Andreas Krause

The irrelevance of capital structure decisions

- We will now seek to determine that capital structure decisions do not affect the value of companies.
- To achieve this, we will assume that markets are perfect and there is no moral hazard or adverse selection in the market, not that there are tax differences in the treatment of debt and equity.

Company value

Company value

- → In order to establish that capital structure does not affect the value of a company as a whole, we first define what we mean by company value.
- ▶ We assess different capital structures by determining the company value in each case and then comparing these values.
- ▶ While the usual emphasis is on the value of equity, we here look at the value of the company as whole by combining the value of debt and equity.
- ▶ Similar to equity, this value will be generated from the investment companies make and the surplus such investments generate.
- → We can now compare companies with different capital structures.

Company value

- To assess the impact capital structure decisions have on company value, we can compare the value of companies with different levels of debt

Company value

- → In order to establish that capital structure does not affect the value of a company as a whole, we first define what we mean by company value.
- ▶ We assess different capital structures by determining the company value in each case and then comparing these values.
- ▶ While the usual emphasis is on the value of equity, we here look at the value of the company as whole by combining the value of debt and equity.
- ▶ Similar to equity, this value will be generated from the investment companies make and the surplus such investments generate.
- → We can now compare companies with different capital structures.

Company value

- To assess the impact capital structure decisions have on company value, we can compare the value of companies with different levels of debt
- Company value here refers to the value of debt and equity combined

Company value

- → In order to establish that capital structure does not affect the value of a company as a whole, we first define what we mean by company value.
- ▶ We assess different capital structures by determining the company value in each case and then comparing these values.
- ▶ While the usual emphasis is on the value of equity, we here look at the value of the company as whole by combining the value of debt and equity.
- ▶ Similar to equity, this value will be generated from the investment companies make and the surplus such investments generate.
- → We can now compare companies with different capital structures.

Company value

- To assess the impact capital structure decisions have on company value, we can compare the value of companies with different levels of debt
- Company value here refers to the value of debt and equity combined
- This value will be based on the ability of the company to generate profits

Company value

- → In order to establish that capital structure does not affect the value of a company as a whole, we first define what we mean by company value.
- ▶ We assess different capital structures by determining the company value in each case and then comparing these values.
- ▶ While the usual emphasis is on the value of equity, we here look at the value of the company as whole by combining the value of debt and equity.
- ▶ Similar to equity, this value will be generated from the investment companies make and the surplus such investments generate.
- → We can now compare companies with different capital structures.

Company value

- To assess the impact capital structure decisions have on company value, we can compare the value of companies with different levels of debt
- Company value here refers to the value of debt and equity combined
- This value will be based on the ability of the company to generate profits

Company value

→ In order to establish that capital structure does not affect the value of a company as a whole, we first define what we mean by company value.

- We assess different capital structures by determining the company value in each case and then comparing these values.
- While the usual emphasis is on the value of equity, we here look at the value of the company as whole by combining the value of debt and equity.
- Similar to equity, this value will be generated from the investment companies make and the surplus such investments generate.

→ We can now compare companies with different capital structures.

Levered and unlevered companies

→ We can now look at how any surplus the investment generates is distributed within a company.
- We assume that regardless of their capital structure, the investments generate the same surplus V. Thus we only look at the combination of debt and equity for a given size of the investment, not investment of different sizes. This implicitly assumes that companies can rise sufficient equity they need for investments and have equally access to loans.
- A company without debt is called 'unlevered' and the entire surplus accrues to the equity holders.
- *Formula*
-
  - A company having debt is called 'levered' and the surplus it generates is accruing to equity holders,
  - but also their lenders.
- *Formula*

→ We can now see how much of the surplus is allocated to equity holders.

Levered and unlevered companies

- Companies generate an identical surplus from its investment in the next time period
- V

Levered and unlevered companies

- → We can now look at how any surplus the investment generates is distributed within a company.
- We assume that regardless of their capital structure, the investments generate the same surplus V. Thus we only look at the combination of debt and equity for a given size of the investment, not investment of different sizes. This implicitly assumes that companies can rise sufficient equity they need for investments and have equally access to loans.
- A company without debt is called 'unlevered' and the entire surplus accrues to the equity holders.
- *Formula*
-
  - A company having debt is called 'levered' and the surplus it generates is accruing to equity holders,
  - but also their lenders.
- *Formula*
- → We can now see how much of the surplus is allocated to equity holders.

Levered and unlevered companies

- Companies generate an identical surplus from its investment in the next time period
- If the company has no debt, this surplus is accruing to its equity holders
- $V \to \hat{E}$

Levered and unlevered companies

- → We can now look at how any surplus the investment generates is distributed within a company.
- We assume that regardless of their capital structure, the investments generate the same surplus V. Thus we only look at the combination of debt and equity for a given size of the investment, not investment of different sizes. This implicitly assumes that companies can rise sufficient equity they need for investments and have equally access to loans.
- A company without debt is called 'unlevered' and the entire surplus accrues to the equity holders.
- *Formula*
-
    - A company having debt is called 'levered' and the surplus it generates is accruing to equity holders,
    - but also their lenders.
- *Formula*
- → We can now see how much of the surplus is allocated to equity holders.

Levered and unlevered companies

- Companies generate an identical surplus from its investment in the next time period
- If the company has no debt, this surplus is accruing to its equity holders
- $V \rightarrow \hat{E}$
- If the company has debt, this surplus is accruing to equity holders
- $V \rightarrow E$

Levered and unlevered companies

- → We can now look at how any surplus the investment generates is distributed within a company.
- We assume that regardless of their capital structure, the investments generate the same surplus V. Thus we only look at the combination of debt and equity for a given size of the investment, not investment of different sizes. This implicitly assumes that companies can rise sufficient equity they need for investments and have equally access to loans.
- A company without debt is called 'unlevered' and the entire surplus accrues to the equity holders.
- *Formula*
-
    - A company having debt is called 'levered' and the surplus it generates is accruing to equity holders,
    - but also their lenders.
- *Formula*
- → We can now see how much of the surplus is allocated to equity holders.

Levered and unlevered companies

- Companies generate an identical surplus from its investment in the next time period
- If the company has no debt, this surplus is accruing to its equity holders
- $V \rightarrow \hat{E}$
- If the company has debt, this surplus is accruing to equity holders and lenders
- $V \rightarrow E + L$

- → We can now look at how any surplus the investment generates is distributed within a company.
- We assume that regardless of their capital structure, the investments generate the same surplus V. Thus we only look at the combination of debt and equity for a given size of the investment, not investment of different sizes. This implicitly assumes that companies can rise sufficient equity they need for investments and have equally access to loans.
- A company without debt is called 'unlevered' and the entire surplus accrues to the equity holders.
- *Formula*
-
  - A company having debt is called 'levered' and the surplus it generates is accruing to equity holders,
  - but also their lenders.
- *Formula*
- → We can now see how much of the surplus is allocated to equity holders.

Levered and unlevered companies

- Companies generate an identical surplus from its investment in the next time period
- If the company has no debt, this surplus is accruing to its equity holders
- $V \to \hat{E}$
- If the company has debt, this surplus is accruing to equity holders and lenders
- $V \to E + L$

Levered and unlevered companies

- → We can now look at how any surplus the investment generates is distributed within a company.
- We assume that regardless of their capital structure, the investments generate the same surplus V. Thus we only look at the combination of debt and equity for a given size of the investment, not investment of different sizes. This implicitly assumes that companies can rise sufficient equity they need for investments and have equally access to loans.
- A company without debt is called 'unlevered' and the entire surplus accrues to the equity holders.
- *Formula*
-
  - A company having debt is called 'levered' and the surplus it generates is accruing to equity holders,
  - but also their lenders.
- *Formula*
- → We can now see how much of the surplus is allocated to equity holders.

Investing into the levered company

→ We first look at the allocation of surplus to equity holders in a levered company.

- - With debt, the equity holders are allocated the surplus the company generates
  - but from this surplus have to repay the loan
  - including any interest that has been agreed.
- *Formula*

→ We can now turn our attention to the unlevered company.

Investing into the levered company

- If a company has debt, its equity holders will only obtain the surplus
- $E \to V$

Investing into the levered company

→ We first look at the allocation of surplus to equity holders in a levered company.

-
  - With debt, the equity holders are allocated the surplus the company generates
  - but from this surplus have to repay the loan
  - including any interest that has been agreed.
- *Formula*

→ We can now turn our attention to the unlevered company.

Investing into the levered company

- If a company has debt, its equity holders will only obtain the surplus less the loan repayment
- $E \to V - \qquad L$

Investing into the levered company

→ We first look at the allocation of surplus to equity holders in a levered company.

- - With debt, the equity holders are allocated the surplus the company generates
  - but from this surplus have to repay the loan
  - including any interest that has been agreed.
- *Formula*

→ We can now turn our attention to the unlevered company.

Investing into the levered company

- If a company has debt, its equity holders will only obtain the surplus less the loan repayment including interest
- $E \to V - (1 + r_L) L$

Investing into the levered company

→ We first look at the allocation of surplus to equity holders in a levered company.

- ▶
  - With debt, the equity holders are allocated the surplus the company generates
  - but from this surplus have to repay the loan
  - including any interest that has been agreed.
- ▶ *Formula*

→ We can now turn our attention to the unlevered company.

Investing into the levered company

- If a company has debt, its equity holders will only obtain the surplus less the loan repayment including interest
- $E \to V - (1 + r_L) L$

Investing into the levered company

→ We first look at the allocation of surplus to equity holders in a levered company.

- ▶
  - With debt, the equity holders are allocated the surplus the company generates
  - but from this surplus have to repay the loan
  - including any interest that has been agreed.
- ▶ *Formula*

→ We can now turn our attention to the unlevered company.

Investing into the unlevered company

Investing into the unlevered company

→ When considering an unlevered company we need to ensure that we use the same amount of initial equity and makes the same investment as the levered company.

- We therefore assume that the investor makes the same equity investment into both companies.
-
  - As to allow the company to make the same investment as the levered company, it is now the investor taking up a loan to increase the equity of the company to the same level as the debt an equity of the levered company.
  - The equity of the company now consists of the equity that is also invested into the levered company and the equity that financed by the loan. This gives total equity in this company of $\hat{E}$. Thus the equity investors owns the entire company, but we can rewrite his fraction of the company he owns as $\frac{E+L}{\hat{E}}$.
-
  - Based on his initial equity, he will be allocated the full surplus of the company, but we write this now as receiving his fraction $\frac{E+L}{\hat{E}}$ of the company, even though this fraction is 1.
  - From this surplus he now has to repay the loan he has taken out to increase the equity of the company.
- *Formula*

→ We can now compare the equity value of the levered and the unlevered companies.

Investing into the unlevered company

- Suppose an investor makes the same equity investment into the unlevered company as in the levered company

Investing into the unlevered company

→ When considering an unlevered company we need to ensure that we use the same amount of initial equity and makes the same investment as the levered company.

- We therefore assume that the investor makes the same equity investment into both companies.
-
  - As to allow the company to make the same investment as the levered company, it is now the investor taking up a loan to increase the equity of the company to the same level as the debt an equity of the levered company.
  - The equity of the company now consists of the equity that is also invested into the levered company and the equity that financed by the loan. This gives total equity in this company of $\hat{E}$. Thus the equity investors owns the entire company, but we can rewrite his fraction of the company he owns as $\frac{E+L}{\hat{E}}$.
-
  - Based on his initial equity, he will be allocated the full surplus of the company, but we write this now as receiving his fraction $\frac{E+L}{\hat{E}}$ of the company, even though this fraction is 1.
  - From this surplus he now has to repay the loan he has taken out to increase the equity of the company.
- *Formula*

→ We can now compare the equity value of the levered and the unlevered companies.

Investing into the unlevered company

- Suppose an investor makes the same equity investment into the unlevered company as in the levered company
- When investing into the unlevered company he also takes out a loan

Investing into the unlevered company

→ When considering an unlevered company we need to ensure that we use the same amount of initial equity and makes the same investment as the levered company.

- We therefore assume that the investor makes the same equity investment into both companies.
-
  - As to allow the company to make the same investment as the levered company, it is now the investor taking up a loan to increase the equity of the company to the same level as the debt an equity of the levered company.
  - The equity of the company now consists of the equity that is also invested into the levered company and the equity that financed by the loan. This gives total equity in this company of $\hat{E}$. Thus the equity investors owns the entire company, but we can rewrite his fraction of the company he owns as $\frac{E+L}{\hat{E}}$.
-
  - Based on his initial equity, he will be allocated the full surplus of the company, but we write this now as receiving his fraction $\frac{E+L}{\hat{E}}$ of the company, even though this fraction is 1.
  - From this surplus he now has to repay the loan he has taken out to increase the equity of the company.
- *Formula*

→ We can now compare the equity value of the levered and the unlevered companies.

Investing into the unlevered company

- Suppose an investor makes the same equity investment into the unlevered company as in the levered company
- When investing into the unlevered company he also takes out a loan such that he obtains a fraction $\frac{E+L}{\hat{E}}$ of the company

Investing into the unlevered company

→ When considering an unlevered company we need to ensure that we use the same amount of initial equity and makes the same investment as the levered company.

- We therefore assume that the investor makes the same equity investment into both companies.
-
  - As to allow the company to make the same investment as the levered company, it is now the investor taking up a loan to increase the equity of the company to the same level as the debt an equity of the levered company.
  - The equity of the company now consists of the equity that is also invested into the levered company and the equity that financed by the loan. This gives total equity in this company of $\hat{E}$. Thus the equity investors owns the entire company, but we can rewrite his fraction of the company he owns as $\frac{E+L}{\hat{E}}$.
-
  - Based on his initial equity, he will be allocated the full surplus of the company, but we write this now as receiving his fraction $\frac{E+L}{\hat{E}}$ of the company, even though this fraction is 1.
  - From this surplus he now has to repay the loan he has taken out to increase the equity of the company.
- *Formula*

→ We can now compare the equity value of the levered and the unlevered companies.

Investing into the unlevered company

- Suppose an investor makes the same equity investment into the unlevered company as in the levered company
- When investing into the unlevered company he also takes out a loan such that he obtains a fraction $\frac{E+L}{\hat{E}}$ of the company
- He will receive his fraction of the surplus
- $E \rightarrow \frac{E+L}{\hat{E}} V$

→ When considering an unlevered company we need to ensure that we use the same amount of initial equity and makes the same investment as the levered company.

- We therefore assume that the investor makes the same equity investment into both companies.
-
  - As to allow the company to make the same investment as the levered company, it is now the investor taking up a loan to increase the equity of the company to the same level as the debt an equity of the levered company.
  - The equity of the company now consists of the equity that is also invested into the levered company and the equity that financed by the loan. This gives total equity in this company of $\hat{E}$. Thus the equity investors owns the entire company, but we can rewrite his fraction of the company he owns as $\frac{E+L}{\hat{E}}$.
-
  - Based on his initial equity, he will be allocated the full surplus of the company, but we write this now as receiving his fraction $\frac{E+L}{\hat{E}}$ of the company, even though this fraction is 1.
  - From this surplus he now has to repay the loan he has taken out to increase the equity of the company.
- *Formula*

→ We can now compare the equity value of the levered and the unlevered companies.

Investing into the unlevered company

- Suppose an investor makes the same equity investment into the unlevered company as in the levered company
- When investing into the unlevered company he also takes out a loan such that he obtains a fraction $\frac{E+L}{\hat{E}}$ of the company
- He will receive his fraction of the surplus and repay his loan including interest
- $E \rightarrow \frac{E+L}{\hat{E}} V - (1 + r_L) L$

Investing into the unlevered company

→ When considering an unlevered company we need to ensure that we use the same amount of initial equity and makes the same investment as the levered company.

- We therefore assume that the investor makes the same equity investment into both companies.
-
  - As to allow the company to make the same investment as the levered company, it is now the investor taking up a loan to increase the equity of the company to the same level as the debt an equity of the levered company.
  - The equity of the company now consists of the equity that is also invested into the levered company and the equity that financed by the loan. This gives total equity in this company of $\hat{E}$. Thus the equity investors owns the entire company, but we can rewrite his fraction of the company he owns as $\frac{E+L}{\hat{E}}$.
-
  - Based on his initial equity, he will be allocated the full surplus of the company, but we write this now as receiving his fraction $\frac{E+L}{\hat{E}}$ of the company, even though this fraction is 1.
  - From this surplus he now has to repay the loan he has taken out to increase the equity of the company.
- *Formula*

→ We can now compare the equity value of the levered and the unlevered companies.

Investing into the unlevered company

- Suppose an investor makes the same equity investment into the unlevered company as in the levered company
- When investing into the unlevered company he also takes out a loan such that he obtains a fraction $\frac{E+L}{\hat{E}}$ of the company
- He will receive his fraction of the surplus and repay his loan including interest
- $E \rightarrow \frac{E+L}{\hat{E}} V - (1 + r_L) L$

Investing into the unlevered company

→ When considering an unlevered company we need to ensure that we use the same amount of initial equity and makes the same investment as the levered company.

- We therefore assume that the investor makes the same equity investment into both companies.
-
  - As to allow the company to make the same investment as the levered company, it is now the investor taking up a loan to increase the equity of the company to the same level as the debt an equity of the levered company.
  - The equity of the company now consists of the equity that is also invested into the levered company and the equity that financed by the loan. This gives total equity in this company of $\hat{E}$. Thus the equity investors owns the entire company, but we can rewrite his fraction of the company he owns as $\frac{E+L}{\hat{E}}$.
-
  - Based on his initial equity, he will be allocated the full surplus of the company, but we write this now as receiving his fraction $\frac{E+L}{\hat{E}}$ of the company, even though this fraction is 1.
  - From this surplus he now has to repay the loan he has taken out to increase the equity of the company.
- *Formula*

→ We can now compare the equity value of the levered and the unlevered companies.

Indifference to the form of financing

→ We

- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
  - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
  - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Indifference to the form of financing

- As both initial investments were identical, the outcome must be identical

→ We

- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
    - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
    - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Indifference to the form of financing

- As both initial investments were identical, the outcome must be identical
- $V-(1+r_L)L=\frac{E+L}{\hat{E}}V-(1+r_L)L$

Indifference to the form of financing

→ We

- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
  - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
  - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Indifference to the form of financing

- As both initial investments were identical, the outcome must be identical
- $V-(1+r_L)L=\frac{E+L}{\hat{E}}V-(1+r_L)L$

$\Rightarrow$ $\hat{E}=E+L=V$

Indifference to the form of financing

→ We
- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
    - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
    - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Indifference to the form of financing

- As both initial investments were identical, the outcome must be identical
- $V - (1 + r_L)\,L = \frac{E+L}{\hat{E}} V - (1 + r_L)\,L$

$\Rightarrow$ $\hat{E} = E + L = V$

$\Rightarrow$ The company value is independent of the capital structure

→ We

- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
  - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
  - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Indifference to the form of financing

- As both initial investments were identical, the outcome must be identical
- $V - (1 + r_L)\, L = \frac{E+L}{\hat{E}} V - (1 + r_L)\, L$

$\Rightarrow$ $\hat{E} = E + L = V$

$\Rightarrow$ The company value is independent of the capital structure

- This is known as the Modigliani-Miller theorem

→ We

- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
    - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
    - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Indifference to the form of financing

- As both initial investments were identical, the outcome must be identical
- $V-(1+r_L)\,L=\frac{E+L}{\hat{E}}V-(1+r_L)\,L$

$\Rightarrow$ $\hat{E}=E+L=V$

$\Rightarrow$ The company value is independent of the capital structure

- This is known as the Modigliani-Miller theorem or the irrelevance of the capital structure

Indifference to the form of financing

→ We

- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
    - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
    - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Indifference to the form of financing

- As both initial investments were identical, the outcome must be identical
- $V-(1+r_L)L=\frac{E+L}{\hat{E}}V-(1+r_L)L$

$\Rightarrow$ $\hat{E}=E+L=V$

$\Rightarrow$ The company value is independent of the capital structure

- This is known as the Modigliani-Miller theorem or the irrelevance of the capital structure

Indifference to the form of financing

→ We

- In both cases, the equity holders invest the same amount E into the company. In the case of the levered company, the company took a loan to increase its investments and in case of the unlevered company the equity holder personally took out a loan to increase the equity. As equity holders have made the same initial investment, the outc9ome for them should be identical.
- *Formula*
- [⇒] Solving this equality we recover that the total equity of the unlevered company is equal to the debt and equity of the levered company. This must be equal to the surplus the company generates as all surplus is allocated to the equity holder in the unlevered company.
- [⇒] The regardless of the combination of equity (E) and debt (L), the thew total value remains unchanged. It is thus that how the investment is financed, byt debt or equity, is irrelevant.
-
  - This result is one of the cornerstones in corporate finance. It is known as the Modigliani-Miller theorem.
  - Often this is also referred to as the irrelevance of the capital structure.

→ We can now take a closer look at the way we determine the surplus an investment generates.

Equity values

Equity values

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- ▶
  - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- ▶ Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- ▶
  - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶
  - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- ▶
  - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶ We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity

Equity values

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- ▶
  - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- ▶ Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- ▶
  - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶
  - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- ▶
  - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶ We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity
- For the unlevered company the surplus consists of the total surplus the company generates
- $\hat{E} = \sum_{\tau=1}^{+\infty} \frac{V}{(1+\hat{r}_E)^\tau}$

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- ▶
  - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- ▶ Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- ▶
  - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶
  - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- ▶
  - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶ We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity
- For the unlevered company the surplus consists of the total surplus the company generates
- $\hat{E} = \sum_{\tau=1}^{+\infty} \frac{V}{(1+\hat{r}_E)^\tau} = \frac{V}{\hat{r}_E}$

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- ▶
  - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- ▶ Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- ▶
  - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶
  - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- ▶
  - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶ We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity
- For the unlevered company the surplus consists of the total surplus the company generates
- $\hat{E} = \sum_{\tau=1}^{+\infty} \frac{V}{(1+\hat{r}_E)^\tau} = \frac{V}{\hat{r}_E}$
- For the levered company the surplus consists of the total surplus the company generates
- $E = \sum_{\tau=1}^{+\infty} \frac{V}{(1+r_E)^\tau}$

Equity values

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- ▶
  - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- ▶ Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- ▶
  - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶
  - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- ▶
  - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶ We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity
- For the unlevered company the surplus consists of the total surplus the company generates
- $\hat{E} = \sum_{\tau=1}^{+\infty} \frac{V}{(1+\hat{r}_E)^\tau} = \frac{V}{\hat{r}_E}$
- For the levered company the surplus consists of the total surplus the company generates less the interest on the loan
- $E = \sum_{\tau=1}^{+\infty} \frac{V - r_L L}{(1+r_E)^\tau}$

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- ▶
  - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- ▶ Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- ▶
  - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶
  - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- ▶
  - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶ We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity
- For the unlevered company the surplus consists of the total surplus the company generates
- $\hat{E} = \sum_{\tau=1}^{+\infty} \frac{V}{(1+\hat{r}_E)^\tau} = \frac{V}{\hat{r}_E}$
- For the levered company the surplus consists of the total surplus the company generates less the interest on the loan
- $E = \sum_{\tau=1}^{+\infty} \frac{V-r_L L}{(1+r_E)^\tau} = \frac{V-r_L L}{r_E}$

Equity values

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity
- For the unlevered company the surplus consists of the total surplus the company generates
- $\hat{E} = \sum_{\tau=1}^{+\infty} \frac{V}{(1+\hat{r}_E)^\tau} = \frac{V}{\hat{r}_E}$
- For the levered company the surplus consists of the total surplus the company generates less the interest on the loan
- $E = \sum_{\tau=1}^{+\infty} \frac{V-r_L L}{(1+r_E)^\tau} = \frac{V-r_L L}{r_E}$
- The discount rates of the two companies might be different as they are financed in a different way

Equity values

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- • Equity is valued using the surplus that accrues to equity holders,
  • where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- • The value of the equity is then the present value of all these future surpluses.
  • We can simplify this geometric series as indicated in this *formula*.
- • The levered company similarly obtains the surplus V,
  • but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- • The value of the equity is then the present value of all these future surpluses, less the interest payments.
  • We can simplify this geometric series as indicated in this *formula*.
- We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Equity values

- The value of equity is given from the surplus accruing to the equity holders, discounted at the appropriate cost of equity
- For the unlevered company the surplus consists of the total surplus the company generates
- $\hat{E} = \sum_{\tau=1}^{+\infty} \frac{V}{(1+\hat{r}_E)^{\tau}} = \frac{V}{\hat{r}_E}$
- For the levered company the surplus consists of the total surplus the company generates less the interest on the loan
- $E = \sum_{\tau=1}^{+\infty} \frac{V-r_L L}{(1+r_E)^{\tau}} = \frac{V-r_L L}{r_E}$
- The discount rates of the two companies might be different as they are financed in a different way

→ We had assumed that the surplus of investments is identical regardless of the capital structure. We have, however, to consider that the surplus is generated in the future and we therefore have to consider its present value. We therefore have to establish not only that the surplus itself is identical for investments of a given size regardless of the way it is financed, which is easily justifiable, but also that the discount rate to obtain its present value is identical across capital structures.

- ▶
  - Equity is valued using the surplus that accrues to equity holders,
  - where future surplus is discounted at the cost of equity. In general that would be the expected return of the equity as determined by asset pricing models.
- ▶ Let us assume that unlevered companies have a cost of equity of $\hat{r}_E$, which is given from the risks of the investment and that the (expected) surpluses in each time period are V.
- ▶
  - The value of the equity is then the present value of all these future surpluses.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶
  - The levered company similarly obtains the surplus V,
  - but from this surplus has to pay interest on the loan. With the investment generating surplus forever, we assume that the loan also has an infinite maturity and hence neglect the loan repayment. The discount rate for the levered company might be different at r_E.
- ▶
  - The value of the equity is then the present value of all these future surpluses, less the interest payments.
  - We can simplify this geometric series as indicated in this *formula*.
- ▶ We explicitly allow for different discount rates for levered and unlevered companies.

→ We will now seek to determine the relationship between the discount rates for levered and unlevered companies.

Cost of equity

Cost of equity

→ We now determine the discount rate for equity for the levered company.

- We use the equity calue of the levered company as a starting point.
-
  - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
-
  - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.

→ We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value

- → We now determine the discount rate for equity for the levered company.
- We use the equity calue of the levered company as a starting point.
-
  - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
-
  - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.
- → We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value
- $E = \frac{V - r_L L}{r_E}$

Cost of equity

- → We now determine the discount rate for equity for the levered company.
- We use the equity calue of the levered company as a starting point.
-
  - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
-
  - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.
- → We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value
- $E = \frac{V - r_L L}{r_E} = \frac{\hat{r}_E}{r_E} \frac{V}{\hat{r}_E} - \frac{r_L}{r_E} L$

Cost of equity

- → We now determine the discount rate for equity for the levered company.
- We use the equity calue of the levered company as a starting point.
-
  - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
-
  - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.
- → We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value
- $E = \frac{V - r_L L}{r_E} = \frac{\hat{r}_E}{r_E} \frac{V}{\hat{r}_E} - \frac{r_L}{r_E} L$
  $= \frac{\hat{r}_E}{r_E} \hat{E} - \frac{r_L}{r_E} L$

Cost of equity

- → We now determine the discount rate for equity for the levered company.
- ▶ We use the equity calue of the levered company as a starting point.
- ▶
  - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
- ▶
  - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- ▶ [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- ▶ [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.
- → We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value
- $E = \frac{V - r_L L}{r_E} = \frac{\hat{r}_E}{r_E} \frac{V}{\hat{r}_E} - \frac{r_L}{r_E} L$
  $= \frac{\hat{r}_E}{r_E} \hat{E} - \frac{r_L}{r_E} L = \frac{\hat{r}_E}{r_E} (E + L) - \frac{r_L}{r_E} L$

Cost of equity

→ We now determine the discount rate for equity for the levered company.

- We use the equity calue of the levered company as a starting point.
-
  - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
-
  - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.

→ We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value
- $E = \frac{V - r_L L}{r_E} = \frac{\hat{r}_E}{r_E} \frac{V}{\hat{r}_E} - \frac{r_L}{r_E} L$
  $= \frac{\hat{r}_E}{r_E} \hat{E} - \frac{r_L}{r_E} L = \frac{\hat{r}_E}{r_E} (E + L) - \frac{r_L}{r_E} L$

$\Rightarrow$ $r_E = \hat{r}_E + (\hat{r}_E - r_L) \frac{L}{E}$

Cost of equity

- → We now determine the discount rate for equity for the levered company.
- ▶ We use the equity calue of the levered company as a starting point.
- ▶
  - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
- ▶
  - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- ▶ [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- ▶ [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.
- → We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value
- $E = \frac{V - r_L L}{r_E} = \frac{\hat{r}_E}{r_E} \frac{V}{\hat{r}_E} - \frac{r_L}{r_E} L$
$$= \frac{\hat{r}_E}{r_E} \hat{E} - \frac{r_L}{r_E} L = \frac{\hat{r}_E}{r_E} (E + L) - \frac{r_L}{r_E} L$$

⇒ $r_E = \hat{r}_E + (\hat{r}_E - r_L) \frac{L}{E}$

⇒ The cost of equity increases with leverage

Cost of equity

- → We now determine the discount rate for equity for the levered company.
- We use the equity calue of the levered company as a starting point.
- - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
- - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.
- → We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Cost of equity

- The relationship between these discount rates can now be obtained from the equity value
- $E = \frac{V - r_L L}{r_E} = \frac{\hat{r}_E}{r_E}\frac{V}{\hat{r}_E} - \frac{r_L}{r_E} L$
  $= \frac{\hat{r}_E}{r_E}\hat{E} - \frac{r_L}{r_E} L = \frac{\hat{r}_E}{r_E}(E + L) - \frac{r_L}{r_E} L$

$\Rightarrow$ $r_E = \hat{r}_E + (\hat{r}_E - r_L)\frac{L}{E}$

$\Rightarrow$ The cost of equity increases with leverage

Cost of equity

- → We now determine the discount rate for equity for the levered company.
- We use the equity calue of the levered company as a starting point.
- - The equity value of the levered company is from above as given in the *formula*.
  - We can now split this fraction and expand the first term by multiplying and then dividing by $\hat{r}_E$.
- - We now know that $\frac{V}{\hat{r}_E} = \hat{E}$ from the valuation of the unlevered company
  - We can now further use from above that $\hat{E} = E + L$.
- [⇒] We can solve this final relationship for the cost of equity of the levered company and obtain this *formula*.
- [⇒] We see that the cost of equity is increasing the higher the leverage is; leverage is the ratio of debt and equity. For this result we assume that the loan rate is below the cost of equity for the unlevered company. We thus also find that the cost of equity, the required rate of return, or the expected return, is higher than that of an unlevered company.
- → We have now established the cost of equity, but if the company uses debt to finance its investments, its total financing costs will not be the cost of debt, but the weighted average cost of capital. We will seek to determine this next.

Weighted average cost of capital

Weighted average cost of capital

→ The weighted average cost of capital (WACC) is used to determine the required return of an investment as this is the rate at which the investment would break even.

- - The WACC is determined by the cost of equity
  - and the cost of debt. The weights are given by the weight of equity and debt, respectively, in the financing of the investment.
- *Formula*
- [⇒] If we insert for the cost of equity from above, we obtain the weighted average cost of capital is identical to the cost of equity in the unlevered company. These costs of equity is determined by the risks of the investment only, and not by how it is financed.
- We thus see that the WACC are identical regardless of the way the investment is financed. This also implies that the present value of future surpluses is the same, regardless of the way the investment is financed, justifying our above approach to treat the value of the surplus as given for levered and unlevered companies.
- With higher leverage the costs of equity increases, this is to compensate for the higher risk of the investment for the equity holders. The investments are larger, relative to the equity, and can therefore create higher profits but also higher losses. These higher costs are exactly offset with the lower costs of loans.

→ We have therefore established that the capital structure is irrelevant for the value of the company as a whole and that the weighted average costs of capital to assess investments are identical for all companies.

Weighted average cost of capital

- The weighted average cost of capital consist of the cost of equity and costs of debt
- $r_{\text{WACC}} = r_E \qquad + r_L$

Weighted average cost of capital

→ The weighted average cost of capital (WACC) is used to determine the required return of an investment as this is the rate at which the investment would break even.

- - The WACC is determined by the cost of equity
  - and the cost of debt. The weights are given by the weight of equity and debt, respectively, in the financing of the investment.
- *Formula*
- [⇒] If we insert for the cost of equity from above, we obtain the weighted average cost of capital is identical to the cost of equity in the unlevered company. These costs of equity is determined by the risks of the investment only, and not by how it is financed.
- We thus see that the WACC are identical regardless of the way the investment is financed. This also implies that the present value of future surpluses is the same, regardless of the way the investment is financed, justifying our above approach to treat the value of the surplus as given for levered and unlevered companies.
- With higher leverage the costs of equity increases, this is to compensate for the higher risk of the investment for the equity holders. The investments are larger, relative to the equity, and can therefore create higher profits but also higher losses. These higher costs are exactly offset with the lower costs of loans.

→ We have therefore established that the capital structure is irrelevant for the value of the company as a whole and that the weighted average costs of capital to assess investments are identical for all companies.

Weighted average cost of capital

- The weighted average cost of capital consist of the cost of equity and costs of debt, with their weights in the financing of the company
- $r_{\text{WACC}} = r_E \frac{E}{E+L} + r_L \frac{L}{E+L}$

Weighted average cost of capital

→ The weighted average cost of capital (WACC) is used to determine the required return of an investment as this is the rate at which the investment would break even.

- - The WACC is determined by the cost of equity
  - and the cost of debt. The weights are given by the weight of equity and debt, respectively, in the financing of the investment.
- *Formula*
- [⇒] If we insert for the cost of equity from above, we obtain the weighted average cost of capital is identical to the cost of equity in the unlevered company. These costs of equity is determined by the risks of the investment only, and not by how it is financed.
- We thus see that the WACC are identical regardless of the way the investment is financed. This also implies that the present value of future surpluses is the same, regardless of the way the investment is financed, justifying our above approach to treat the value of the surplus as given for levered and unlevered companies.
- With higher leverage the costs of equity increases, this is to compensate for the higher risk of the investment for the equity holders. The investments are larger, relative to the equity, and can therefore create higher profits but also higher losses. These higher costs are exactly offset with the lower costs of loans.

→ We have therefore established that the capital structure is irrelevant for the value of the company as a whole and that the weighted average costs of capital to assess investments are identical for all companies.

Weighted average cost of capital

- The weighted average cost of capital consist of the cost of equity and costs of debt, with their weights in the financing of the company
- $r_{\text{WACC}} = r_E \frac{E}{E+L} + r_L \frac{L}{E+L}$

$\Rightarrow$ $r_{\text{WACC}} = \hat{r}_E$

Weighted average cost of capital

- → The weighted average cost of capital (WACC) is used to determine the required return of an investment as this is the rate at which the investment would break even.
-
  - The WACC is determined by the cost of equity
  - and the cost of debt. The weights are given by the weight of equity and debt, respectively, in the financing of the investment.
- *Formula*
- [⇒] If we insert for the cost of equity from above, we obtain the weighted average cost of capital is identical to the cost of equity in the unlevered company. These costs of equity is determined by the risks of the investment only, and not by how it is financed.
- We thus see that the WACC are identical regardless of the way the investment is financed. This also implies that the present value of future surpluses is the same, regardless of the way the investment is financed, justifying our above approach to treat the value of the surplus as given for levered and unlevered companies.
- With higher leverage the costs of equity increases, this is to compensate for the higher risk of the investment for the equity holders. The investments are larger, relative to the equity, and can therefore create higher profits but also higher losses. These higher costs are exactly offset with the lower costs of loans.
- → We have therefore established that the capital structure is irrelevant for the value of the company as a whole and that the weighted average costs of capital to assess investments are identical for all companies.

Weighted average cost of capital

- The weighted average cost of capital consist of the cost of equity and costs of debt, with their weights in the financing of the company
- $r_{\text{WACC}} = r_E \frac{E}{E+L} + r_L \frac{L}{E+L}$

$\Rightarrow$ $r_{\text{WACC}} = \hat{r}_E$

- The weighted average cost of capital is unaffected by the capital structure

Weighted average cost of capital

- → The weighted average cost of capital (WACC) is used to determine the required return of an investment as this is the rate at which the investment would break even.
-
  - The WACC is determined by the cost of equity
  - and the cost of debt. The weights are given by the weight of equity and debt, respectively, in the financing of the investment.
- *Formula*
- [⇒] If we insert for the cost of equity from above, we obtain the weighted average cost of capital is identical to the cost of equity in the unlevered company. These costs of equity is determined by the risks of the investment only, and not by how it is financed.
- We thus see that the WACC are identical regardless of the way the investment is financed. This also implies that the present value of future surpluses is the same, regardless of the way the investment is financed, justifying our above approach to treat the value of the surplus as given for levered and unlevered companies.
- With higher leverage the costs of equity increases, this is to compensate for the higher risk of the investment for the equity holders. The investments are larger, relative to the equity, and can therefore create higher profits but also higher losses. These higher costs are exactly offset with the lower costs of loans.
- → We have therefore established that the capital structure is irrelevant for the value of the company as a whole and that the weighted average costs of capital to assess investments are identical for all companies.

Weighted average cost of capital

- The weighted average cost of capital consist of the cost of equity and costs of debt, with their weights in the financing of the company
- $r_{\text{WACC}} = r_E \frac{E}{E+L} + r_L \frac{L}{E+L}$

$\Rightarrow$ $r_{\text{WACC}} = \hat{r}_E$

- The weighted average cost of capital is unaffected by the capital structure
- The increase in the cost of equity as leverage increases is compensated for by the use of lower-cost loans

Weighted average cost of capital

- → The weighted average cost of capital (WACC) is used to determine the required return of an investment as this is the rate at which the investment would break even.
-
  - The WACC is determined by the cost of equity
  - and the cost of debt. The weights are given by the weight of equity and debt, respectively, in the financing of the investment.
- *Formula*
- [⇒] If we insert for the cost of equity from above, we obtain the weighted average cost of capital is identical to the cost of equity in the unlevered company. These costs of equity is determined by the risks of the investment only, and not by how it is financed.
- We thus see that the WACC are identical regardless of the way the investment is financed. This also implies that the present value of future surpluses is the same, regardless of the way the investment is financed, justifying our above approach to treat the value of the surplus as given for levered and unlevered companies.
- With higher leverage the costs of equity increases, this is to compensate for the higher risk of the investment for the equity holders. The investments are larger, relative to the equity, and can therefore create higher profits but also higher losses. These higher costs are exactly offset with the lower costs of loans.
- → We have therefore established that the capital structure is irrelevant for the value of the company as a whole and that the weighted average costs of capital to assess investments are identical for all companies.

Weighted average cost of capital

- The weighted average cost of capital consist of the cost of equity and costs of debt, with their weights in the financing of the company
- $r_{\text{WACC}} = r_E \frac{E}{E+L} + r_L \frac{L}{E+L}$

$\Rightarrow$ $r_{\text{WACC}} = \hat{r}_E$

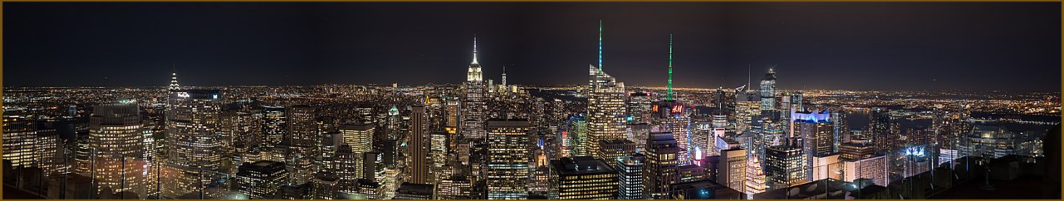
- The weighted average cost of capital is unaffected by the capital structure
- The increase in the cost of equity as leverage increases is compensated for by the use of lower-cost loans

Weighted average cost of capital

→ The weighted average cost of capital (WACC) is used to determine the required return of an investment as this is the rate at which the investment would break even.

-
  - The WACC is determined by the cost of equity
  - and the cost of debt. The weights are given by the weight of equity and debt, respectively, in the financing of the investment.
- *Formula*
- [⇒] If we insert for the cost of equity from above, we obtain the weighted average cost of capital is identical to the cost of equity in the unlevered company. These costs of equity is determined by the risks of the investment only, and not by how it is financed.
- We thus see that the WACC are identical regardless of the way the investment is financed. This also implies that the present value of future surpluses is the same, regardless of the way the investment is financed, justifying our above approach to treat the value of the surplus as given for levered and unlevered companies.
- With higher leverage the costs of equity increases, this is to compensate for the higher risk of the investment for the equity holders. The investments are larger, relative to the equity, and can therefore create higher profits but also higher losses. These higher costs are exactly offset with the lower costs of loans.

→ We have therefore established that the capital structure is irrelevant for the value of the company as a whole and that the weighted average costs of capital to assess investments are identical for all companies.



Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
United Kingdom

E-mail: mnsak@bath.ac.uk