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Rational bubbles

Existence of bubbles

- Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are not observed
- Similar observations are also made in real estate, exchange rates, and commodities (mainly in precious metals)

Rationality of bubbles

- Stock market bubbles can be rational if the expected returns of investors are identical to alternative investments without bubbles
- An investor might buy a stock that is overvalued in the anticipation that he can sell it at an even higher price in the future
- The bubble might burst and the investor make significant losses, which needs to be considered
- As long as the expected return from a stock with a bubble is identical to that of a stock without a bubble, investing into these stocks is rational

Bubble specification

- The stock price consists of the fundamental value and the bubble
- $P_t = P_t^* + B_t$
- The fundamental value increases with its expected return: $P_{t+1}^* = (1+\mu)\,P_t^*$
- A bubble may burst and the price reverts to its fundamental value, or the bubble continues to exist
- $B_{t+1} = \begin{cases} (1+R)\,B_t & \text{with probability} \quad \pi \\ 0 & \text{with probability} \quad 1-\pi \end{cases}$
- The expected return of the bubble has to yield the same return as investing into the stock
- $\mathsf{E}\left[B_{t+1}\right] = \pi\,(1+R)\,B_t = (1+\mu)\,B_t$

$\Rightarrow$ $1 + R = \frac{1+\mu}{\pi}$

Increasing deviations of stock price and fundamental value

- As long as the bubble continues, it will grow faster than the stock price
- This compensates for the risk of the bubble bursting

$\Rightarrow$ The stock price deviates ever further from the fundamental value

- Bubbles grow faster the more likely they are to burst
- The bursting of a bubble is purely stochastic

Starting of a bubble

- A bubble can only start if a bubble emerges exogenously, $B_0 > 0$
- This might arise from random misvaluations due to incomplete information
- Random demand by noise traders might also give rise to the initial bubble

Impossibility of negative bubbles in stock markets

- Stocks have limited liability and a stock price below zero is not possible
- As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts

$\Rightarrow$ Stock price increases and the bubble bursts

- Investors anticipate this increase and will invest in the time period before,...
- A negative bubble cannot emerge
- The same applies to commodities and real estate as their value has a lower floor as well

Impossibility bubbles in fixed income securities

- Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- Investors anticipate this burst and will sell in the time period before

$\Rightarrow$ The bubble bursts a time period before maturity, but now investors know the bubble will burst a time period prior to maturity

$\Rightarrow$ The bubble will burst two time periods before maturity

$\Rightarrow$ ...

$\Rightarrow$ Bubbles cannot emerge

Positive and negative bubbles in exchange rates

- In foreign exchange markets positive and negative bubbles can emerge
- A positive bubble for one investor is a negative bubble in the other country
- In principle positive and negative bubbles can emerge in derivatives markets
- Arbitrage with the underlying asset should prevent any such mispricing

Summary

- Bubbles can emerge as long as it is unknown when they will burst
- Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- As long as they exist, bubbles will grow faster than the asset value to compensate for the risk of the bubble bursting
- Bubbles are mostly reported in stock markets, real estate, and sometimes in exchange rates and commodities

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