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Performance evaluation

# Outline

- Exploiting market inefficiencies
- Sharpe ratio
- Jensen's  $\alpha$
- Portfolio return decomposition
- Summary

- Investors seek to exploit their perceived informational advantage over other investors and make a profits from this information.
- While they might generate returns that are higher than other investments, this return needs to put into context of the risk the investor is taking.
- As a higher risk implies that higher returns should be achieved to compensate for this additional risk, we need to consider the risks investors are taking.
- We will here see how the performance of investors can be measured and the performance can be assessed against relevant benchmarks.

# Outline

- We will look at the two most common performance measures, the Sharpe ratio and Jensen's  $\alpha$ , but then also look at ways to decompose portfolio returns into different component to give a more comprehensive view of the performance of investors, taking elements of both performance measures into account.

## ■ Exploiting market inefficiencies

■ Sharpe ratio

■ Jensen's  $\alpha$

■ Portfolio return decomposition

■ Summary

- We can now briefly outline the way investors seek to exploit any market inefficiencies and point out the different risks they are taking.

# Trading on information

- ▶ Investors may seek to exploit information they have obtained, believing markets are inefficient
- ▶ They might buy assets they believe are undervalued or will increase in value
- ▶ They might sell assets they believe are overvalued or will decrease in value

- Information is obtained with the aim of using it in order to make profits from trading.
- - Investors might have obtained information which they seek to trade on to make profits.
  - This implies that they believe that this information is not yet included into the price and hence markets are inefficient.
- If the information suggests that the future price will be higher than the current price the investor would buy the asset. If we assume that the market will be efficient in the future, the future price will equal the value of the asset and hence we often talk of 'undervalued' assets.
- If the information suggests that the future price will be lower than the current price the investor would sell the asset. If we assume that the market will be efficient in the future, the future price will equal the value of the asset and hence we often talk of 'overvalued' assets.
- Hence in order for an investor to believe he can make profits, he must believe that the market is inefficient and the information he has received is not included into the price yet, but that it will be included into the price in the future. If the information were never included into the price, the price would not move and there are profits the investor could make.

## Considering risks

- ▶ Trying to exploit perceived market inefficiencies will change the return on their investments
- ▶ As they will deviate from their optimal portfolio, the risk of their portfolio will also change
- ▶ To assess the performance of investors, we need to take this risk into account

- Information is rarely perfect and there remains a risk that the information was not correct, or that subsequent information causes an adverse price movement before the price could adjust to the information concerned.
- ▶ The consequence of trading on information is that the return on investment will change compared to the investor not trading.
- ▶ Another consequence of trading on information is that the portfolio the investor holds will deviate from his long-term optimal portfolio.
- ▶ This will also mean that the risks of the portfolio the investor holds will change. We have to take into account the different risk such a portfolio is exposed to and how this affects the utility derived from it/.
- We thus cannot rely only on returns to assess the performance of an investor, but have to consider the risks as well.

# Risk types

**Systematic risk** The risk affecting the market as a whole

**Unsystematic risk** The risk affecting only an individual asset

- ▶ To assess the performance of an investor, we need to adjust their returns by the risk they are taking
- ▶ Depending on the relevant risk, we need to make different adjustments

- Depending on the context of our assessment of the investor's performance, different types of risks are relevant and need to be considered.
- ▶ The first type of risk is systematic risk, which is the risk arising from the co-movement with the market (the  $\beta$ -risk from the Capital Asset Pricing Model) and which cannot be diversified.
- ▶ The other risk is the unsystematic or idiosyncratic risk, which is the risk relevant only to a single asset; this risk can be diversified. The total risk of a portfolio will consist of the combinations of all the systematic and unsystematic risks. The total risk of a single asset is the combination of systematic and unsystematic risk.
- ▶ A good performance measure will take into the risks the investor is taking and the performance measures will adjust returns accordingly.
- ▶ How such adjustments are made will depend on the risks we have to consider. And which risks are relevant will in turn depend on the objectives and concerns of the person assessing the performance, which do not necessarily coincide with that of the investor.
- We will now consider two of the most common such performance measures. Although many other performance measures have been developed, often for use in specific contexts, these are the most widely used measures and illustrate the idea of adjusting returns for risk.

## ■ Exploiting market inefficiencies

### ■ Sharpe ratio

#### ■ Jensen's $\alpha$

#### ■ Portfolio return decomposition

#### ■ Summary

- We first consider the Sharpe ratio, which uses the total risk of an asset or portfolio as the relevant risk for adjusting the return.
- Using the Sharpe ratio allows us to compare the performance of two portfolios, such as a benchmark portfolio.

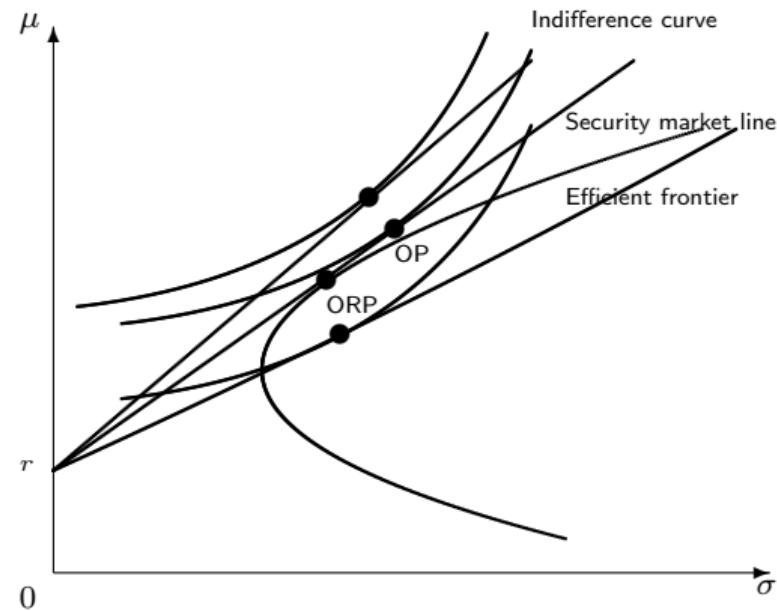
# Considering total risk

- ▶ If we are concerned about the total risk, we use variance as a risk measure
- ▶ From portfolio theory we know that decisions are made using the mean return and return variance
- ▶ We can compare returns and risks building on portfolio selection theory

## Considering total risk

- We consider the total risk of a portfolio as the relevant risk, thus we use the combination of systematic and unsystematic risk.
- ▶ If we are concerned about the total risk of a portfolio, we can use the variance of the return on the portfolio as our risk measure.
- ▶ Portfolio theory tells us that investment decisions are based on the expected return and the variance of this return, in addition to the covariances in a portfolio. If we only consider a portfolio and interpret this as a single asset, covariances can be ignored. The actually realised return of a given period of time can then be used to estimate the expected return and variance.
- ▶ The idea of the Sharpe ratio is to assess the risk-return relationship using portfolio theory.
- We will see how the Sharpe ratio relates to portfolio theory

## Portfolio selection



- We now briefly recap the basics of portfolio theory, which then allows us to derive the performance measure.
- ▶ We consider decisions using the mean (expected return) and the risk (standard deviation).
- ▶ We then know that the efficient frontier, that is the collection of all portfolios that cannot be ranked without use of more than the assumption of risk aversion, is this hyperbolic curve.
- ▶ If we now introduce a risk-free asset, which we know gives us the straight security market line, which is tangential to the efficient frontier and crosses the vertical axis at the risk-free rate.
- ▶ The tangential point is the location of the Optimal Risky Portfolio (ORP), representing the portfolio of risky assets that all risk-averse investors hold, as long as they agree on the mean, variances, and covariances of the assets in the portfolio. This portfolio is independent of the specifics of the preferences, we only need investors to be risk-averse.
- ▶ With a higher mean increasing the utility of investors, but a higher risk reducing the utility, the indifference curve will be positively sloped.
- ▶ The optimal portfolio (OP) will then be a combination of the ORP and the risk-free asset and it is located where the indifference curve is tangential to the security market line.
- ▶ Let us now consider a portfolio that is located above the security market line; it will have a higher return and/or a lower risk than the OP.
- ▶ We can draw a straight line, similar to the security market line through this portfolio. This line will have a higher slope than the security market line.
- ▶ We can now find the indifference curve at this portfolio. We see that the indifference curve is to the upper left of the indifference curve giving us the OP. This implies a higher utility level.
- ▶ Let us now consider a portfolio that is located below the security market line; it will have a lower return and/or a higher risk than the OP.
- ▶ We can draw a straight line, similar to the security market line through this portfolio. This line will have a lower slope than the security market line.
- ▶ We can now find the indifference curve at this portfolio. We see that the indifference curve is to the lower right of the indifference curve giving us the OP. This implies a lower utility level.
- We can now develop a performance measure based on the slope of this straight line through the portfolio. We have seen that a higher slope corresponds to a higher utility level, which indicates a better performance.

# Slope as performance measure

- ▶ A higher slope corresponds to a higher utility level
- ▶ We use the slope as a performance measure
- ▶ The vertical direction gives the excess return of the investor over the risk-free rate
- ▶ The horizontal direction represents the risk
- ▶  $SR_i = \frac{\mu_i - r}{\sigma_i}$
- ▶ The Sharpe ratio measures the excess return relative to the total risk the investor takes

- Using the intuition of exploiting the slope as an indicator which portfolio is referred, we can now proceed to define this slope more formally as a performance measure.
- ▶ We have seen that a higher slope of the line from the risk-free asset to the portfolio corresponds to a higher utility level
- ▶ We therefore use this slope as our performance measure, a higher value will indicate a higher performance.
- ▶ The length of the vertical direction is the difference between the expected return of the portfolio,  $\mu_i$  and the risk-free rate,  $r$ . This is often referred to as the excess return the investor generates over the risk-free rate.
- ▶ The length of the horizontal direction is the risk (standard deviation) of the portfolio, given that the risk of the risk-free asset is nil.
- ▶ *Formula*
- ▶ The Sharpe ratio is simply the excess return divided by the standard deviation.
- The Sharpe ratio itself is not meaningful, it always needs to be compared with another portfolio; this might be a benchmark portfolio for the investors, such as the market portfolio. As we use the standard deviation of returns, the risks considered are the total risks of both portfolios.

■ Exploiting market inefficiencies

■ Sharpe ratio

■ Jensen's  $\alpha$

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■ Summary

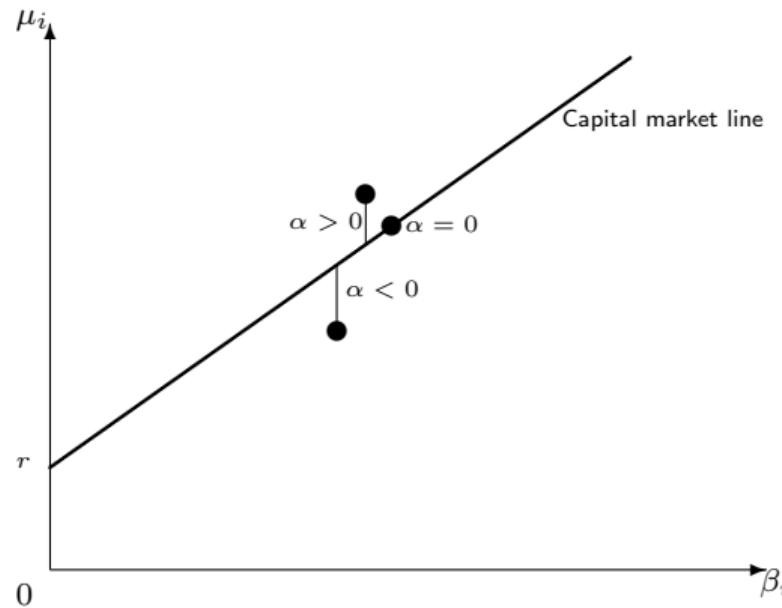
- We can now look at an alternative performance measure that takes into account only systematic risk.

# Using asset pricing benchmark

- ▶ If their portfolio is well-diversified, the investor will not be concerned with idiosyncratic risk
- ▶ Rather than total risk, the risk assessment will be based on systematic risk only
- ▶ We can use the Capital Asset Pricing Model (CAPM) to determine the return required for a given systematic risk

- We will consider the reasons for focussing on systematic risk first.
- An investor who holds a well-diversified portfolio will have diversified away unsystematic risk and will therefore not be much concerned about such risk. This might not be the case for the investor directly, but, for example, for a client is the investor is an asset manager; even if the portfolio itself is not well-diversified, the client might be through holding other investments besides those considered here. In this case they would want to use a performance measure that excludes unsystematic risk as they are not concerned about it.
- If we exclude unsystematic risk, we only have to be concerned about systematic risk.
- With the Capital Asset Pricing Model serving as a benchmark for the return that should be generated for a given level of systematic risk, we can compare the return of the portfolio with that implied by the CAPM.
- We can now develop a performance measure intuitively based on these ideas.

# Using the CAPM as a benchmark



# Using the CAPM as a benchmark

- We now show the intuitive idea behind using the CAPM as a benchmark.
- We consider the expected return ( $\mu_i$ ) and the systematic risk  $\beta_i$  of a portfolio.
- The CAPM gives us the market line as a straight line crossing the vertical axis at the risk-free rate.
- A portfolio that generates the required rate of return will be located on the capital market line.
- We define this situation as the portfolio producing no return beyond that required by the CAPM and set it as  $\alpha = 0$ .
- Consider now a portfolio above the Capital market line. Such a portfolio will have a higher return and/or a lower systematic risk.
- The performance measure will now be the difference between the return of this portfolio and the return that it should generate according to the CAPM. In this case the difference is positive and the portfolio performs better than it should according to the CAPM.
- Consider now a portfolio below the Capital market line. Such a portfolio will have a lower return and/or a higher systematic risk.
- The performance measure will now be the difference between the return of this portfolio and the return that it should generate according to the CAPM. In this case the difference is negative and the portfolio performs worse than it should according to the CAPM.
- Thus the larger the difference to the return implied by the CAPM, the better the performance of the portfolio. We have a natural benchmark here to compare the performance with and that is the market portfolio, which is located at  $\beta_1 = 1$ .

## Differences in returns

- ▶ The CAPM gives the excess return of an asset as the excess return of the market, adjusted for systematic risk
- ▶  $\hat{\mu}_i - r = \beta_i (\mu_M - r)$
- ▶ The performance measure is the difference of the actual return and the return implied by the CAPM
- ▶  $\alpha = \mu_i - \hat{\mu}_i$
- ▶ Jensen's  $\alpha$  only considers systematic risk, any idiosyncratic risk will be ignored

- Using the intuition of using the difference between the return and its CAPM benchmark, we can now define this performance measure more formally.
  - ▶
    - The CAPM gives us the return an asset, or any portfolio, should achieve over and above the risk-free rate.
    - This excess return of the asset or portfolio is determined by the excess return of the market portfolio,
    - which is then adjusted for the systematic risk of the asset or portfolio.
  - ▶ *Formula*
  - ▶ The Return suggested by the CAPM serves as the benchmark return for assessing the return of a portfolio, which is simply the difference in the actual return and the benchmark return.
  - ▶ *Formula*
  - ▶
    - This performance measure, known as Jensen's  $\alpha$  uses the return of the portfolio, adjusted with an expression including  $\beta_i$ , which represents the systematic risk of the portfolio. Hence the return is adjusted only for systematic risk.
    - Any idiosyncratic (unsystematic) risk will not be considered.
- We thus have two performance measures, the Sharpe ratio considering all risks, while Jensen's  $\alpha$  only considers systematic risk. Which risk measure is to be chosen, will depend on the purpose of the performance evaluation and which risks are relevant to the person conducting the analysis. If they are concerned about unsystematic risk as well as systematic risk, they should choose the Sharpe ratio; if they are concerned only about systematic risk, Jensen's  $\alpha$  is the better alternative.

■ Exploiting market inefficiencies

■ Sharpe ratio

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- We will now address the lack of concern for unsystematic risk when using Jensen's  $\alpha$ .
- Ignoring unsystematic risk can give incentives to portfolio managers to choose portfolios that have low systematic risk, but high unsystematic risk, as long as such risk is rewarded in the market through a higher return. This would imply that the CAPM does not hold.

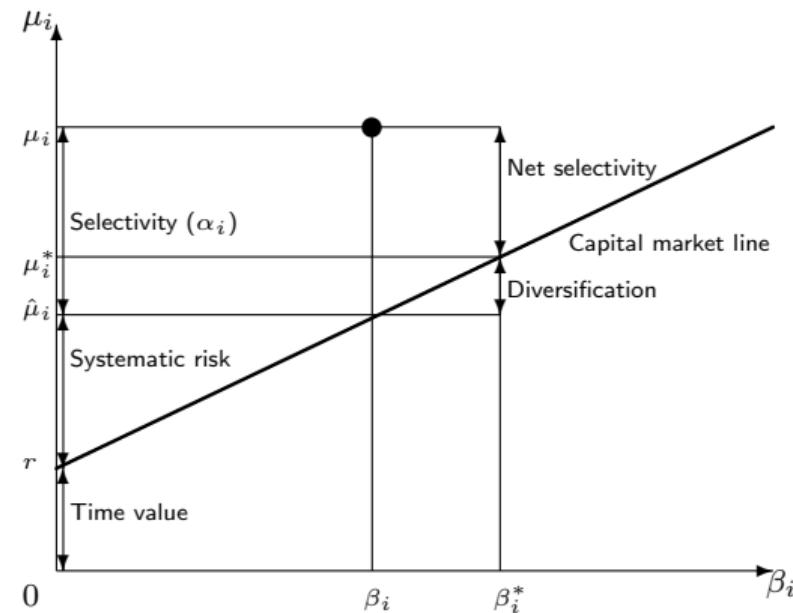
# Adjusting performance for idiosyncratic risk

- ▶ The Sharpe ratio takes into account systematic and idiosyncratic risk, while Jensen's  $\alpha$  only takes into account systematic risk
- ▶ A main shortcoming using Jensen's  $\alpha$  is that it does not consider the potential for an investor incurring additional idiosyncratic risks
- ▶ If actual returns follow the CAPM, then  $\hat{R}_i = r + \beta_i (R_M - r)$
- ⇒ Total risk:  $\sigma_i^2 = \beta_i^2 \sigma_M^2$
- ▶ If an investor now incurs total risk  $\sigma_i$ , then the systematic risk equivalent to this total risk is  $\beta_i^* = \frac{\sigma_i}{\sigma_M}$

# Adjusting performance for idiosyncratic risk

- We seek to make an adjustment to Jensen's  $\alpha$  that takes into account unsystematic (idiosyncratic) risk.
- ▶ • We have seen that the Sharpe ratio takes into account unsystematic and systematic risk,
  - and Jensen's  $\alpha$  only takes into account systematic risk.
- ▶ The benefit of using Jensen's  $\alpha$  is that it is more intuitively interpreted as the additional return the portfolio generates, while the Sharpe ratio is a slope which lacks any direct interpretation. However, when using Jensen's  $\alpha$ , we cannot consider unsystematic risk, which in some portfolios can be substantial. This might be particularly the case if we were to assess the strategy of using information on an individual asset; in this case unsystematic risk is not diversified at all.
- ▶ Let us assume that actual returns follow the CAPM perfectly, thus the actual return (and not only the expected return) will be determined by the actual return of the market (rather than only its expected value) in line with the CAPM equation.
- ⇒ We can now take the variance of the returns of the portfolio. Taking into account that the risk-free rate is constant, the variance will depend on the variance of the market and  $\text{beta}_i$  of the portfolio.
- ▶ The variance, or standard deviation, was used as a measure of the total risk of the portfolio. If we know the total risk of the portfolio (standard deviation), we can determine the amount of systematic risk that would generate the same amount of risk. by solving for  $\beta_i$ . A portfolio with systematic risk  $\beta_i^*$  has the same amount of systematic risk as the total risk of a portfolio with standard deviation  $\sigma_i$ .
- We can now intuitively determine an adjustment to Jensen's  $\alpha$  that takes unsystematic risk into account.

## Determining net selectivity



# Determining net selectivity

- We will now take into account the unsystematic risk in a portfolio and adjust Jensen's  $\alpha$  accordingly.
- We consider the expected return ( $\mu_i$ ) and the systematic risk  $\beta_i$  of a portfolio.
- The CAPM gives us the market line as a straight line crossing the vertical axis at the risk-free rate.
- We consider some portfolio with given characteristics.
- We can determine the return and systematic risk.
- Given the systematic risk, we can determine the benchmark return using the CAPM.
- We can now divide the total return in three components. The first component represents compensation for giving up liquidity by making the investment. This is often referred to as 'time value' and would accrue from investing into a risk-free asset.
- The return to the benchmark return is now the compensation for the portfolio being exposed to systematic risk.
- The final element is now the additional value added by from managing the portfolio, usually attributed to selecting investment profitably and is referred to as 'selectivity'. This is exactly Jensen's  $\alpha$ .
- We now take into account the unsystematic risk and determine the equivalent amount of systematic risk required such that it would be equal to the total risk of the portfolio. Note that this value is always higher than the actual systematic risk as unsystematic risk cannot be negative.
- We can use the CAPM again to determine the return that is required in a portfolio that exhibits this amount of systematic risk.
- We can now attribute the difference between the two CAPM benchmarks as the compensation for risks the portfolio incurs by not diversifying fully; equivalent this part of the return would not be incurred if the portfolio was properly diversified.
- The performance measure is here called 'net selectivity' and represents the return made in excess of the return required from taking the equivalent systematic risk.
- Note that even if selectivity positive, net selectivity may be negative; net selectivity is always lower than selectivity as the equivalent systemic risk is always higher than the actual systemic risk.

# Net selectivity

- ▶ The selectivity is Jensen's  $\alpha$
- ▶ Investors may take additional idiosyncratic risk, equivalent to a total systematic risk of  $\beta_i^*$
- ▶ This will increase the benchmark return to  $\mu_i^*$
- ▶ Taking into account the idiosyncratic risk, the net selectivity represents the value added by investors

- We have thus modified Jensen's  $\alpha$  to take into account the idiosyncratic risk of the portfolio.
- ▶ What in this context is termed selectivity is identical to Jensen's  $\alpha$ . We will now adjust this value to take into account the unsystematic risk incurred in the portfolio.
  - ▶ A portfolio that is not fully diversified will be exposed to unsystematic (idiosyncratic) risk.
  - We have used the total risk of the portfolio, consisting of systematic and unsystematic risk, to determine the  $\beta_i$  that would generate the same amount of systematic risk as the total risk.
- ▶ We use this hypothetical systematic risk to determine a benchmark return using the CAPM.
- ▶ Using this revised benchmark, we then use the idea from Jensen's  $\alpha$  to determine the net selectivity as our performance measure, defined as the difference between the actual return and the revised benchmark return. This return is the value added by the investor due to his ability to select assets and time investments, using information he will have obtained.
- Net selectivity retains the properties of Jensen's  $\alpha$  in being a return in excess of the return required to compensate for the risk taken, but includes systematic and unsystematic risk. While the Sharpe ratio is more commonly used to assess portfolio performance against the total risk of a portfolio, net selectivity might be the more intuitive measure. A drawback is, however, that the benchmark return cannot be chosen freely as it is set to the market portfolio; however, the net selectivity of another portfolio can be determined and the two net selectivities can be compared, giving the same effect.

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- We can now summarise the key ideas about the performance evaluation of portfolios.

# Importance of risk adjustments

- ▶ Investors exploiting market inefficiencies may generate higher returns
- ▶ Higher returns might be associated with higher risks
- ▶ Taking into account the additional risks, shows if investors generate economic profits
- ▶ Risk adjustments can be made for systematic risk only or for all risks
- ▶ Systematic risk is appropriate if a portfolio is well diversified
- ▶ Total risk should be used if a portfolio is not well diversified

# Importance of risk adjustments

- Investors might have additional information and if the market is not fully efficient, they will seek to exploit their knowledge by trading on the information they hold. As information is not perfect, investors are exposed to risks and when determining whether the information was used effectively, the risks have to be taken into account.
- ▶ When making use of information, assuming that the information is overall reliable and the investor has sufficient skills to exploit their informational advantage, higher returns should be generated than from not seeking to exploit the information.
- ▶ Financial economics has established that typically higher returns and higher risks are highly correlated. We therefore have to consider whether the higher returns the investor generates are the result of taking higher risks and merely compensation for these risks, or whether they actually represent added value.
- ▶ Thus we have to consider the risks investors take to determine whether the return generated is compensation for additional risks or the results of the skills and information of the investor.
- ▶
  - We may make adjustment only for systematic risk with the argument that other risks are irrelevant as they can be diversified away.
  - On the other hand, we may want to consider all risks using the argument that these are the risks investors are actually exposed to and which will reduce the utility of risk-averse investors, regardless of whether they could be diversified away.
- ▶ In this context if a well-diversified portfolio is held, unsystematic risk can be ignored and the risk adjustment should only include the systematic risk.
- ▶ In the other case, all risk should be considered. If a portfolio is well-diversified, there is no unsystematic risk and the total and systematic risks are identical, making the choice of risk measure arbitrary.
- In the case where the portfolio is not well-diversified, which will be more realistic, the choice of risk measure and hence the performance will become important.

# Choice of performance measure

- ▶ Adjusting returns with different risk measures can lead to different results
- ▶ An investor might generate a high Jensen's  $\alpha$
- ▶ The same investor might increase its idiosyncratic risk and this can lead to a low performance if measured by the Sharpe ratio
- ▶ Determining which performance measure to use will depend on which type of risk is relevant

- The choice of an appropriate risk measure can affect the assessment of the performance, thus it is important to be aware of the type of risks the different risk measures include and how the performance is determined.
- ▶ Using different risk measures can give different results in the performance assessment. It might be that portfolio *A* is performing better than portfolio *B* using one performance measure, but will perform worse when using another performance measure.
- ▶ Let us assume we have portfolio *A* which generates a high Jensen's  $\alpha$  as the investors makes use of the information he holds, while the original portfolio *B* generates a much lower Jensen's  $\alpha$ .
- ▶
  - However, portfolio *A* might have a high unsystematic risk as a result of the portfolio becoming much less well diversified as the investor sought to trade on his information.
  - This high level of unsystematic risk will increase its overall risk and thus when using the Sharpe ratio will reduce its performance, resulting in a lower performance than the original portfolio *B*.
- ▶ We need to select the performance measure accordingly and take into account which risks have to be considered in the performance assessment.
- There is no general answer to which risk should be considered as it depends on the specific circumstances of the evaluator. The risks considered have to be those that are relevant to the person making (or receiving) the performance evaluation.



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