a continuous adjustment of the weights between risky and risk-free assets

## CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of  $\alpha V_0$
- ▶ As not the entire investment is in the risky asset, the payoff at maturity will be different to that of a hedge with put options, but no premium is payable either
- ▶ Transaction costs of adjusting the portfolio constantly can be prohibitive
- ▶ CPPI can be used as an alternative to the use of derivatives or where derivatives are not available



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