



Andreas Krause

Rational bubbles

- Asset prices deviate often substantially from their fundamental value for prolonged times. This is often referred to as 'bubble'.
- While such bubbles persist for a long time, it can happen that the asset price adjusts suddenly, leading to a crash of the asset price.
- We will look at why bubbles may emerge and why they might crash.

Existence of bubbles

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶ • We often observe that stock prices in particular deviate a lot from the fundamental value.
 - ▶ • Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶ • We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - ▶ • It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶ • Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - ▶ • We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - ▶ • Commodities can also show significant and prolonged overvaluations.
 - ▶ • This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often **deviate substantially** from their fundamental value

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - Commodities can also show significant and prolonged overvaluations.
 - This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a **bubble**

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - Commodities can also show significant and prolonged overvaluations.
 - This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are **exceeding** their fundamental value substantially

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - **We observe that stock prices are above the fundamental value, thus stocks are overvalued.**
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - Commodities can also show significant and prolonged overvaluations.
 - This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are **not observed**

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - **It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.**
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - Commodities can also show significant and prolonged overvaluations.
 - This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are not observed
- ▶ Similar observations are also made in **real estate**

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - Commodities can also show significant and prolonged overvaluations.
 - This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are not observed
- ▶ Similar observations are also made in real estate, **exchange rates**

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - **We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.**
 - Commodities can also show significant and prolonged overvaluations.
 - This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are not observed
- ▶ Similar observations are also made in real estate, exchange rates, and **commodities**

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - **Commodities can also show significant and prolonged overvaluations.**
 - This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are not observed
- ▶ Similar observations are also made in real estate, exchange rates, and commodities (mainly in **precious metals**)

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - Commodities can also show significant and prolonged overvaluations.
 - **This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.**
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Existence of bubbles

- ▶ Stock prices often deviate substantially from their fundamental value, which is known as a bubble
- ▶ Stock prices are exceeding their fundamental value substantially, but substantial undervaluations are not observed
- ▶ Similar observations are also made in real estate, exchange rates, and commodities (mainly in precious metals)

Existence of bubbles

- We have strong empirical evidence that the prices of assets and its fundamental value deviate substantially from each other for longer periods of time.
 - ▶
 - We often observe that stock prices in particular deviate a lot from the fundamental value.
 - Such deviations which are persisting for a long period of time are called a 'bubble'.
 - ▶
 - We observe that stock prices are above the fundamental value, thus stocks are overvalued.
 - It is very rarely the case that stocks are undervalued for a long period of time, the price is below the fundamental value. These assertions hold for stocks where there is no information that is not available to traders, or which is barely known. We here assume that most, if not all, traders agree on the fundamental value and the price is nevertheless different.
 - ▶
 - Such observations are not limited to stock markets. We also find them in property, residential houses as well as commercial property, such as office spaces or property in industrial estates.
 - We also often find that one currency is significantly overvalued or undervalued; this would be an example of an asset with a bubble being undervalued.
 - Commodities can also show significant and prolonged overvaluations.
 - **This is particularly observed in precious metals, such as gold, sometimes also in silver. It might also occur in rare earth metals (like Lithium) or other metals such as cobalt, mainly arising from potential demand in computer chips.**
- We will seek to explain the existence of such bubbles and show that their existence is rational.

Rationality of bubbles

- We now discuss conditions under which investing into stocks having a bubble can be rational.
- Investing into a stock which is known to be in a bubble, that is its price is above the fundamental value, can be rational if the expected return on this stock is identical to other investments (adjusting for risks). In that case, even if a stock (or another asset) is overvalued, would it be rational to invest into this stock.
- The idea is that even though the stock is bought at a price exceeding the fundamental value, it would be rational to do so if the investor expects the stock price to increase even further. In that case he would be able to sell the stock at a higher price and the fundamental value becomes irrelevant.
- However, the bubble might burst and the stock price return to the fundamental value of the asset. In that case the investor would face a significant loss. The possibility of such a loss needs to be considered when assessing the returns of the investment.
- If the expected return of investing into a stock during its bubble is the same as that of the same stock not in a bubble, the investment would be rational. We choose the same stock here such that we do not need to adjust for different risks.
- Formally, the risks of a stock in a bubble and a stock not in a bubble are not the same, the possibility of the bubble bursting will increase risks. We neglect this aspect here, but note that for risk averse investors, this would reduce the utility of investing into a stock during a bubble; however, we could easily compensate the investor for this additional risk by a slightly higher return on the stock in a bubble without changing arguments.

Rationality of bubbles

- ▶ Stock market bubbles can be **rational** if the expected returns of investors are identical to alternative investments without bubbles

- We now discuss conditions under which investing into stocks having a bubble can be rational.
- Investing into a stock which is known to be in a bubble, that is its price is above the fundamental value, can be rational if the expected return on this stock is identical to other investments (adjusting for risks). In that case, even if a stock (or another asset) is overvalued, would it be rational to invest into this stock.
- The idea is that even though the stock is bought at a price exceeding the fundamental value, it would be rational to do so if the investor expects the stock price to increase even further. In that case he would be able to sell the stock at a higher price and the fundamental value becomes irrelevant.
- However, the bubble might burst and the stock price return to the fundamental value of the asset. In that case the investor would face a significant loss. The possibility of such a loss needs to be considered when assessing the returns of the investment.
- If the expected return of investing into a stock during its bubble is the same as that of the same stock not in a bubble, the investment would be rational. We choose the same stock here such that we do not need to adjust for different risks.
- Formally, the risks of a stock in a bubble and a stock not in a bubble are not the same, the possibility of the bubble bursting will increase risks. We neglect this aspect here, but note that for risk averse investors, this would reduce the utility of investing into a stock during a bubble; however, we could easily compensate the investor for this additional risk by a slightly higher return on the stock in a bubble without changing arguments.

Rationality of bubbles

- ▶ Stock market bubbles can be rational if the expected returns of investors are identical to alternative investments without bubbles
- ▶ An investor might buy a stock that is overvalued in the anticipation that he can **sell it at an even higher price** in the future

- We now discuss conditions under which investing into stocks having a bubble can be rational.
- Investing into a stock which is known to be in a bubble, that is its price is above the fundamental value, can be rational if the expected return on this stock is identical to other investments (adjusting for risks). In that case, even if a stock (or another asset) is overvalued, would it be rational to invest into this stock.
- The idea is that even though the stock is bought at a price exceeding the fundamental value, it would be rational to do so if the investor expects the stock price to increase even further. In that case he would be able to sell the stock at a higher price and the fundamental value becomes irrelevant.
- However, the bubble might burst and the stock price return to the fundamental value of the asset. In that case the investor would face a significant loss. The possibility of such a loss needs to be considered when assessing the returns of the investment.
- If the expected return of investing into a stock during its bubble is the same as that of the same stock not in a bubble, the investment would be rational. We choose the same stock here such that we do not need to adjust for different risks.
- Formally, the risks of a stock in a bubble and a stock not in a bubble are not the same, the possibility of the bubble bursting will increase risks. We neglect this aspect here, but note that for risk averse investors, this would reduce the utility of investing into a stock during a bubble; however, we could easily compensate the investor for this additional risk by a slightly higher return on the stock in a bubble without changing arguments.

Rationality of bubbles

- ▶ Stock market bubbles can be rational if the expected returns of investors are identical to alternative investments without bubbles
- ▶ An investor might buy a stock that is overvalued in the anticipation that he can sell it at an even higher price in the future
- ▶ The bubble might **burst** and the investor make significant losses, which needs to be considered

- We now discuss conditions under which investing into stocks having a bubble can be rational.
- Investing into a stock which is known to be in a bubble, that is its price is above the fundamental value, can be rational if the expected return on this stock is identical to other investments (adjusting for risks). In that case, even if a stock (or another asset) is overvalued, would it be rational to invest into this stock.
- The idea is that even though the stock is bought at a price exceeding the fundamental value, it would be rational to do so if the investor expects the stock price to increase even further. In that case he would be able to sell the stock at a higher price and the fundamental value becomes irrelevant.
- **However, the bubble might burst and the stock price return to the fundamental value of the asset. In that case the investor would face a significant loss. The possibility of such a loss needs to be considered when assessing the returns of the investment.**
- If the expected return of investing into a stock during its bubble is the same as that of the same stock not in a bubble, the investment would be rational. We choose the same stock here such that we do not need to adjust for different risks.
- Formally, the risks of a stock in a bubble and a stock not in a bubble are not the same, the possibility of the bubble bursting will increase risks. We neglect this aspect here, but note that for risk averse investors, this would reduce the utility of investing into a stock during a bubble; however, we could easily compensate the investor for this additional risk by a slightly higher return on the stock in a bubble without changing arguments.

Rationality of bubbles

- ▶ Stock market bubbles can be rational if the expected returns of investors are identical to alternative investments without bubbles
- ▶ An investor might buy a stock that is overvalued in the anticipation that he can sell it at an even higher price in the future
- ▶ The bubble might burst and the investor make significant losses, which needs to be considered
- ▶ As long as the expected return from a stock with a bubble is identical to that of a stock without a bubble, investing into these stocks is **rational**

- We now discuss conditions under which investing into stocks having a bubble can be rational.
- Investing into a stock which is known to be in a bubble, that is its price is above the fundamental value, can be rational if the expected return on this stock is identical to other investments (adjusting for risks). In that case, even if a stock (or another asset) is overvalued, would it be rational to invest into this stock.
- The idea is that even though the stock is bought at a price exceeding the fundamental value, it would be rational to do so if the investor expects the stock price to increase even further. In that case he would be able to sell the stock at a higher price and the fundamental value becomes irrelevant.
- However, the bubble might burst and the stock price return to the fundamental value of the asset. In that case the investor would face a significant loss. The possibility of such a loss needs to be considered when assessing the returns of the investment.
- **If the expected return of investing into a stock during its bubble is the same as that of the same stock not in a bubble, the investment would be rational. We choose the same stock here such that we do not need to adjust for different risks.**
- Formally, the risks of a stock in a bubble and a stock not in a bubble are not the same, the possibility of the bubble bursting will increase risks. We neglect this aspect here, but note that for risk averse investors, this would reduce the utility of investing into a stock during a bubble; however, we could easily compensate the investor for this additional risk by a slightly higher return on the stock in a bubble without changing arguments.

Rationality of bubbles

- ▶ Stock market bubbles can be rational if the expected returns of investors are identical to alternative investments without bubbles
- ▶ An investor might buy a stock that is overvalued in the anticipation that he can sell it at an even higher price in the future
- ▶ The bubble might burst and the investor make significant losses, which needs to be considered
- ▶ As long as the expected return from a stock with a bubble is identical to that of a stock without a bubble, investing into these stocks is rational

- We now discuss conditions under which investing into stocks having a bubble can be rational.
- ▶ Investing into a stock which is known to be in a bubble, that is its price is above the fundamental value, can be rational if the expected return on this stock is identical to other investments (adjusting for risks). In that case, even if a stock (or another asset) is overvalued, would it be rational to invest into this stock.
- ▶ The idea is that even though the stock is bought at a price exceeding the fundamental value, it would be rational to do so if the investor expects the stock price to increase even further. In that case he would be able to sell the stock at a higher price and the fundamental value becomes irrelevant.
- ▶ However, the bubble might burst and the stock price return to the fundamental value of the asset. In that case the investor would face a significant loss. The possibility of such a loss needs to be considered when assessing the returns of the investment.
- ▶ If the expected return of investing into a stock during its bubble is the same as that of the same stock not in a bubble, the investment would be rational. We choose the same stock here such that we do not need to adjust for different risks.
- Formally, the risks of a stock in a bubble and a stock not in a bubble are not the same, the possibility of the bubble bursting will increase risks. We neglect this aspect here, but note that for risk averse investors, this would reduce the utility of investing into a stock during a bubble; however, we could easily compensate the investor for this additional risk by a slightly higher return on the stock in a bubble without changing arguments.

Bubble specification

- We can now develop a simple model of a bubble that meets all these requirements.
- - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- *Formula*
- The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- *Formula*
- After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value
- ▶ $P_t = P_t^*$

Bubble specification

- We can now develop a simple model of a bubble that meets all these requirements.
- - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- *Formula*
- The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- *Formula*
- After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
- ▶ $P_t = P_t^* + B_t$

- We can now develop a simple model of a bubble that meets all these requirements.
- ▶
 - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- ▶ *Formula*
- ▶ The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- ▶
 - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- ▶ *Formula*
- ▶ After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- ▶ *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
- ▶ $P_t = P_t^* + B_t$
- ▶ The fundamental value increases with its **expected return**: $P_{t+1}^* = (1 + \mu) P_t^*$

- We can now develop a simple model of a bubble that meets all these requirements.
- - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- *Formula*
- **The fundamental value increases by its expected return, which could be determined from the CAPM, for example.**
- - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- *Formula*
- After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
- ▶ $P_t = P_t^* + B_t$
- ▶ The fundamental value increases with its expected return: $P_{t+1}^* = (1 + \mu) P_t^*$
- ▶ A bubble may **burst** and the price reverts to its fundamental value
- ▶
$$B_{t+1} = \begin{cases} 0 & \text{with probability } 1 - \pi \end{cases}$$

- We can now develop a simple model of a bubble that meets all these requirements.
- ▶
 - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- ▶ *Formula*
- ▶ The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- ▶
 - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- ▶ *Formula*
- ▶ After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- ▶ *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
- ▶ $P_t = P_t^* + B_t$
- ▶ The fundamental value increases with its expected return: $P_{t+1}^* = (1 + \mu) P_t^*$
- ▶ A bubble may **burst** and the price reverts to its fundamental value, or the **bubble continues** to exist
- ▶
$$B_{t+1} = \begin{cases} (1 + R) B_t & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases}$$

Bubble specification

- We can now develop a simple model of a bubble that meets all these requirements.
- ▶
 - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- ▶ *Formula*
- ▶ The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- ▶
 - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - **Alternatively, the bubble will continue to exist and grow at a some rate R .**
- ▶ *Formula*
- ▶ After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- ▶ *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
- ▶ $P_t = P_t^* + B_t$
- ▶ The fundamental value increases with its expected return: $P_{t+1}^* = (1 + \mu) P_t^*$
- ▶ A bubble may burst and the price reverts to its fundamental value, or the bubble continues to exist
- ▶
$$B_{t+1} = \begin{cases} (1 + R) B_t & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases}$$
- ▶ The **expected return of the bubble** has to yield the same return as **investing into the stock**
- ▶ $E[B_{t+1}] = \pi (1 + R) B_t = (1 + \mu) B_t$

Bubble specification

- We can now develop a simple model of a bubble that meets all these requirements.
- ▶
 - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- ▶ *Formula*
- ▶ The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- ▶
 - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- ▶ *Formula*
- ▶ After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- ▶ *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
- ▶ $P_t = P_t^* + B_t$
- ▶ The fundamental value increases with its expected return: $P_{t+1}^* = (1 + \mu) P_t^*$
- ▶ A bubble may burst and the price reverts to its fundamental value, or the bubble continues to exist
- ▶
$$B_{t+1} = \begin{cases} (1 + R) B_t & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases}$$
- ▶ The expected return of the bubble has to yield the same return as investing into the stock
- ▶
$$\mathbb{E}[B_{t+1}] = \pi (1 + R) B_t = (1 + \mu) B_t$$

⇒ $1 + R = \frac{1 + \mu}{\pi}$

Bubble specification

- We can now develop a simple model of a bubble that meets all these requirements.
- - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- *Formula*
- The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- *Formula*
- After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- *Formula*
- ⇒ **This easily solves for the required return of the bubble, as long as it persists.**
- We can now analyse the implications of our findings.

Bubble specification

- ▶ The stock price consists of the fundamental value and the bubble
- ▶ $P_t = P_t^* + B_t$
- ▶ The fundamental value increases with its expected return: $P_{t+1}^* = (1 + \mu) P_t^*$
- ▶ A bubble may burst and the price reverts to its fundamental value, or the bubble continues to exist
- ▶
$$B_{t+1} = \begin{cases} (1 + R) B_t & \text{with probability } \pi \\ 0 & \text{with probability } 1 - \pi \end{cases}$$
- ▶ The expected return of the bubble has to yield the same return as investing into the stock
- ▶
$$\mathbb{E}[B_{t+1}] = \pi (1 + R) B_t = (1 + \mu) B_t$$
- ⇒
$$1 + R = \frac{1 + \mu}{\pi}$$

Bubble specification

- We can now develop a simple model of a bubble that meets all these requirements.
- ▶
 - We divide the price of a stock up into two components; the first component is the fundamental value of the stock.
 - The second component would be any deviation from this fundamental value, which we call a 'bubble'.
- ▶ *Formula*
- ▶ The fundamental value increases by its expected return, which could be determined from the CAPM, for example.
- ▶
 - We assume that if a bubble bursts, the price changes instantly to be the fundamental value, this implies that the bubble becomes zero. We assume that the bubble bursts with some probability $1 - \pi$.
 - Alternatively, the bubble will continue to exist and grow at a some rate R .
- ▶ *Formula*
- ▶ After the afore mentioned, the expected return of the stock in a bubble has to be equal to the expected return of a stock without the bubble. The expected return of the stock in a bubble is given by the case where the bubble persists (when the bubble bursts the bubble becomes zero) and this needs to generate the same return μ as the fundamental value of the stock (the stock without the bubble).
- ▶ *Formula*
- ⇒ This easily solves for the required return of the bubble, as long as it persists.
- **We can now analyse the implications of our findings.**

Increasing deviations of stock price and fundamental value

Increasing deviations of stock price and fundamental value

- We can now see that over time stock price increase faster than the fundamental value, at least until the bubble bursts.
- We see that the return of the bubble term for as long as it persists is higher than the return of the fundamental value. This implies that the bubble term grows faster than the fundamental value.
- This higher return is the compensation for the large negative return in the case that the bubble bursts. Thus a higher return if the bubble persists offsets a negative return if the bubble bursts.
- ⇒ With the bubble term becoming ever bigger, the stock price becomes more and more detached from the fundamental value.
- We also see that bubbles that are more likely to burst (π) have a higher growth rate. This higher growth rate compensates for the higher risk of the bubble bursting. It is thus that the more unstable a bubble is, the faster it grows.
- The reason the bubble bursts is not considered here, it is a purely stochastic process in which in each time period the bubble will burst with probability $1 - \pi$.
- Having considered the properties of the bubble, we can look at the properties of these.

Increasing deviations of stock price and fundamental value

- ▶ As long as the bubble continues, it will **grow faster** than the stock price

Increasing deviations of stock price and fundamental value

- We can now see that over time stock price increase faster than the fundamental value, at least until the bubble bursts.
- We see that the return of the bubble term for as long as it persists is higher than the return of the fundamental value. This implies that the bubble term grows faster than the fundamental value.
- This higher return is the compensation for the large negative return in the case that the bubble bursts. Thus a higher return if the bubble persists offsets a negative return if the bubble bursts.
- ⇒ With the bubble term becoming ever bigger, the stock price becomes more and more detached from the fundamental value.
- We also see that bubbles that are more likely to burst (π) have a higher growth rate. This higher growth rate compensates for the higher risk of the bubble bursting. It is thus that the more unstable a bubble is, the faster it grows.
- The reason the bubble bursts is not considered here, it is a purely stochastic process in which in each time period the bubble will burst with probability $1 - \pi$.
- Having considered the properties of the bubble, we can look at the properties of these.

Increasing deviations of stock price and fundamental value

- ▶ As long as the bubble continues, it will grow faster than the stock price
- ▶ This compensates for the **risk** of the bubble bursting

Increasing deviations of stock price and fundamental value

- We can now see that over time stock price increase faster than the fundamental value, at least until the bubble bursts.
- We see that the return of the bubble term for as long as it persists is higher than the return of the fundamental value. This implies that the bubble term grows faster than the fundamental value.
- **This higher return is the compensation for the large negative return in the case that the bubble bursts. Thus a higher return if the bubble persists offsets a negative return if the bubble bursts.**
- ⇒ With the bubble term becoming ever bigger, the stock price becomes more and more detached from the fundamental value.
- We also see that bubbles that are more likely to burst (π) have a higher growth rate. This higher growth rate compensates for the higher risk of the bubble bursting. It is thus that the more unstable a bubble is, the faster it grows.
- The reason the bubble bursts is not considered here, it is a purely stochastic process in which in each time period the bubble will burst with probability $1 - \pi$.
- Having considered the properties of the bubble, we can look at the properties of these.

Increasing deviations of stock price and fundamental value

- ▶ As long as the bubble continues, it will grow faster than the stock price
- ▶ This compensates for the risk of the bubble bursting
- ⇒ The stock price **deviates ever further** from the fundamental value

Increasing deviations of stock price and fundamental value

- We can now see that over time stock price increase faster than the fundamental value, at least until the bubble bursts.
- We see that the return of the bubble term for as long as it persists is higher than the return of the fundamental value. This implies that the bubble term grows faster than the fundamental value.
- This higher return is the compensation for the large negative return in the case that the bubble bursts. Thus a higher return if the bubble persists offsets a negative return if the bubble bursts.
- ⇒ **With the bubble term becoming ever bigger, the stock price becomes more and more detached from the fundamental value.**
- We also see that bubbles that are more likely to burst (π) have a higher growth rate. This higher growth rate compensates for the higher risk of the bubble bursting. It is thus that the more unstable a bubble is, the faster it grows.
- The reason the bubble bursts is not considered here, it is a purely stochastic process in which in each time period the bubble will burst with probability $1 - \pi$.
- Having considered the properties of the bubble, we can look at the properties of these.

Increasing deviations of stock price and fundamental value

- ▶ As long as the bubble continues, it will grow faster than the stock price
- ▶ This compensates for the risk of the bubble bursting
- ⇒ The stock price deviates ever further from the fundamental value
- ▶ Bubbles grow faster the **more likely** they are to burst

Increasing deviations of stock price and fundamental value

- We can now see that over time stock price increase faster than the fundamental value, at least until the bubble bursts.
- We see that the return of the bubble term for as long as it persists is higher than the return of the fundamental value. This implies that the bubble term grows faster than the fundamental value.
- This higher return is the compensation for the large negative return in the case that the bubble bursts. Thus a higher return if the bubble persists offsets a negative return if the bubble bursts.
- ⇒ With the bubble term becoming ever bigger, the stock price becomes more and more detached from the fundamental value.
- We also see that bubbles that are more likely to burst (π) have a higher growth rate. This higher growth rate compensates for the higher risk of the bubble bursting. It is thus that the more unstable a bubble is, the faster it grows.
- The reason the bubble bursts is not considered here, it is a purely stochastic process in which in each time period the bubble will burst with probability $1 - \pi$.
- Having considered the properties of the bubble, we can look at the properties of these.

Increasing deviations of stock price and fundamental value

- ▶ As long as the bubble continues, it will grow faster than the stock price
- ▶ This compensates for the risk of the bubble bursting
- ⇒ The stock price deviates ever further from the fundamental value
- ▶ Bubbles grow faster the more likely they are to burst
- ▶ The bursting of a bubble is purely **stochastic**

Increasing deviations of stock price and fundamental value

- We can now see that over time stock price increase faster than the fundamental value, at least until the bubble bursts.
- We see that the return of the bubble term for as long as it persists is higher than the return of the fundamental value. This implies that the bubble term grows faster than the fundamental value.
- This higher return is the compensation for the large negative return in the case that the bubble bursts. Thus a higher return if the bubble persists offsets a negative return if the bubble bursts.
- ⇒ With the bubble term becoming ever bigger, the stock price becomes more and more detached from the fundamental value.
- We also see that bubbles that are more likely to burst (π) have a higher growth rate. This higher growth rate compensates for the higher risk of the bubble bursting. It is thus that the more unstable a bubble is, the faster it grows.
- **The reason the bubble bursts is not considered here, it is a purely stochastic process in which in each time period the bubble will burst with probability $1 - \pi$.**
- Having considered the properties of the bubble, we can look at the properties of these.

Increasing deviations of stock price and fundamental value

- ▶ As long as the bubble continues, it will grow faster than the stock price
- ▶ This compensates for the risk of the bubble bursting
- ⇒ The stock price deviates ever further from the fundamental value
- ▶ Bubbles grow faster the more likely they are to burst
- ▶ The bursting of a bubble is purely stochastic

Increasing deviations of stock price and fundamental value

- We can now see that over time stock price increase faster than the fundamental value, at least until the bubble bursts.
- We see that the return of the bubble term for as long as it persists is higher than the return of the fundamental value. This implies that the bubble term grows faster than the fundamental value.
- This higher return is the compensation for the large negative return in the case that the bubble bursts. Thus a higher return if the bubble persists offsets a negative return if the bubble bursts.
- ⇒ With the bubble term becoming ever bigger, the stock price becomes more and more detached from the fundamental value.
- We also see that bubbles that are more likely to burst (π) have a higher growth rate. This higher growth rate compensates for the higher risk of the bubble bursting. It is thus that the more unstable a bubble is, the faster it grows.
- The reason the bubble bursts is not considered here, it is a purely stochastic process in which in each time period the bubble will burst with probability $1 - \pi$.
- Having considered the properties of the bubble, we can look at the properties of these.

Starting of a bubble

Starting of a bubble

- The first point to consider is how a bubble can start. From our model it is apparent that if $B_0 = 0$, no bubble will emerge.
- ▶
 - A bubble cannot emerge endogenously from the simple model we have established here.
 - We need to introduce an initial bubble term exogenously.
- ▶ If information on the fundamental value is not perfect, the signals traders receive might give rise to misvaluations, which could then form part of the initial bubble.
- ▶ Noise traders with their random demand can also create a bubble term purely by chance, which then can persists in the manner described here.
- While the start of a bubble can have many reasons, we predominantly see positive bubble terms in stock markets. We will now explore the reason for this observation.

Starting of a bubble

- ▶ A bubble can only start if a bubble emerges **exogenously**

Starting of a bubble

- The first point to consider is how a bubble can start. From our model it is apparent that if $B_0 = 0$, no bubble will emerge.
- ▶
 - A bubble cannot emerge endogenously from the simple model we have established here.
 - We need to introduce an initial bubble term exogenously.
- ▶ If information on the fundamental value is not perfect, the signals traders receive might give rise to misvaluations, which could then form part of the initial bubble.
- ▶ Noise traders with their random demand can also create a bubble term purely by chance, which then can persists in the manner described here.
- While the start of a bubble can have many reasons, we predominantly see positive bubble terms in stock markets. We will now explore the reason for this observation.

Starting of a bubble

- ▶ A bubble can only start if a bubble emerges exogenously, $B_0 > 0$

Starting of a bubble

- The first point to consider is how a bubble can start. From our model it is apparent that if $B_0 = 0$, no bubble will emerge.
- ▶
 - A bubble cannot emerge endogenously from the simple model we have established here.
 - **We need to introduce an initial bubble term exogenously.**
- ▶ If information on the fundamental value is not perfect, the signals traders receive might give rise to misvaluations, which could then form part of the initial bubble.
- ▶ Noise traders with their random demand can also create a bubble term purely by chance, which then can persists in the manner described here.
- While the start of a bubble can have many reasons, we predominantly see positive bubble terms in stock markets. We will now explore the reason for this observation.

Starting of a bubble

- ▶ A bubble can only start if a bubble emerges exogenously, $B_0 > 0$
- ▶ This might arise from **random misvaluations** due to incomplete information

Starting of a bubble

- The first point to consider is how a bubble can start. From our model it is apparent that if $B_0 = 0$, no bubble will emerge.
- ▶
 - A bubble cannot emerge endogenously from the simple model we have established here.
 - We need to introduce an initial bubble term exogenously.
- ▶ **If information on the fundamental value is not perfect, the signals traders receive might give rise to misvaluations, which could then form part of the initial bubble.**
- ▶ Noise traders with their random demand can also create a bubble term purely by chance, which then can persists in the manner described here.
- While the start of a bubble can have many reasons, we predominantly see positive bubble terms in stock markets. We will now explore the reason for this observation.

Starting of a bubble

- ▶ A bubble can only start if a bubble emerges exogenously, $B_0 > 0$
- ▶ This might arise from random misvaluations due to incomplete information
- ▶ Random demand by **noise traders** might also give rise to the initial bubble

Starting of a bubble

- The first point to consider is how a bubble can start. From our model it is apparent that if $B_0 = 0$, no bubble will emerge.
- ▶
 - A bubble cannot emerge endogenously from the simple model we have established here.
 - We need to introduce an initial bubble term exogenously.
- ▶ If information on the fundamental value is not perfect, the signals traders receive might give rise to misvaluations, which could then form part of the initial bubble.
- ▶ **Noise traders with their random demand can also create a bubble term purely by chance, which then can persists in the manner described here.**
- While the start of a bubble can have many reasons, we predominantly see positive bubble terms in stock markets. We will now explore the reason for this observation.

Starting of a bubble

- ▶ A bubble can only start if a bubble emerges exogenously, $B_0 > 0$
- ▶ This might arise from random misvaluations due to incomplete information
- ▶ Random demand by noise traders might also give rise to the initial bubble

Starting of a bubble

- The first point to consider is how a bubble can start. From our model it is apparent that if $B_0 = 0$, no bubble will emerge.
- ▶
 - A bubble cannot emerge endogenously from the simple model we have established here.
 - We need to introduce an initial bubble term exogenously.
- ▶ If information on the fundamental value is not perfect, the signals traders receive might give rise to misvaluations, which could then form part of the initial bubble.
- ▶ Noise traders with their random demand can also create a bubble term purely by chance, which then can persists in the manner described here.
- **While the start of a bubble can have many reasons, we predominantly see positive bubble terms in stock markets. We will now explore the reason for this observation.**

Impossibility of negative bubbles in stock markets

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have **limited liability** and a stock price below zero is not possible

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know **when it will stop** to grow

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- ▶ Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- ▶
 - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- ▶
 - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- ▶
 - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- ▶ We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- ▶ Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- ▶ Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- ▶
 - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - **The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.**
- ▶
 - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- ▶
 - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- ▶ We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- ▶ Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest **at this moment** as the stock price cannot reduce

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- ▶ Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- ▶
 - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- ▶
 - **Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.**
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- ▶
 - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- ▶ We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- ▶ Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but **only increase** if the bubble bursts

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - **However, if the bubble bursts, it will increase and the investor makes a large profit.**
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts

⇒ Stock price increases and the **bubble bursts**

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts

⇒ Stock price increases and the bubble bursts

- ▶ Investors **anticipate this increase** and will invest in the time period before

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- - **A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.**
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts

⇒ Stock price increases and the bubble bursts

- ▶ Investors anticipate this increase and will invest in the time period before,...

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- ▶ Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- ▶
 - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- ▶
 - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- ▶
 - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - **This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...**
- ▶ We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- ▶ Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts

⇒ Stock price increases and the bubble bursts

- ▶ Investors anticipate this increase and will invest in the time period before,...
- ▶ A negative bubble **cannot** emerge

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- ▶ Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- ▶
 - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- ▶
 - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- ▶
 - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- ▶ **We can take this logic back to the initial time period and we can never have a negative bubble to commence.**
- ▶ Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts

⇒ Stock price increases and the bubble bursts

- ▶ Investors anticipate this increase and will invest in the time period before,...
- ▶ A negative bubble cannot emerge
- ▶ The same applies to **commodities** and **real estate** as their value has a lower floor as well

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- **Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.**
- We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.

Impossibility of negative bubbles in stock markets

- ▶ Stocks have limited liability and a stock price below zero is not possible
- ▶ As the bubbles has to increase as long as it persists, investors know when it will stop to grow, at $B_t = -P_t$
- ▶ Investors would invest at this moment as the stock price cannot reduce, but only increase if the bubble bursts

⇒ Stock price increases and the bubble bursts

- ▶ Investors anticipate this increase and will invest in the time period before,...
- ▶ A negative bubble cannot emerge
- ▶ The same applies to commodities and real estate as their value has a lower floor as well

Impossibility of negative bubbles in stock markets

- We will now show that in many markets, negative bubbles cannot emerge.
- Stocks have limited liability, that is shareholders have no obligations to cover any shortfall to creditors if the company fails. It is therefore that the value of the stock can never become negative.
- - We had established that for as long as bubbles do not burst, they are increasing. Suppose now we have a negative bubble. As the stock price can never become negative, there is a limit to the size of the bubble.
 - The largest possible bubble is when the bubble has reached the fundamental value of the stock as then the stock price is zero.
- - Any investor who buys the stock at the price zero, can be sure that it will not be reducing any further.
 - However, if the bubble bursts, it will increase and the investor makes a large profit.
- ⇒ The stock price has to increase as such a stock would be very attractive to investors and there would be a large demand. This demand will increase the stock price, thus reduce the bubble, which in our model implies that the bubble bursts.
- - A time period earlier, investors will anticipate that the bubble will burst in the next time period for sure and hence the profits will be large.
 - This will then lead to investors having a high demand for the stock in the time period before, causing the bubble to burst. The time period before that now, the investors know it will burst in the next time period, and so on and so on...
- We can take this logic back to the initial time period and we can never have a negative bubble to commence.
- Commodities also have a lower price limit (which might be slightly negative due to storage costs) and real estate.
- **We have thus seen that in many markets negative bubbles cannot emerge as prices have a lower limit.**

Impossibility bubbles in fixed income securities

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a **fixed price** and **fixed date**

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble **must burst**

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - **As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.**
- - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors **anticipate** this burst

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- ▶
 - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- ▶
 - **Investors know that the bubble will burst at maturity**
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period **before**

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- - Investors know that the bubble will burst at maturity
 - **and hence they would sell the bond in the time period before to avoid the certain loss.**
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period before
- ⇒ The bubble bursts a time period **before maturity**

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - **As investors sell the bond, the bubble will burst a time period prior to maturity.**
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period before
- ⇒ The bubble bursts a time period before maturity, but now investors **know** the bubble will burst a time period prior to maturity

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - **Now investors know that the bubble will burst a time period prior to maturity.**
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period before
 - ⇒ The bubble bursts a time period before maturity, but now investors know the bubble will burst a time period prior to maturity
 - ⇒ The bubble will burst **two time periods** before maturity

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ **The same logic now applies and the bubble will burst two time periods prior to maturity**
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period before
 - ⇒ The bubble bursts a time period before maturity, but now investors know the bubble will burst a time period prior to maturity
 - ⇒ The bubble will burst two time periods before maturity
 - ⇒ ...

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- ▶
 - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- ▶
 - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ **This logic now applies all the way back to the current time period.**
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period before
 - ⇒ The bubble bursts a time period before maturity, but now investors know the bubble will burst a time period prior to maturity
 - ⇒ The bubble will burst two time periods before maturity
 - ⇒ ...
 - ⇒ Bubbles **cannot** emerge

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- ▶
 - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- ▶
 - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ **Hence we cannot have a bubble, positive or negative, in fixed income markets.**
- The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.

Impossibility bubbles in fixed income securities

- ▶ Fixed income securities are repaid at a fixed price and fixed date, at this point the bubble must burst
- ▶ Investors anticipate this burst and will sell in the time period before
 - ⇒ The bubble bursts a time period before maturity, but now investors know the bubble will burst a time period prior to maturity
 - ⇒ The bubble will burst two time periods before maturity
 - ⇒ ...
 - ⇒ Bubbles cannot emerge

Impossibility bubbles in fixed income securities

- We will establish that bubbles in fixed income securities are impossible as well.
- - Fixed income securities (bonds) are repaid at a specific date and the amount that is repaid is fixed as well (usually the face value of the bond).
 - As this payment is known and neglecting the possibility of default, there cannot be a bubble at the maturity of the bond. Even if the issuer can default on the bond, the size of the bubble will be strictly limited as the most that will be repaid is the agreed amount and the least repaid is zero, giving maximum sizes for the size of the bubble.
- - Investors know that the bubble will burst at maturity
 - and hence they would sell the bond in the time period before to avoid the certain loss.
- ⇒
 - As investors sell the bond, the bubble will burst a time period prior to maturity.
 - Now investors know that the bubble will burst a time period prior to maturity.
- ⇒ The same logic now applies and the bubble will burst two time periods prior to maturity
- ⇒ This logic now applies all the way back to the current time period.
- ⇒ Hence we cannot have a bubble, positive or negative, in fixed income markets.
- **The fixed repayment amount and maturity date are certainties and this does not allow the emergence of any bubble, positive or negative.**

Positive and negative bubbles in exchange rates

Positive and negative bubbles in exchange rates

- However, we will now argue that negative bubbles can exist, at least in currencies.
- Foreign exchange markets are different in that no lower or upper limitation of the price applies, even though exchange rates cannot be negative.
- A bubble in a currency can be a positive bubble for one investor (overvaluation of their currency), but for an investor in the other currency it would be a negative bubble (undervaluation of the other currency). The exchange rates are e and $\frac{1}{e}$, respectively, thus they can never reach zero. Thus a positive bubble is always also a negative bubble and vice versa.
- In derivatives markets, negative values are possible (futures/forwards and swaps) and hence we have no lower limit and negative bubble should also be possible.
- However, derivatives are different in that their value is tied to the value of the underlying asset and arbitrage between the derivative and the underlying asset should make any bubbles impossible.
- It is thus only foreign exchange markets that can have negative bubbles, otherwise only positive bubbles can be observed.

Positive and negative bubbles in exchange rates

- ▶ In foreign exchange markets positive **and** negative bubbles can emerge

Positive and negative bubbles in exchange rates

- However, we will now argue that negative bubbles can exist, at least in currencies.
- Foreign exchange markets are different in that no lower or upper limitation of the price applies, even though exchange rates cannot be negative.
- A bubble in a currency can be a positive bubble for one investor (overvaluation of their currency), but for an investor in the other currency it would be a negative bubble (undervaluation of the other currency). The exchange rates are e and $\frac{1}{e}$, respectively, thus they can never reach zero. Thus a positive bubble is always also a negative bubble and vice versa.
- In derivatives markets, negative values are possible (futures/forwards and swaps) and hence we have no lower limit and negative bubble should also be possible.
- However, derivatives are different in that their value is tied to the value of the underlying asset and arbitrage between the derivative and the underlying asset should make any bubbles impossible.
- It is thus only foreign exchange markets that can have negative bubbles, otherwise only positive bubbles can be observed.

Positive and negative bubbles in exchange rates

- ▶ In foreign exchange markets positive and negative bubbles can emerge
- ▶ A **positive bubble** for one investor is a **negative bubble** in the other country

Positive and negative bubbles in exchange rates

- However, we will now argue that negative bubbles can exist, at least in currencies.
- Foreign exchange markets are different in that no lower or upper limitation of the price applies, even though exchange rates cannot be negative.
- A bubble in a currency can be a positive bubble for one investor (overvaluation of their currency), but for an investor in the other currency it would be a negative bubble (undervaluation of the other currency). The exchange rates are e and $\frac{1}{e}$, respectively, thus they can never reach zero. Thus a positive bubble is always also a negative bubble and vice versa.
- In derivatives markets, negative values are possible (futures/forwards and swaps) and hence we have no lower limit and negative bubble should also be possible.
- However, derivatives are different in that their value is tied to the value of the underlying asset and arbitrage between the derivative and the underlying asset should make any bubbles impossible.
- It is thus only foreign exchange markets that can have negative bubbles, otherwise only positive bubbles can be observed.

Positive and negative bubbles in exchange rates

- ▶ In foreign exchange markets positive and negative bubbles can emerge
- ▶ A positive bubble for one investor is a negative bubble in the other country
- ▶ **In principle** positive and negative bubbles can emerge in derivatives markets

Positive and negative bubbles in exchange rates

- However, we will now argue that negative bubbles can exist, at least in currencies.
- Foreign exchange markets are different in that no lower or upper limitation of the price applies, even though exchange rates cannot be negative.
- A bubble in a currency can be a positive bubble for one investor (overvaluation of their currency), but for an investor in the other currency it would be a negative bubble (undervaluation of the other currency). The exchange rates are e and $\frac{1}{e}$, respectively, thus they can never reach zero. Thus a positive bubble is always also a negative bubble and vice versa.
- In derivatives markets, negative values are possible (futures/forwards and swaps) and hence we have no lower limit and negative bubble should also be possible.
- However, derivatives are different in that their value is tied to the value of the underlying asset and arbitrage between the derivative and the underlying asset should make any bubbles impossible.
- It is thus only foreign exchange markets that can have negative bubbles, otherwise only positive bubbles can be observed.

Positive and negative bubbles in exchange rates

- ▶ In foreign exchange markets positive and negative bubbles can emerge
- ▶ A positive bubble for one investor is a negative bubble in the other country
- ▶ In principle positive and negative bubbles can emerge in derivatives markets
- ▶ **Arbitrage** with the underlying asset should **prevent** any such **mispicing**

Positive and negative bubbles in exchange rates

- However, we will now argue that negative bubbles can exist, at least in currencies.
- Foreign exchange markets are different in that no lower or upper limitation of the price applies, even though exchange rates cannot be negative.
- A bubble in a currency can be a positive bubble for one investor (overvaluation of their currency), but for an investor in the other currency it would be a negative bubble (undervaluation of the other currency). The exchange rates are e and $\frac{1}{e}$, respectively, thus they can never reach zero. Thus a positive bubble is always also a negative bubble and vice versa.
- In derivatives markets, negative values are possible (futures/forwards and swaps) and hence we have no lower limit and negative bubble should also be possible.
- **However, derivatives are different in that their value is tied to the value of the underlying asset and arbitrage between the derivative and the underlying asset should make any bubbles impossible.**
- It is thus only foreign exchange markets that can have negative bubbles, otherwise only positive bubbles can be observed.

Positive and negative bubbles in exchange rates

- ▶ In foreign exchange markets positive and negative bubbles can emerge
- ▶ A positive bubble for one investor is a negative bubble in the other country
- ▶ In principle positive and negative bubbles can emerge in derivatives markets
- ▶ Arbitrage with the underlying asset should prevent any such mispricing

Positive and negative bubbles in exchange rates

- However, we will now argue that negative bubbles can exist, at least in currencies.
- Foreign exchange markets are different in that no lower or upper limitation of the price applies, even though exchange rates cannot be negative.
- A bubble in a currency can be a positive bubble for one investor (overvaluation of their currency), but for an investor in the other currency it would be a negative bubble (undervaluation of the other currency). The exchange rates are e and $\frac{1}{e}$, respectively, thus they can never reach zero. Thus a positive bubble is always also a negative bubble and vice versa.
- In derivatives markets, negative values are possible (futures/forwards and swaps) and hence we have no lower limit and negative bubble should also be possible.
- However, derivatives are different in that their value is tied to the value of the underlying asset and arbitrage between the derivative and the underlying asset should make any bubbles impossible.
- **It is thus only foreign exchange markets that can have negative bubbles, otherwise only positive bubbles can be observed.**

Summary

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is **unknown** when they will burst

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use **backward induction** on when to sell or buy the asset

Summary

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - **Is an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.**
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will **not** emerge

Summary

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - **The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.**
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- ▶ As long as they exist, bubbles will **grow faster** than the asset value to compensate for the risk of the bubble bursting

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- **Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.**
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- ▶ As long as they exist, bubbles will grow faster than the asset value to compensate for the risk of the bubble bursting
- ▶ Bubbles are mostly reported in **stock markets**

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - **Most commonly bubbles are discussed in the stock market,**
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- ▶ As long as they exist, bubbles will grow faster than the asset value to compensate for the risk of the bubble bursting
- ▶ Bubbles are mostly reported in stock markets, **real estate**

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - **but often also in real estate markets, which can also see an unsustainable increase in prices.**
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- ▶ As long as they exist, bubbles will grow faster than the asset value to compensate for the risk of the bubble bursting
- ▶ Bubbles are mostly reported in stock markets, real estate, and sometimes in **exchange rates**

Summary

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - **At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.**
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- ▶ As long as they exist, bubbles will grow faster than the asset value to compensate for the risk of the bubble bursting
- ▶ Bubbles are mostly reported in stock markets, real estate, and sometimes in exchange rates and **commodities**

Summary

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - **Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).**
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').

Summary

- ▶ Bubbles can emerge as long as it is unknown when they will burst
- ▶ Knowing a time by which they will burst, will allow investors to use backward induction on when to sell or buy the asset, the bubble will not emerge
- ▶ As long as they exist, bubbles will grow faster than the asset value to compensate for the risk of the bubble bursting
- ▶ Bubbles are mostly reported in stock markets, real estate, and sometimes in exchange rates and commodities

Summary

- We can now summarize our main finding about the existence and properties of bubbles.
- We have established that bubbles can emerge in any asset where there is no certain time at which it will burst, such as the maturity of the asset or the reaching of a certain price threshold (zero price).
- - If an investor knows when the bubble will burst, he can work backwards and sell (or buy for a negative bubble) the asset a time period before that.
 - The resulting demand for the asset will cause the bubble to burst earlier and earlier, such that the bubble does not emerge in the first place as it would burst immediately.
- Bubbles that persist are growing faster than the value of the asset; it is this high growth rate that compensates for the possibility of the bubble bursting.
- - Most commonly bubbles are discussed in the stock market,
 - but often also in real estate markets, which can also see an unsustainable increase in prices.
 - At times exchange rates are persistently undervalued or overvalued and this can be interpreted as a bubble.
 - Much rarer are bubbles in commodities (some times observed in gold, particularly during times of political or wider economic uncertainty).
- As assets with bubbles give the same return as the fundamental value, it is rational to invest into such assets, even if the investor knows that there is a bubble. He hopes to be able to sell the asset later, before the bubble bursts ('ride the wave').



Copyright © by Andreas Krause

Picture credits:

Cover: Tobias Deml, CC BY-SA 4.0 <https://creativecommons.org/licenses/by-sa/4.0/>, via Wikimedia Commons, <https://upload.wikimedia.org/wikipedia/commons/2/26/Gaming-Wall-Street.BTS.Prodigium-266.jpg>

Back: Michael Vadon, CC BY 2.0 <https://creativecommons.org/licenses/by/2.0/>, via Wikimedia Commons, [https://upload.wikimedia.org/wikipedia/commons/9/97/Manhattan\(NYC-New_York_City\)Skyline\(31769153946\).jpg](https://upload.wikimedia.org/wikipedia/commons/9/97/Manhattan(NYC-New_York_City)Skyline(31769153946).jpg)

Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk