



Rational bubbles

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are not observed
- ▶ Similar observations are also made in real estate, exchange rates, and commodities (mainly in precious metals)

Rationality of bubbles

- ▶ Stock market bubbles can be rational if the expected returns of investors are identical to alternative investments without bubbles
- ▶ An investor might buy a stock that is overvalued in the anticipation that he can sell it at an even higher price in the future
- ▶ The bubble might burst and the investor make significant losses, which needs to be considered
- ▶ As long as the expected return from a stock with a bubble is identical to that of a stock without a bubble, investing into these stocks is rational

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
 - ▶ $P_t = P_t^* + B_t$
 - ▶ The fundamental value increases with its expected return: $P_{t+1}^* = (1 + \mu) P_t^*$
 - ▶ A bubble may burst and the price reverts to its fundamental value, or the bubble continues to exist
 - ▶
$$B_{t+1} = \begin{cases} (1 + R) B_t & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases}$$
 - ▶ The expected return of the bubble has to yield the same return as investing into the stock
 - ▶ $E[B_{t+1}] = \pi (1 + R) B_t = (1 + \mu) B_t$
- $\Rightarrow 1 + R = \frac{1 + \mu}{\pi}$

Increasing deviations of stock price and fundamental value

- ▶ As long as the bubble continues, it will grow faster than the stock price
- ▶ This compensates for the risk of the bubble bursting
- ⇒ The stock price deviates ever further from the fundamental value
- ▶ Bubbles grow faster the more likely they are to burst
- ▶ The bursting of a bubble is purely stochastic

Starting of a bubble

- ▶ A bubble can only start if a bubble emerges exogenously, $B_0 > 0$
- ▶ This might arise from random misvaluations due to incomplete information
- ▶ Random demand by noise traders might also give rise to the initial bubble

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts
- ⇒ Stock price increases and the bubble bursts
- ▶ Investors anticipate this increase and will invest in the time period before,...
- ▶ A negative bubble cannot emerge
- ▶ The same applies to commodities and real estate as their value has a lower floor as well

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period before
- ⇒ The bubble bursts a time period before maturity, but now investors know the bubble will burst a time period prior to maturity
- ⇒ The bubble will burst two time periods before maturity
- ⇒ ...
- ⇒ Bubbles cannot emerge

Positive and negative bubbles in exchange rates

- ▶ In foreign exchange markets positive and negative bubbles can emerge
- ▶ A positive bubble for one investor is a negative bubble in the other country
- ▶ In principle positive and negative bubbles can emerge in derivatives markets
- ▶ Arbitrage with the underlying asset should prevent any such mispricing

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- ▶ As long as they exist, bubbles will grow faster than the asset value to compensate for the risk of the bubble bursting
- ▶ Bubbles are mostly reported in stock markets, real estate, and sometimes in exchange rates and commodities



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