



Andreas Krause

Portfolio insurance

- When having a portfolio, it is often desirable to ensure that its value does not below a certain threshold.
- Such an investment strategy can be achieved in principle by using options to hedge positions, but if the portfolio consists of many different assets on which options are not available and it is not closely matching an index on which options are traded, this strategy is not possible.
- we will therefore here look at a different investment strategy to achieve the desired outcome, called 'portfolio insurance'.

Aim of portfolio insurance

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- We now briefly discuss what portfolio insurance seeks to achieve.
- ▶ Risk management alone usually does not eliminate losses with certainty. Using, for example Value-at-Risk as a risk measure and adjusting the risk of the portfolio, will not eliminate the possibility of losses, it might only reduce the probability of these occurring. Thus these measures are ineffective to obtain certainty that losses beyond a certain threshold do not occur at all.
 - ▶ One not unusual scenario is that at a given point in the future, the value of a portfolio must be at least a certain amount.
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 - There might be regulatory reasons why losses cannot exceed a certain threshold, such as capital requirements for banks or solvency regulation for insurance companies. Thus their investment strategies must ensure that at the relevant point in the future, the losses accumulated are not exceeding their limit.
 - Even where no regulatory restrictions exist, investors might want to ensure the portfolio has a minimum value, that is a losses are limited, to meet any future obligations they have.
 - These obligations might be payments to pensioners, but also the repayment of a mortgage for individual investors.
 - ▶ Derivatives, in particular options, can be used in principle to achieved this desired outcome.
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 - Using derivatives is often not feasible, as derivatives might not be available for the assets held,
 - and holding a large number of assets would require holding a large number of different derivatives, which increases costs substantially,
- Thus in order to achieve the aim of portfolio insurance a different methodology needs to be used.

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- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does **not prevent losses**

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Buy-and-hold strategy

- After setting out the aim of portfolio insurance more formally, we look at the properties of a buy-and-hold strategy that yield the desired outcome.
 - ▶ We define the minimum value of the portfolio that the investor needs to achieve as a fraction α of the current value of the portfolio.
 - ▶ Thus at the specified time in the future, the value of the portfolio has to exceed this threshold.
 - ▶ Let us now assume that the investor would invest into the risk-free asset. The value of risk-free asset is given by the present value of the repayment at maturity (the time horizon relevant for the portfolio insurance). The amount to be repaid we suggest to be the threshold of the investor value.
- ⇒ At maturity, the relevant date for the portfolio insurance, the risk-free bond would be worth exactly this threshold.
 - ▶ We now propose that the investor invests the remainder of his funds into a risky asset. This remainder at any time, C_t , will be the difference of the value of the portfolio less the investment into the risk-free bond. We can insert for the value of the risk-free bond to obtain the final equality.
 - If the threshold is not too low, thus α significantly above 0, the investment into the risk-free asset will be substantial, leaving not much investment into the risky asset.
 - If α is close to 1, it might even be that the investment required is a short position as C_t becomes negative.
- Such a buy-and-hold strategy seems not to be very efficient in that the use of risky assets, which usually have higher returns, is very limited.

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- ▶ An investor wants to ensure that the **final value** of his **investment** exceeds some **threshold**, expressed relative to his current value
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 - If α is close to 1, it might even be that the investment required is a short position as C_t becomes negative.
- Such a buy-and-hold strategy seems not to be very efficient in that the use of risky assets, which usually have higher returns, is very limited.

Maximum loss of the risky asset

Maximum loss of the risky asset

- We will now introduce an additional assumption on the losses of the risky asset and develop an alternative investment strategy.
- ▶ We will assume that losses on risky assets are limited; if negative information about an asset is revealed, we assume that while the investor would make some losses, these losses are limited as he would be able to sell the asset quickly. In a well-diversified portfolio, losses might also be limited as idiosyncratic risk only has a small impact on losses.
- ▶ How we determine the maximum loss we attribute to the risky asset is to be decided by the investor. It might be that they choose the value-at-Risk as the reasonable loss an asset might accumulate over a short time period until it can be sold. Other risk measures can also be used..
- ▶
 - Using value-at-Risk would allow us to use the quantile chosen as the threshold below which we find it unlikely that the value of the risky asset would fall.
 - The possible losses would be the Value-at-Risk.
- With this assumption, we can now revisit our buy-and-hold strategy and improve on it.

Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses **exceeding a certain threshold** before it can be liquidated

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Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses exceeding a certain threshold before it can be liquidated
- ▶ This risk of such a loss may be evaluated using **Value-at-Risk** or other risk measures

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- ▶ With Value-at-Risk as a risk measure this threshold would be the **reasonable loss** the investor could make

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An alternative investment strategy

An alternative investment strategy

→ We now develop the idea of an investment strategy that allows a larger investment into the risky asset.

- ▶
 - In terms of terminology, the in portfolio insurance it has become common to call the amount invested into the risky asset the 'cushion'
 - and the investment into the risk-free asset the 'floor'. The floor gets its name from the fact that this value is guaranteed, it being a risk-free asset and the investment in the risky asset is the amount that exceed this threshold and can thus be lost without affecting the minimum requirement. It provides a cushion for any losses the investments might make.
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 - As the floor consists of the risk-free asset, this amount is guaranteed to the investors,
 - we have to add any accumulated interest until the time horizon; we will therefore take the present value of all such payments.
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 - We now propose that the investment into the risky asset is not only the cushion, C_t , but a larger amount by adding a factor m .
 - Correspondingly less is invested into the risk-free asset.

→ We will now have to determine what the optimal value for this multiplier m is.

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a **cushion**

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- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the **floor**

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An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be **guaranteed** to obtain the floor

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An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be guaranteed to obtain the floor, plus any accumulated interest

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- ▶ The investment strategy is to invest mC_t into the **risky asset**

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Worst case scenario

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- We have to ensure that the minimum value of the portfolio is always achieved. Thus we have to look at a worst-case scenario and if the value of the portfolio exceeds its threshold, the investment strategy is viable.
- ▶ Such a worst case scenario is if the risky asset incurs a loss equal to its maximum possible loss as determined above. The risk-free asset does not incur any loss, hence its value remains stable.
- ▶ We now know that the value of the portfolio in the next time period will be at least the value of the portfolio now, less the worst-case losses of the investment into the risky asset, where the worst-case loss is γ . The investment into the risk-free asset is accumulating interest during this time period, which will increase the value of the portfolio.
- ⇒ We know that the portfolio will consist of the cushion and the risk-free asset.
 - We can now also replace the current value of the portfolio as being a combination of the cushion and risk-free asset (floor).
 - Collecting terms, gives us the *formula*.
- ▶ Let us assume we do not make any changes to investments, then the value of the risk-free asset increases by the interest that has been paid for that time period. This amount is then re-invested into the risk-free asset.
- ⇒ If we insert this last relationship, we can get the cushion in the next period must be at least the expression in the *formula*.
- We can now use these values to determine the optimal multiplier to invest into the risky asset.

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain
- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r(V_t - m C_t)$

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- $\Rightarrow V_{t+1} = C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$

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- ▶ If we keep the investments constant, then $B_{t+1} = (1 + r) B_t$

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$$\Rightarrow C_{t+1} \geq C_t (1 + r - m (\gamma + r))$$

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Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain

- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$

$$\begin{aligned}\Rightarrow V_{t+1} = C_{t+1} + B_{t+1} &\geq V_t - \gamma m C_t + r (V_t - m C_t) \\ &= C_t + B_t - \gamma m C_t + r (C_t + B_t - m C_t) \\ &= C_t (1 + r - m (\gamma + r)) + (1 + r) B_t\end{aligned}$$

- ▶ If we keep the investments constant, then $B_{t+1} = (1 + r) B_t$

$$\Rightarrow C_{t+1} \geq C_t (1 + r - m (\gamma + r))$$

Worst case scenario

- We have to ensure that the minimum value of the portfolio is always achieved. Thus we have to look at a worst-case scenario and if the value of the portfolio exceeds its threshold, the investment strategy is viable.
- ▶ Such a worst case scenario is if the risky asset incurs a loss equal to its maximum possible loss as determined above. The risk-free asset does not incur any loss, hence its value remains stable.
- ▶ We now know that the value of the portfolio in the next time period will be at least the value of the portfolio now, less the worst-case losses of the investment into the risky asset, where the worst-case loss is γ . The investment into the risk-free asset is accumulating interest during this time period, which will increase the value of the portfolio.
- ⇒ We know that the portfolio will consist of the cushion and the risk-free asset.
 - We can now also replace the current value of the portfolio as being a combination of the cushion and risk-free asset (floor).
 - Collecting terms, gives us the *formula*.
- ▶ Let us assume we do not make any changes to investments, then the value of the risk-free asset increases by the interest that has been paid for that time period. This amount is then re-invested into the risk-free asset.
- ⇒ If we insert this last relationship, we can get the cushion in the next period must be at least the expression in the *formula*.
- We can now use these values to determine the optimal multiplier to invest into the risky asset.

Optimal investment into the risky asset

- We can now determine how much should optimally be invested into the risky asset.
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 - The initial cushion will be the value of the portfolio less a full investment into the risk-free asset such that at the time horizon the threshold is guaranteed.
 - We can collect terms and obtain the *formula*.
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- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a **positive cushion** ensures that the total investments meet the condition that $V_T \geq \alpha V_0$

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Constant proportion portfolio insurance

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- We can now look at a key property of this investment strategy, namely that it requires constant adjustment of the holding of risky assets.
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 - The multiplier m is assumed to be fixed, or constant, and this investment strategy is therefore called Constant Proportion Portfolio Insurance,
 - or often known under its abbreviation.
- ▶ As the value of the risky asset changes all the time, and the risk-free asset accumulated interest, the value of the portfolio changes and hence the amount invested into risky assets change. Note that we assumed that the investment into the risk-free asset was constant, we only re-invest the accumulated interest. Therefore, the amount invested into the risky asset changes every time period.
- ▶ Investors therefore have to adjust their investments into the risky asset constantly, imposing potential transaction costs.
- This constant adjustment is similar to the constant adjustment needed when using options in Δ -hedging to perfectly hedge a risky asset.

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- ▶ As the investment consists of a fixed multiple m of the cushion invested into the risky asset, it is called the Constant Proportion Portfolio Insurance (CPPI)
- ▶ As the cushion **changes every time period** due to the value of the risky asset changing, the amounts invested into risky assets changes constantly

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- ▶ This requires a **continuous adjustment** of the weights between risky and risk-free assets

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CPPI as a hedging tool

- We now put CPPI in context of the hedging strategies involving derivatives.
- ▶ We argue that CPPI is similar to using options to hedge; while the minimum value is guaranteed, a positive development of the value of the risky asset will generate a higher portfolio value. This is similar to the use of options, where losses are limited, but profits are retained by the investor. Thus CPPI is like purchasing a put option with a strike price at the threshold.
- ▶
 - The investment into the risk-free asset will alter the final value of the portfolio compared to the use of put options, hence the two are not the same strategy.
 - In addition, there is no option premium payable when using CPPI.
- ▶ The need to adjust the holding of the risky asset(s) constantly can be very costly and may make CPPI not sustainable.
- ▶
 - However, where costs are not prohibitive, CPPI can be an alternative to hedging with put options or other derivatives.
 - It will be particularly attractive if options on the assets held are not available.
- We have thus developed an investment strategy that ensures a minimum value of the portfolio at the end of the time horizon, while maintaining to a large extent the optimal portfolio that reflects the preferences of the investor due to a higher investment into risky assets than a simple buy-and hold strategy. However, to achieve this we had to assume that losses on the risky asset are limited; if this condition is not fulfilled, CPPI has no advantage over a simple buy-and hold strategy.

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a **Put option** having a strike price of αV_0

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