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 Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses

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Portfolio_insurance

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Worst case scenario

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