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Properties of option prices

- We will now investigate the properties of options prices.
- We will see how the option price is affected as we change the parameters, focussing on those of the Black-Scholes model, but the results are equally applicable to the binomial model and other option pricing models.

Influences on option values

- Option values are affected by a large number of parameters and knowing these can help to hedge the exposure of the underlying asset
- Option pricing theory allows to analyse the influence these variables have on option values
- These influences can be used by investors to hedge risks

Influences on option values

→ Looking at the option pricing formula we will investigate how their changes will affect the option value.

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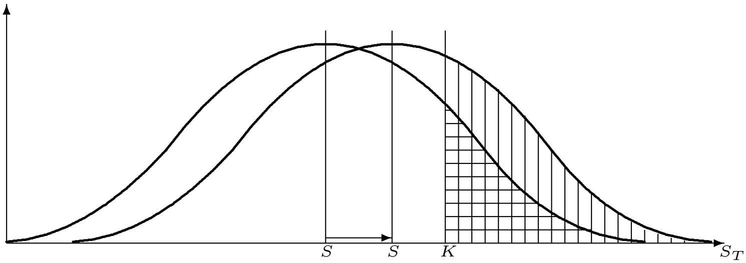
  - Parameters used in option pricing are not remaining are changing over time, certainly does the time to maturity constantly reduce and the price of the underlying asset will also change. However, the volatility of the underlying asset might also change, as might the risk-free rate.
  - If we know how the option value will change, we can better hedge the exposure of any risks from the underlying asset.
- We can use option pricing theories and take the partial derivative of the option price to determine their respective influences.
- We will see how this information can be used to provide a more perfect hedge against risks.

→ We will look at a number of factors that affect the value of options, focussing here only on call options.

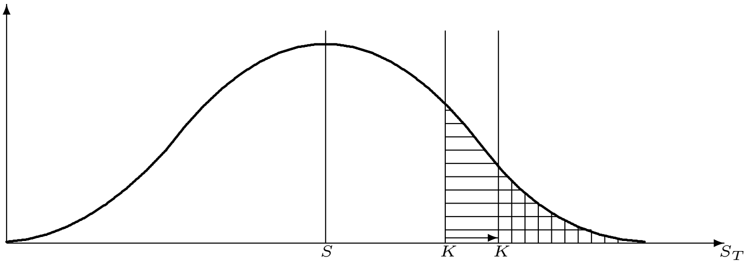
Current asset value



If the current asset value increases, call options become more valuable

- → We will first look at how an increase in the current asset price affects the option value.
- ▶ We will look at the distribution of the price of the underlying asset at maturity of the option.
- ▶ We assume a distribution of the asset price at maturity.
- ▶ The distribution of the asset price at maturity, given the current asset price is roughly symmetric around the current price.
- ▶ We look at a call option with this strike price.
- ▶ We now have a different distribution of the asset price at maturity of the option.
- ▶ The current asset price is higher and hence the distribution has shifted.
- ▶ If the asset price at maturity is above the strike price the call option will be exercised and the buyer will make a profit. The area with horizontal lines shows this region with the original asset price.
- ▶ With a higher asset price, the likelihood of the option being exercised is higher and the profits themselves will be higher as the vertical lines show.
- ▶ This implies that the option has a higher value to the buyer.
- → Hence the option value increases as the price of the underlying asset increases.

Strike price



If the strike price increases, call options become less valuable

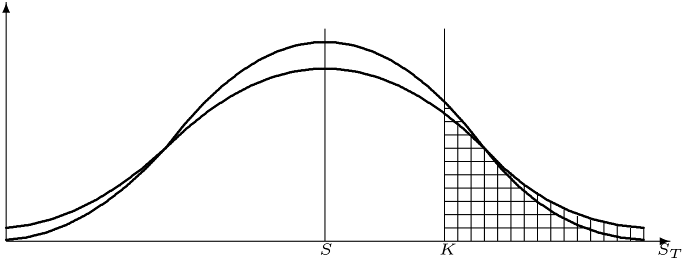
Strike price

→ We will now look at how an increase in the strike price affects the option value. Although the strike price is not a variable that can change, it is interesting to know how the option price will be affected.

- We will look at the distribution of the price of the underlying asset at maturity of the option.
- We assume a distribution of the asset price at maturity.
- The distribution of the asset price at maturity, given the current asset price is roughly symmetric around the current price.
- We look at a call option with this strike price.
- We can now increase this strike price.
- If the asset price at maturity is above the strike price the call option will be exercised and the buyer will make a profit. The area with horizontal lines shows this region with the original asset price.
- With a higher strike price, the likelihood of the option being exercised is lower and the profits themselves will be lower as the vertical lines show.
- This implies that the option has a lower value to the buyer.

→ Hence the option value decreases as the strike price increases.

Volatility



If the volatility increases, call options become more valuable

A long time to maturity will increase the variability of the final value and affect the option price in the same way

Volatility

- → Let us finally look at how an increase in the volatility affects the option value. The volatility of assets might increase if the general economic conditions become more uncertain or companies increase the risks they are taking.
- ▶ We will look at the distribution of the price of the underlying asset at maturity of the option.
- ▶ We assume a distribution of the asset price at maturity.
- ▶ The distribution of the asset price at maturity, given the current asset price is roughly symmetric around the current price.
- ▶ We look at a call option with this strike price.
- ▶ We now increase the volatility of the asset; this implies that more extreme movements are more likely and the distribution of asset prices at maturity has more weight at extreme prices and less weight to prices for smaller movements from the current price.
- ▶ If the asset price at maturity is above the strike price the call option will be exercised and the buyer will make a profit. The area with horizontal lines shows this region with the original asset price.
- ▶ With a higher volatility, the likelihood of the option being exercised is higher and the profits themselves will be higher as the vertical lines show. The reduction for smaller price increases is more than compensated for by the larger increases.
- ▶ This implies that the option has a higher value to the buyer.
- ▶ The same argument can be used in case the time to maturity increases. This is identical to a higher volatility as the longer time to maturity allows the asset pruces at maturity to vary more.
- → The option value in creases with volatility and time to maturity.

Option Greeks

- The effect of variables have on option prices are known as the Greeks
- The Greeks measure the marginal effect a variable has on the option price

	Call options	**Put options**
Delta	$\Delta_C = \frac{\partial C}{\partial S} = N\left(d_1\right)$	$\Delta_P = \frac{\partial P}{\partial S} = N\left(d_1\right) - 1$
Vega	$\nu_C = \frac{\partial C}{\partial \sigma} = Sn\left(d_1\right)\sqrt{T} = \frac{\partial P}{\partial \sigma} = \nu_P$	
Theta	$\theta_C = \frac{\partial C}{\partial T} = -\frac{Sn(d_1)}{2\sqrt{T}} - rKe^{-rT}N\left(d_2\right)$	$\theta_P = \frac{\partial P}{\partial T} = -\frac{Sn(d_1)}{2\sqrt{T}} + rKe^{-rT}N\left(d_2\right)$
Gamma	$\Gamma_C = \frac{\partial^2 C}{\partial S^2} = \frac{n(d_1)}{S\sigma\sqrt{T}} = \frac{\partial^2 P}{\partial S^2} = \Gamma_P$	

Option Greeks

→ We can now explore the the effect of variables on options prices using the Black-Scholes formula.

- The change of the option value if one of the parameters changes is called a 'Greek'.
- The Greeks show the partial derivative of the option value with respect to the parameter and thus show the sensitivity.
- Using the Black-Scholes formula, we can derive these Greeks and we see that some are identical for put and call options, wile other are different. The Greeks are all given Greek symbols from which their names derive and the most common are shown in this table. Of particular importance is the Delta of an option.

→ We will now use the Delta to derive the hedge of a position in the underlying asset that is not only avoiding losses at the maturity of the option, but at any time prior to maturity.

Δ-hedging

- Investors can use options not only to hedge their final payoff, but also the value of their position at any time
- Portfolio value hedged with a put option: $V = S + hP$
- Values do not change as the asset value changes: $\frac{\partial V}{\partial S} = 0$

$\Rightarrow$ $h = \frac{1}{1 - N(d_1)} = -\frac{1}{\Delta_P}$

- This is known as the hedge ratio
- For each asset, the investor should hold h put options and the value of their combined position does not change as the asset value changes

Δ-hedging

- $\rightarrow$ If we use an option to eliminate all risk from a position at any point until maturity, this referred to as Δ-hedging.
- We will see how how to use options to ensure that a position has no volatility at all, thus there is no risk at any point in time.
- We consider a portfolio of the underlying asset and h put options.
- We now want to ensure that this portfolio is risk-free, that means it does not change its value as the value of the asset changes. We achieve this by setting the first derivative zero. This is not a minimisation, but we seek to avoid a change in the value of the portfolio.
- [⇒]
  - Solving this equation for the optimal number of put options, we get this *formula*.
  - This is 1 divided by the Delta of the put option, hence the name Δ-hedging.
- The number of put options required in a Δ-hedge is also called the hedge ratio.
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  - The strategy would be for each asset, the investor hold h put options.
  - In this case the value of the portfolio will not change.
- $\rightarrow$ When using Δ-hedging, it has to be noted that the hedge ratio changes every time the value of the underlying asset changes; thus the number of put options needs to be constantly adjusted, which can be expensive if trading is not free. Similarly can we hedge the risks arising from the change of other parameters and employ a obtain a hedge ration with a different Greek. We can even hedge the risk of multiple parameters changes, but this would require multiple put options with different strike prices to achieve such complex hedging strategies.

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