Andreas Krause

Value-at-risk

Outline

- Measuring risk
- Value-at-Risk
- Managing portfolio risk
- Discussion

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Limits to volatility as a risk measure

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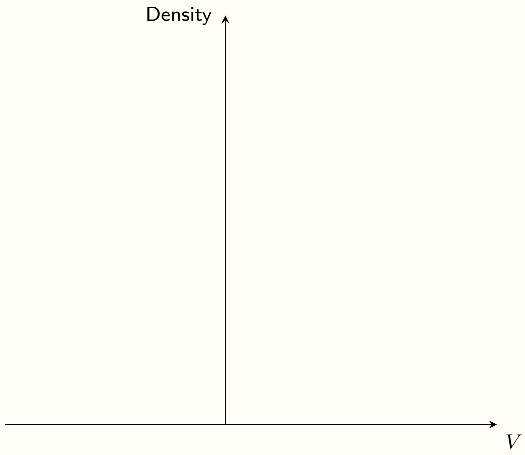

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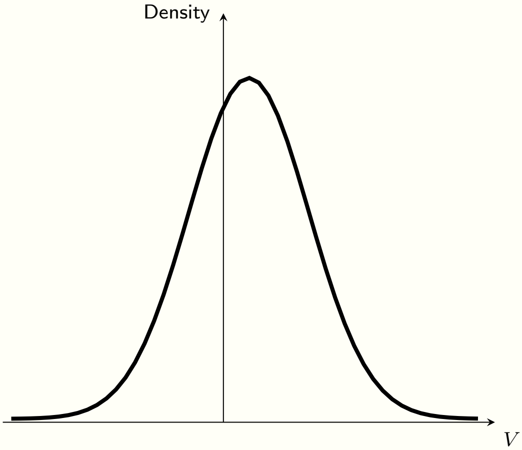

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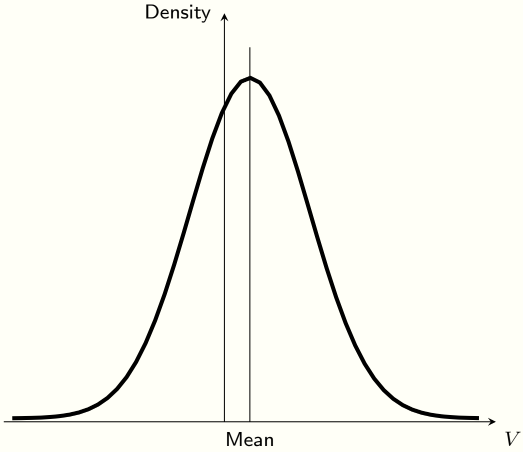

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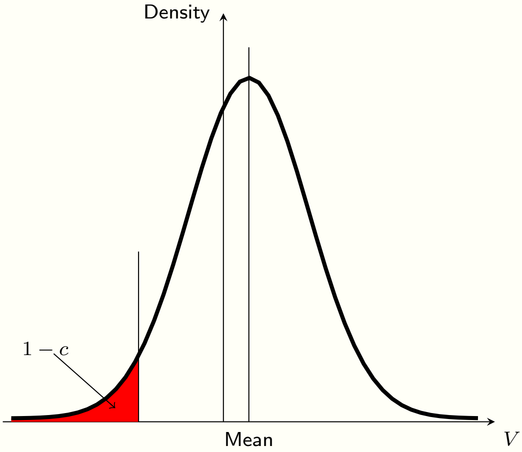

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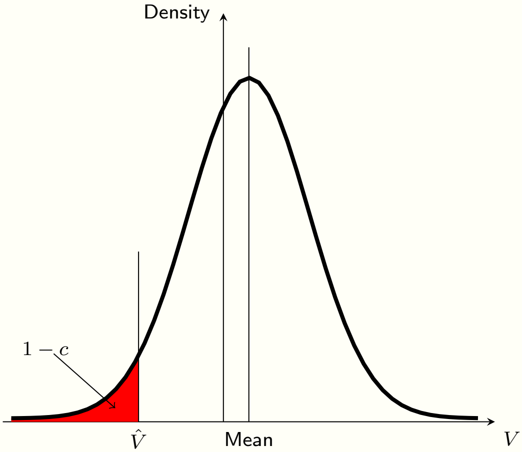

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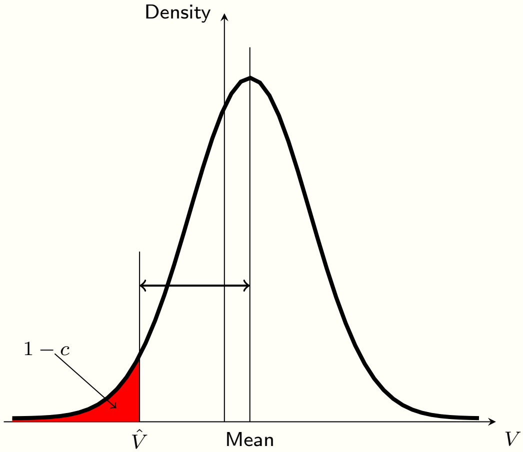

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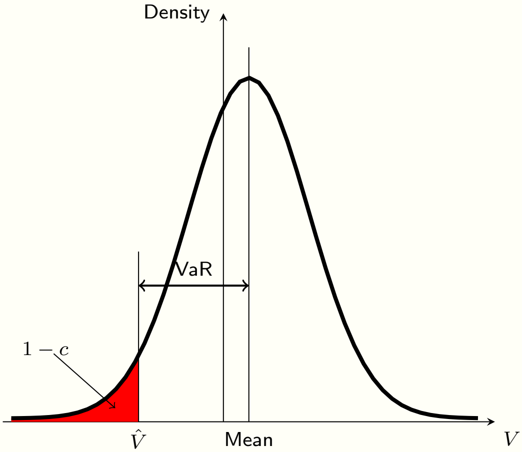

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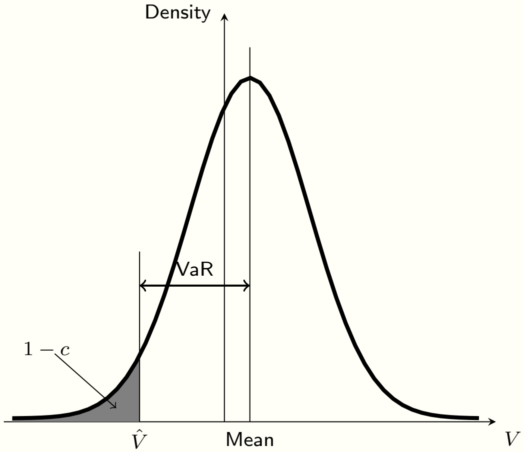

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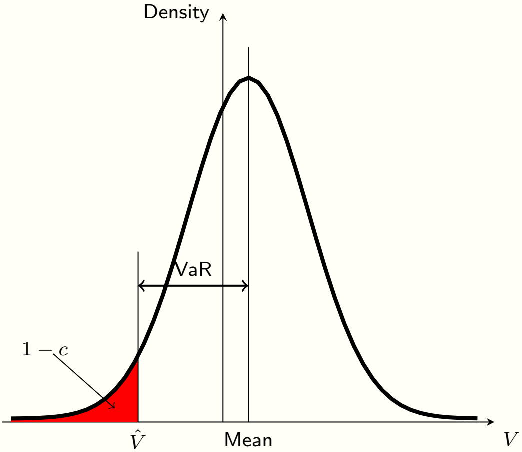

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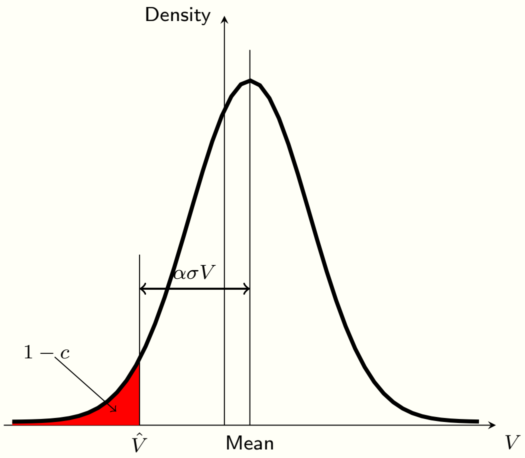

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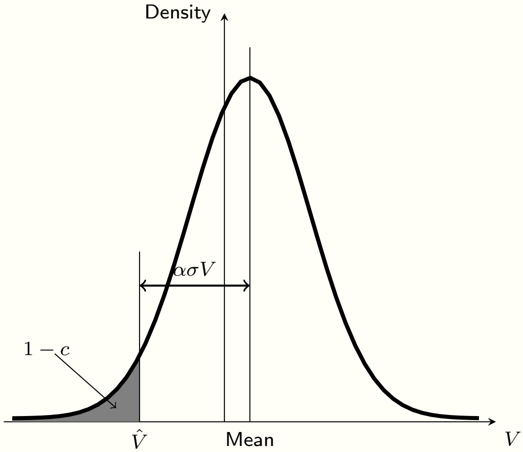

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Changing portfolio risk

- To reduce the risk of a portfolio, reduce the weight of those assets with high marginal Value-at-Risks, high β_i, and increase those with low marginal Value-at-Risk, low β_i
- Reducing the weight of an asset with a high β_i reduces the Value-at-Risk considerably and increasing the weight of an asset with low β_i increases it by less, leading to a reduction in the Value-at-Risk
- The larger the difference between these two assets is, the bigger the impact on the Value-at-Risk
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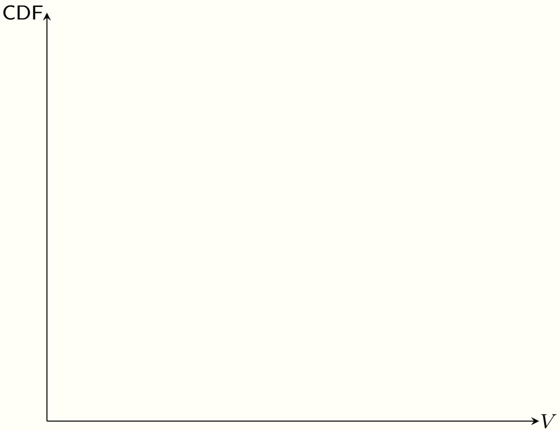
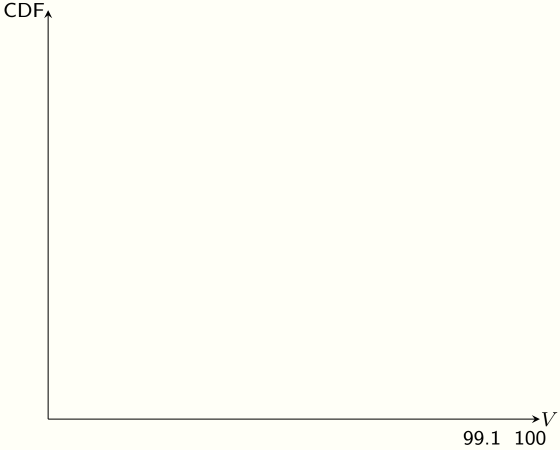


Diversification increasing Value-at-Risk



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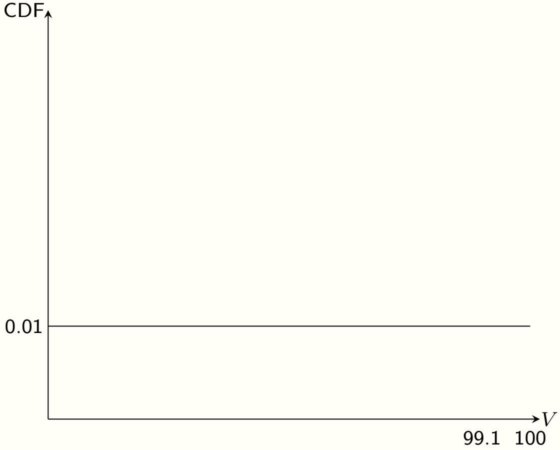

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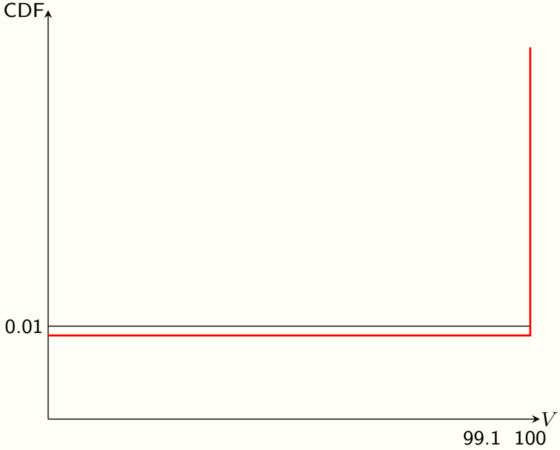

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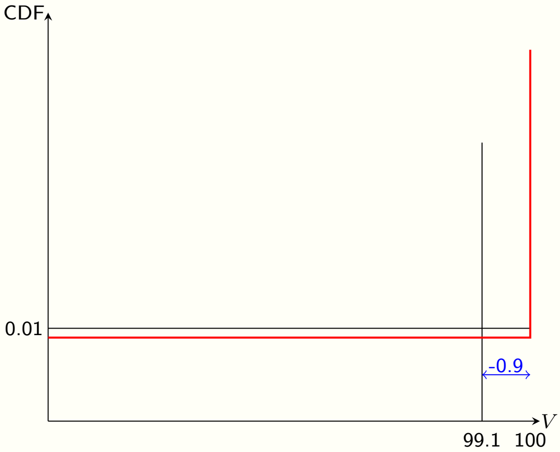

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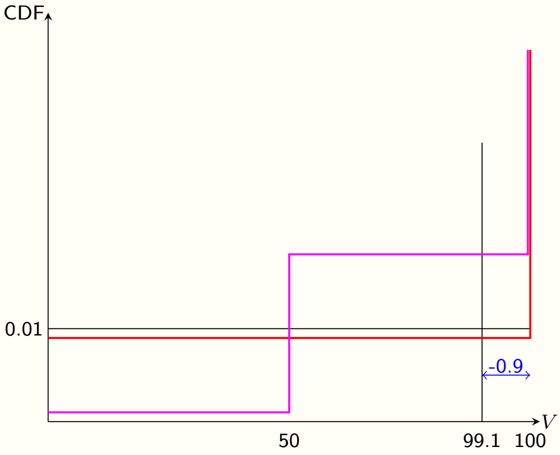

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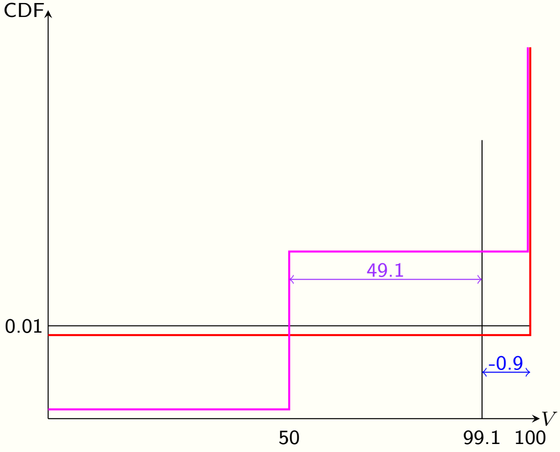

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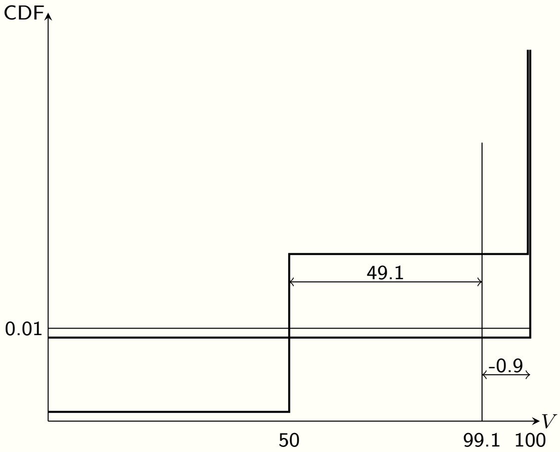

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