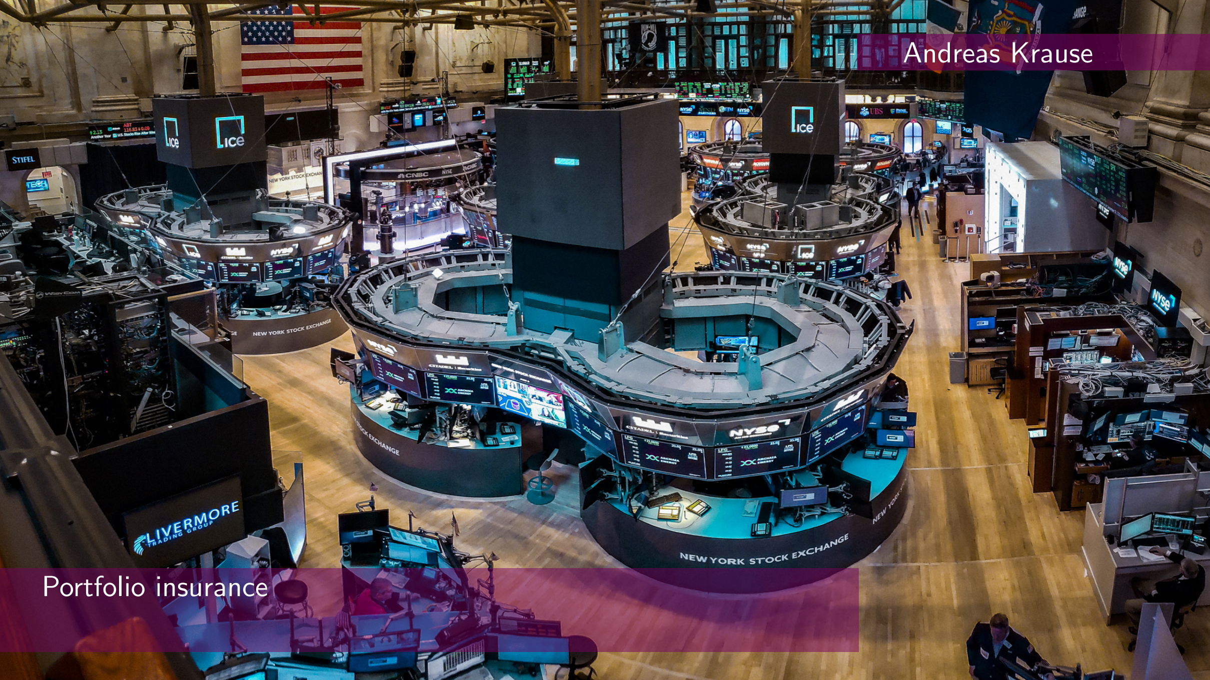
Andreas Krause

Portfolio insurance



- When having a portfolio, it is often desirable to ensure that its value does not below a certain threshold.
- Such an investment strategy can be achieved in principle by using options to hedge positions, but if the portfolio consists of many different assets on which options are not available and it is not closely matching an index on which options are traded, this strategy is not possible.
- we will therefore here look at a different investment strategy to achieve the desired outcome, called 'portfolio insurance'.

Aim of portfolio insurance

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- Derivatives, in particular options, can be used in principle to achieved this desired outcome.
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  - and holding a large number of assets would require holding a large number of different derivatives, which increases costs substantially,

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- We define the minimum value of the portfolio that the investor needs to achieve as a fraction α of the current value of the portfolio.
- Thus at the specified time in the future, the value of the portfolio has to exceed this threshold.
- Let us now assume that the investor would invest into the risk-free asset. The value of risk-free asset is given by the present value of the repayment at maturity (the time horizon relevant for the portfolio insurance). The amount to be repaid we suggest to be the threshold of the investor value.
- [⇒] At maturity, the relevant date for the portfolio insurance, the risk-free bond would be worth exactly this threshold.
- We now propose that the investor investors the remainder of his funds into a risky asset. This remainder at any time, C_t, will be the difference of the value of the portfolio less the investment into the risk-free bond. We can insert for the value of the risk-free bond to obtain the final equality.
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→ Such a buy-and-hold strategy seems not to be very efficient in that the use of risky assets,w hich usually have higher returns, is very limited.

Maximum loss of the risky asset

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→ We will now introduce an additional assumption on the losses of the risky asset and develop an alternative investment strategy.

- We will assume that losses on risky assets are limited; if negative information about an asset is revealed, we assume that while the investor would make some losses, these losses are limited as he would be able to sell the asset quickly. In a well-diversified portfolio, losses might also be limited as idiosyncratic risk only has a small impact on losses.
- How we determine the maximum loss we attribute to the risky asset is to be decided by the investor. It might be that they choose the value-at-Risk as the reasonable loss an asset might accumulate over a short time period until it can be sold. Other risk measures can also be used..
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    - Using value-at-Risk would allow us to use the quantile chosen as the threshold below which we find it unlikely that the value of the risky asset would fall.
    - The possible posses would be the Value-at-Risk.

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An alternative investment strategy

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→ We now develop the idea of an investment strategy that allows a larger investment into the risky asset.

- - In terms of terminology, the in portfolio insurance it has become common to call the amount invested into the risky asset the 'cushion'
  - and the investment into the risk-free asset the 'floor'. The floor gets its name from the fact that this value is guaranteed, it being a risk-free asset and the investment in the risky asset is the amount that exceed this threshold and can thus be lost without affecting the minimum requirement. It provides a cusion for any losses the investemnts might make.
- - As the floor consists of the risk-free asset, this amount is guaranteed to the investors,
  - we have to add any accumulated interest until the time horizon; we will therefore take the present value of all such payments.
- - We now propose that the investment into the risky asset is not only the cushion, C_t, but a larger amount by adding a factor m.
  - Correspondingly less is invested into the risk-free asset.

→ We will now have to determine what the optimal value for this multiplier m is.

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Worst case scenario

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→ We have to ensure that the minimum value of the portfolio is always achieved. Thus we have to look at a worst-case scenario and if the value of the portfolio exceeds its threshold, the investment strategy is viable.

- Such a worst case scenario is if the risky asset incurs a loss equal to its maximum possible loss as determined above. The risk-free asset does not incur any loss, hence its value remains stable.
- We now know that the value of the portfolio in the next time period will be at least the value of the portfolio now, less the worst-case losses of the investment into the risky asset, where the worst-case loss is γ. The investment into the risk-free asset is accumulating interest during this time period, which will increase the value of the portfolio.
- [$\Rightarrow$] We know that the portfolio will consist of the cushion and the risk-free asset.
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- Let us assume we do not make any changes to investments, then the value of the risk-free asset increases by the interest that has been paid for that time period. This amount is then re-invested into the risk-free asset.
- [$\Rightarrow$] If we insert this last relationship, we can get the cushion in the next period must be at least the expression in the *formula*.

→ We can now use these values to determine the optimal multiplier to invest into the risky asset.

Worst case scenario

- The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain
- $V_{t+1} \geq V_t - \gamma m C_t + r\left(V_t - mC_t\right)$

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$$\Rightarrow V_{t+1} = C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r\left(V_t - mC_t\right)$$
$$= C_t + B_t - \gamma m C_t + r\left(C_t + B_t - mC_t\right)$$
$$= C_t\left(1 + r - m\left(\gamma + r\right)\right) + \left(1 + r\right) B_t$$

- If we keep the investments constant, then $B_{t+1} = \left(1 + r\right) B_t$

$$\Rightarrow C_{t+1} \geq C_t\left(1 + r - m\left(\gamma + r\right)\right)$$

Worst case scenario

→ We have to ensure that the minimum value of the portfolio is always achieved. Thus we have to look at a worst-case scenario and if the value of the portfolio exceeds its threshold, the investment strategy is viable.

- Such a worst case scenario is if the risky asset incurs a loss equal to its maximum possible loss as determined above. The risk-free asset does not incur any loss, hence its value remains stable.
- We now know that the value of the portfolio in the next time period will be at least the value of the portfolio now, less the worst-case losses of the investment into the risky asset, where the worst-case loss is γ. The investment into the risk-free asset is accumulating interest during this time period, which will increase the value of the portfolio.
- [⇒] We know that the portfolio will consist of the cushion and the risk-free asset.
- [] We can now also replace the current value of the portfolio as being a combination of the cushion and risk-free asset (floor).
- [] Collecting terms, gives us the *formula*.
- Let us assume we do not make any changes to investments, then the value of the risk-free asset increases by the interest that has been paid for that time period. This amount is then re-invested into the risk-free asset.
- [⇒] If we insert this last relationship, we can get the cushion in the next period must be at least the expression in the *formula*.

→ We can now use these values to determine the optimal multiplier to invest into the risky asset.

Optimal investment into the risky asset

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→ We can now determine how much should optimally be invested into the risky asset.

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Constant proportion portfolio insurance

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    - The multiplier m is assumed to be fixed, or constant, and this investment strategy is therefore called Constant Proportion Portfolio Insurance,
    - or often known under its abbreviation.
- As the value of the risky asset changes all the time, and the risk-free asset accumulated interest, the value of the portfolio changes and hence the amount invested into risky assets change. Note that we assumed that the investment into the risk-free asset was constant, we only re-invest the accumulated interest. Therefore, the amount invested into the risky asset changes every time period.
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CPPI as a hedging tool

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→ We now put CPPI in context of the hedging strategies involving derivatives.

- We argue that CPPI is similar to using options to hedge; while the minimum value is guaranteed, a positive development of the value of the risky asset will generate a higher portfolio value. This is similar to the use of options, where losses are limited, but profits are retained ny the investor. Thus CPPI is like purchasing a put option with a strike price at the threshold.
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  - The investment into the risk-free asset will alter the final value of the portfolio compared to the use of put options, hence the two are not the same startegy.
  - In addition, there is no option premium payable when using CPPI.
- The need to adjust the holding of the risky asset(s) constantly can be very costly and may make CPPI not sustainable.
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  - However, where costs are not prohibitive, CPPI can be an alternative to hedging with put options or other derivatives.
  - It will be particularly attractive if options on the assets held are not available.

→ We have thus developed an investment strategy that ensures a minimum value of the portfolio at the end of the time horizon, while maintaining to a large extend the optimal portfolio that reflects the preferences of the investor due to a higher investment into risky assets than a simple buy-and hold strategy. However, to achieve this we had to assume that losses on the risky asset are limited; if this condition is not fulfilled, CPPI has no advantage over a simple buy-and hold strategy.

CPPI as a hedging tool

- As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0

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