



Andreas Krause

Portfolio insurance

Aim of portfolio insurance

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does **not prevent losses**

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain **minimum value** of their investment at the end of their time horizon

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by **regulatory requirements**

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by regulatory requirements or the need to meet given obligations

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by regulatory requirements or the need to meet given obligations, for example **pension payments**

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by regulatory requirements or the need to meet given obligations, for example pension payments
- ▶ In principle **derivatives** can be used to achieve this aim as they allow to hedge these risks

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by regulatory requirements or the need to meet given obligations, for example pension payments
- ▶ In principle derivatives can be used to achieve this aim as they allow to hedge these risks
- ▶ In many cases derivatives are **not readily available**

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by regulatory requirements or the need to meet given obligations, for example pension payments
- ▶ In principle derivatives can be used to achieve this aim as they allow to hedge these risks
- ▶ In many cases derivatives are not readily available or the number of instruments that need to be used is **prohibitively high**

Aim of portfolio insurance

- ▶ Measuring risk and adjusting portfolios to achieve a certain level of risk does not prevent losses
- ▶ In many cases investors want to ensure a certain minimum value of their investment at the end of their time horizon
- ▶ This can be driven by regulatory requirements or the need to meet given obligations, for example pension payments
- ▶ In principle derivatives can be used to achieve this aim as they allow to hedge these risks
- ▶ In many cases derivatives are not readily available or the number of instruments that need to be used is prohibitively high

Buy-and-hold strategy

Buy-and-hold strategy

- ▶ An investor wants to ensure that the **final value** of his **investment** exceeds some **threshold**, expressed relative to his current value
- ▶ $V_T \geq \alpha V_0$

Buy-and-hold strategy

- ▶ An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
- ▶ $V_T \geq \alpha V_0$
- ▶ The investor invests into the risk-free asset so this final value is ensured is ensured: $B_t = \alpha V_0 e^{-r(T-t)}$

Buy-and-hold strategy

- ▶ An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
- ▶ $V_T \geq \alpha V_0$
- ▶ The investor invests into the risk-free asset so this final value is ensured is ensured: $B_t = \alpha V_0 e^{-r(T-t)}$

$$\Rightarrow B_T = \alpha V_0$$

Buy-and-hold strategy

- ▶ An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
 - ▶ $V_T \geq \alpha V_0$
 - ▶ The investor invests into the risk-free asset so this final value is ensured is ensured: $B_t = \alpha V_0 e^{-r(T-t)}$
- $\Rightarrow B_T = \alpha V_0$
- ▶ The remainder is invested into the risky asset: $C_t = V_t - B_t = V_t - \alpha V_0 e^{-r(T-t)}$

Buy-and-hold strategy

- ▶ An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
- ▶ $V_T \geq \alpha V_0$
- ▶ The investor invests into the risk-free asset so this final value is ensured is ensured: $B_t = \alpha V_0 e^{-r(T-t)}$
- $\Rightarrow B_T = \alpha V_0$
- ▶ The remainder is invested into the risky asset: $C_t = V_t - B_t = V_t - \alpha V_0 e^{-r(T-t)}$
- ▶ This implies usually a **low investment into risky assets**

Buy-and-hold strategy

- ▶ An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
- ▶ $V_T \geq \alpha V_0$
- ▶ The investor invests into the risk-free asset so this final value is ensured is ensured: $B_t = \alpha V_0 e^{-r(T-t)}$
- $\Rightarrow B_T = \alpha V_0$
- ▶ The remainder is invested into the risky asset: $C_t = V_t - B_t = V_t - \alpha V_0 e^{-r(T-t)}$
- ▶ This implies usually a low investment into risky assets or even a **short position**

Buy-and-hold strategy

- ▶ An investor wants to ensure that the final value of his investment exceeds some threshold, expressed relative to his current value
- ▶ $V_T \geq \alpha V_0$
- ▶ The investor invests into the risk-free asset so this final value is ensured is ensured: $B_t = \alpha V_0 e^{-r(T-t)}$
- $\Rightarrow B_T = \alpha V_0$
- ▶ The remainder is invested into the risky asset: $C_t = V_t - B_t = V_t - \alpha V_0 e^{-r(T-t)}$
- ▶ This implies usually a low investment into risky assets or even a short position

Maximum loss of the risky asset

Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses **exceeding a certain threshold** before it can be liquidated

Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses exceeding a certain threshold before it can be liquidated
- ▶ This risk of such a loss may be evaluated using **Value-at-Risk** or other risk measures

Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses exceeding a certain threshold before it can be liquidated
- ▶ This risk of such a loss may be evaluated using Value-at-Risk or other risk measures
- ▶ With Value-at-Risk as a risk measure this threshold would be the **reasonable loss** the investor could make

Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses exceeding a certain threshold before it can be liquidated
- ▶ This risk of such a loss may be evaluated using Value-at-Risk or other risk measures
- ▶ With Value-at-Risk as a risk measure this threshold would be the reasonable loss the investor could make, the **Value-at-Risk** itself

Maximum loss of the risky asset

- ▶ Assume the risky asset cannot make losses exceeding a certain threshold before it can be liquidated
- ▶ This risk of such a loss may be evaluated using Value-at-Risk or other risk measures
- ▶ With Value-at-Risk as a risk measure this threshold would be the reasonable loss the investor could make, the Value-at-Risk itself

An alternative investment strategy

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a **cushion**

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the **floor**

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be **guaranteed** to obtain the floor

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be guaranteed to obtain the floor, plus any accumulated interest

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be guaranteed to obtain the floor, plus any accumulated interest
- ▶ The investment strategy is to invest mC_t into the **risky asset**

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be guaranteed to obtain the floor, plus any accumulated interest
- ▶ The investment strategy is to invest mC_t into the risky asset and $B_t = V_t - mC_t$ into the **risk-free asset**

An alternative investment strategy

- ▶ The amount not invested into the risk-free asset, $C_t = V_t - B_t$, is called a cushion and the investment into the risk-free asset is the floor
- ▶ The investor will always be guaranteed to obtain the floor, plus any accumulated interest
- ▶ The investment strategy is to invest mC_t into the risky asset and $B_t = V_t - mC_t$ into the risk-free asset

Worst case scenario

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain
- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r(V_t - m C_t)$

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain
 - ▶ $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$
- $\Rightarrow V_{t+1} = C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain

- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$

$$\begin{aligned}\Rightarrow V_{t+1} &= C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t) \\ &= C_t + B_t - \gamma m C_t + r (C_t + B_t - m C_t)\end{aligned}$$

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain

- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$

$$\begin{aligned}\Rightarrow V_{t+1} &= C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t) \\ &= C_t + B_t - \gamma m C_t + r (C_t + B_t - m C_t) \\ &= C_t (1 + r - m (\gamma + r)) + (1 + r) B_t\end{aligned}$$

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain
- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$
- ⇒ $V_{t+1} = C_{t+1} + B_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$
$$= C_t + B_t - \gamma m C_t + r (C_t + B_t - m C_t)$$
$$= C_t (1 + r - m (\gamma + r)) + (1 + r) B_t$$
- ▶ If we keep the investments constant, then $B_{t+1} = (1 + r) B_t$

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain

- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$

$$\begin{aligned}\Rightarrow V_{t+1} = C_{t+1} + B_{t+1} &\geq V_t - \gamma m C_t + r (V_t - m C_t) \\ &= C_t + B_t - \gamma m C_t + r (C_t + B_t - m C_t) \\ &= C_t (1 + r - m (\gamma + r)) + (1 + r) B_t\end{aligned}$$

- ▶ If we keep the investments constant, then $B_{t+1} = (1 + r) B_t$

$$\Rightarrow C_{t+1} \geq C_t (1 + r - m (\gamma + r))$$

Worst case scenario

- ▶ The worst case scenario is that the risky asset loses its maximum value and the risk free investment yields its return for certain

- ▶ $V_{t+1} \geq V_t - \gamma m C_t + r (V_t - m C_t)$

$$\begin{aligned}\Rightarrow V_{t+1} = C_{t+1} + B_{t+1} &\geq V_t - \gamma m C_t + r (V_t - m C_t) \\ &= C_t + B_t - \gamma m C_t + r (C_t + B_t - m C_t) \\ &= C_t (1 + r - m (\gamma + r)) + (1 + r) B_t\end{aligned}$$

- ▶ If we keep the investments constant, then $B_{t+1} = (1 + r) B_t$

$$\Rightarrow C_{t+1} \geq C_t (1 + r - m (\gamma + r))$$

Optimal investment into the risky asset

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT}$

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a **positive cushion** ensures that the total investments meet the condition that $V_T \geq \alpha V_0$

Optimal investment into the risky asset

- If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$
 $\Rightarrow 1 + r - m(\gamma + r) > 0$

Optimal investment into the risky asset

- If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$

$$\Rightarrow 1 + r - m(\gamma + r) > 0$$

$$\Rightarrow m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$$

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$
- $\Rightarrow 1 + r - m(\gamma + r) > 0$
- $\Rightarrow m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$
- ▶ As assets can **at most** lose all their value

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$
- ⇒ $1 + r - m(\gamma + r) > 0$
- ⇒ $m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$
- ▶ As assets can at most lose all their value, $\gamma < 1$

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$
- ⇒ $1 + r - m(\gamma + r) > 0$
- ⇒ $m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$
- ▶ As assets can at most lose all their value, $\gamma < 1$ and $m > 1$

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$
- ⇒ $1 + r - m(\gamma + r) > 0$
- ⇒ $m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$
- ▶ As assets can at most lose all their value, $\gamma < 1$ and $m > 1$
- ▶ The investor can **invest more** into the risk-free asset than in a buy-and-hold strategy

Optimal investment into the risky asset

- ▶ If the investor starts with a cushion $C_0 = V_0 - \alpha V_0 e^{-rT} = (1 - \alpha e^{-rT}) V_0$, a positive cushion ensures that the total investments meet the condition that $V_T \geq \alpha V_0$
- ⇒ $1 + r - m(\gamma + r) > 0$
- ⇒ $m < \frac{1+r}{\gamma+r} \approx \frac{1}{\gamma}$
- ▶ As assets can at most lose all their value, $\gamma < 1$ and $m > 1$
- ▶ The investor can invest more into the risk-free asset than in a buy-and-hold strategy

Constant proportion portfolio insurance

Constant proportion portfolio insurance

- ▶ As the investment consists of a fixed multiple m of the cushion invested into the risky asset, it is called the **Constant Proportion Portfolio Insurance**

Constant proportion portfolio insurance

- ▶ As the investment consists of a fixed multiple m of the cushion invested into the risky asset, it is called the Constant Proportion Portfolio Insurance (CPPI)

Constant proportion portfolio insurance

- ▶ As the investment consists of a fixed multiple m of the cushion invested into the risky asset, it is called the Constant Proportion Portfolio Insurance (CPPI)
- ▶ As the cushion **changes every time period** due to the value of the risky asset changing, the amounts invested into risky assets changes constantly

Constant proportion portfolio insurance

- ▶ As the investment consists of a fixed multiple m of the cushion invested into the risky asset, it is called the Constant Proportion Portfolio Insurance (CPPI)
- ▶ As the cushion changes every time period due to the value of the risky asset changing, the amounts invested into risky assets changes constantly
- ▶ This requires a **continuous adjustment** of the weights between risky and risk-free assets

Constant proportion portfolio insurance

- ▶ As the investment consists of a fixed multiple m of the cushion invested into the risky asset, it is called the Constant Proportion Portfolio Insurance (CPPI)
- ▶ As the cushion changes every time period due to the value of the risky asset changing, the amounts invested into risky assets changes constantly
- ▶ This requires a continuous adjustment of the weights between risky and risk-free assets

CPPI as a hedging tool

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a **Put option** having a strike price of αV_0

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0
- ▶ As not the entire investment is in the risky asset, the payoff at maturity will be **different** to that of a hedge with put options

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0
- ▶ As not the entire investment is in the risky asset, the payoff at maturity will be different to that of a hedge with put options, but **no premium** is payable either

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0
- ▶ As not the entire investment is in the risky asset, the payoff at maturity will be different to that of a hedge with put options, but no premium is payable either
- ▶ **Transaction costs** of adjusting the portfolio constantly can be prohibitive

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0
- ▶ As not the entire investment is in the risky asset, the payoff at maturity will be different to that of a hedge with put options, but no premium is payable either
- ▶ Transaction costs of adjusting the portfolio constantly can be prohibitive
- ▶ CPPI can be used as an **alternative** to the use of derivatives

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0
- ▶ As not the entire investment is in the risky asset, the payoff at maturity will be different to that of a hedge with put options, but no premium is payable either
- ▶ Transaction costs of adjusting the portfolio constantly can be prohibitive
- ▶ CPPI can be used as an alternative to the use of derivatives or where derivatives are **not available**

CPPI as a hedging tool

- ▶ As the minimum value of the investment at the end of the time horizon is ensured, but its value can be higher, CPPI is comparable to hedging the portfolio with a Put option having a strike price of αV_0
- ▶ As not the entire investment is in the risky asset, the payoff at maturity will be different to that of a hedge with put options, but no premium is payable either
- ▶ Transaction costs of adjusting the portfolio constantly can be prohibitive
- ▶ CPPI can be used as an alternative to the use of derivatives or where derivatives are not available



Copyright © by Andreas Krause

Picture credits:

Cover: Tobias Deml, CC BY-SA 4.0 (<https://creativecommons.org/licenses/by-sa/4.0>), via Wikimedia Commons, https://upload.wikimedia.org/wikipedia/commons/2/26/Gaming-Wall-Street_BTS_Prodigium-266.jpg

Back: Michael Vadon, CC BY 2.0 (<https://creativecommons.org/licenses/by/2.0/>), via Wikimedia Commons, [https://upload.wikimedia.org/wikipedia/commons/9/97/Manhattan\(NYC-New-York-City\)Skyline\(31769153946\).jpg](https://upload.wikimedia.org/wikipedia/commons/9/97/Manhattan(NYC-New-York-City)Skyline(31769153946).jpg)

Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk