

## Properties of option prices

- We will now investigate the properties of options prices.
- We will see how the option price is affected as we change the parameters, focussing on those of the Black-Scholes model, but the results are equally applicable to the binomial model and other option pricing models.

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  - If we know how the option value will change, we can better hedge the exposure of any risks from the underlying asset.
- We can use option pricing theories and take the partial derivative of the option price to determine their respective influences.
- We will see how this information can be used to provide a more perfect hedge against risks.
- We will look at a number of factors that affect the value of options, focussing here only on call options.

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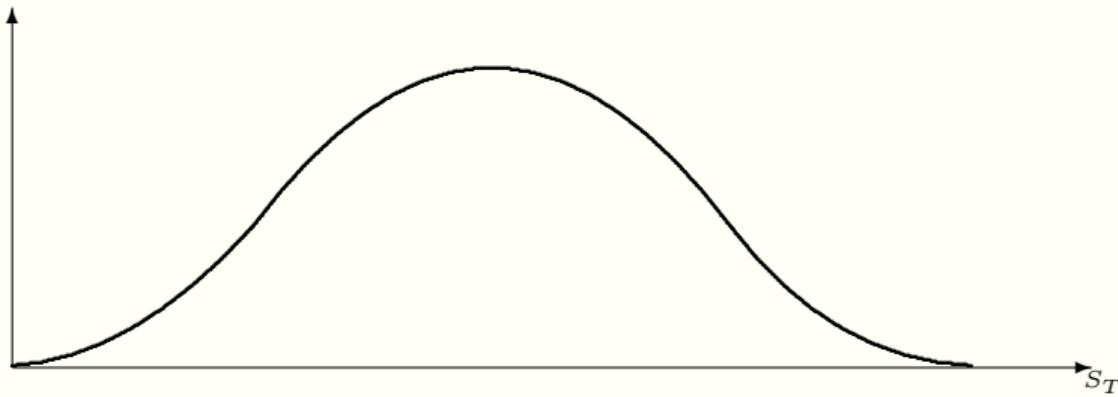
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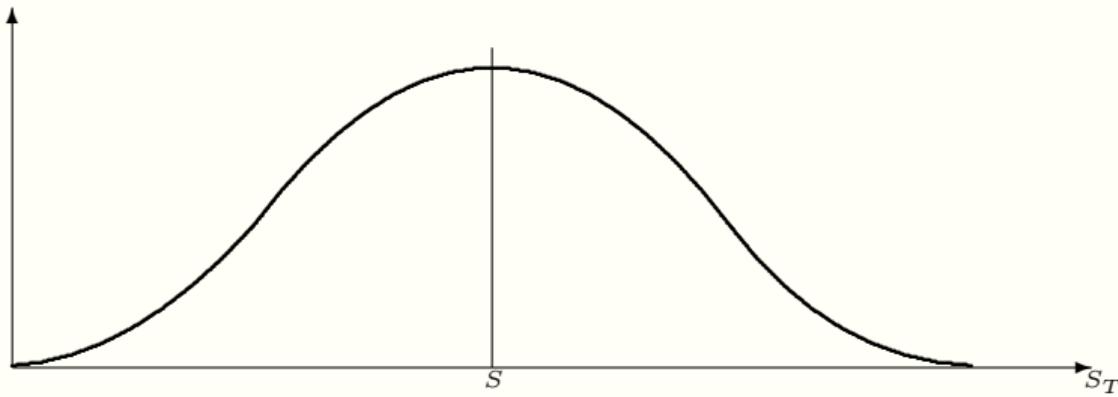
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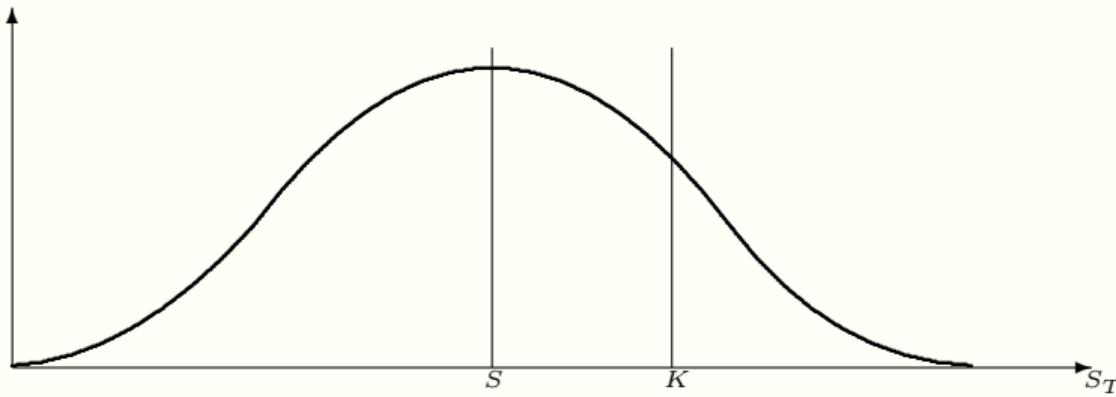
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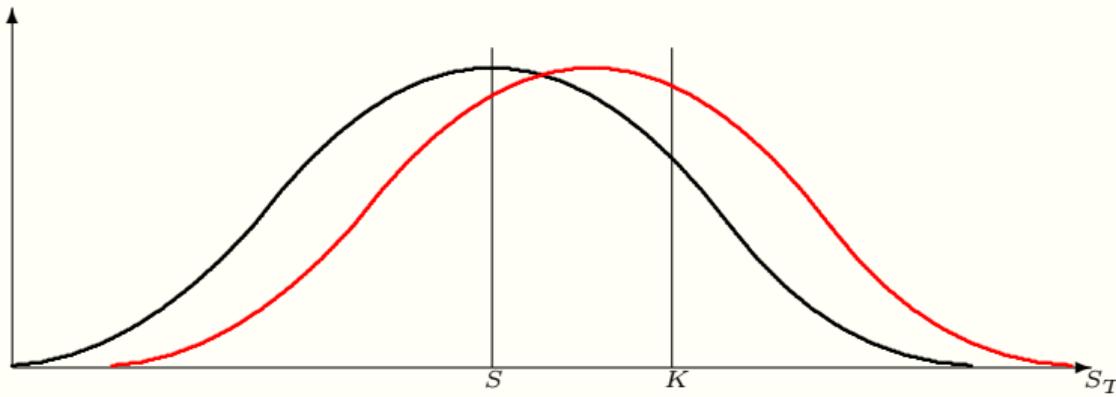
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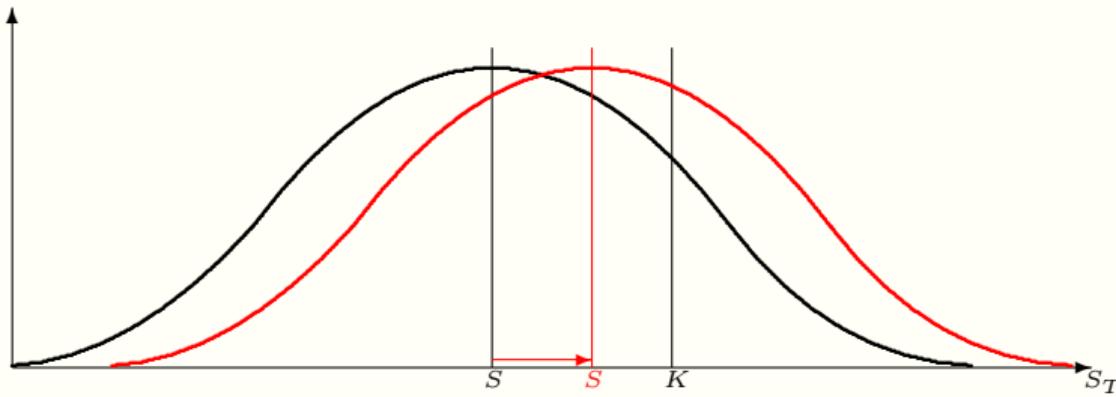
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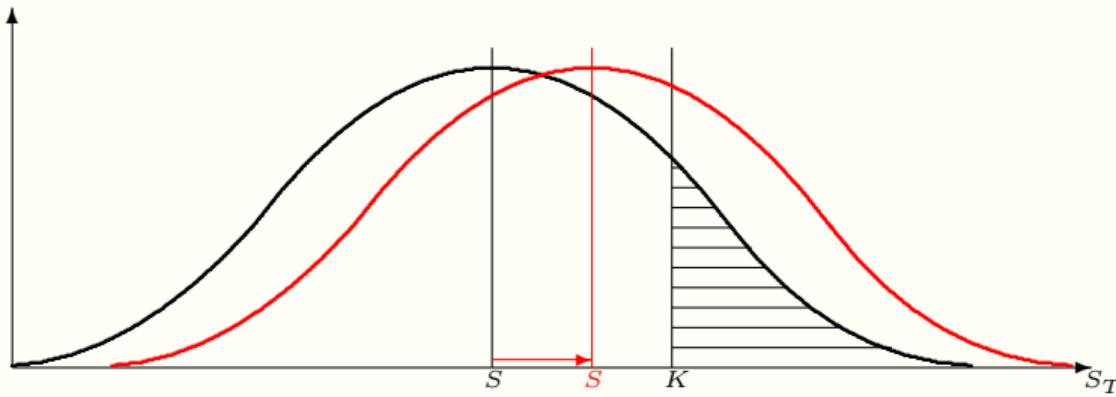
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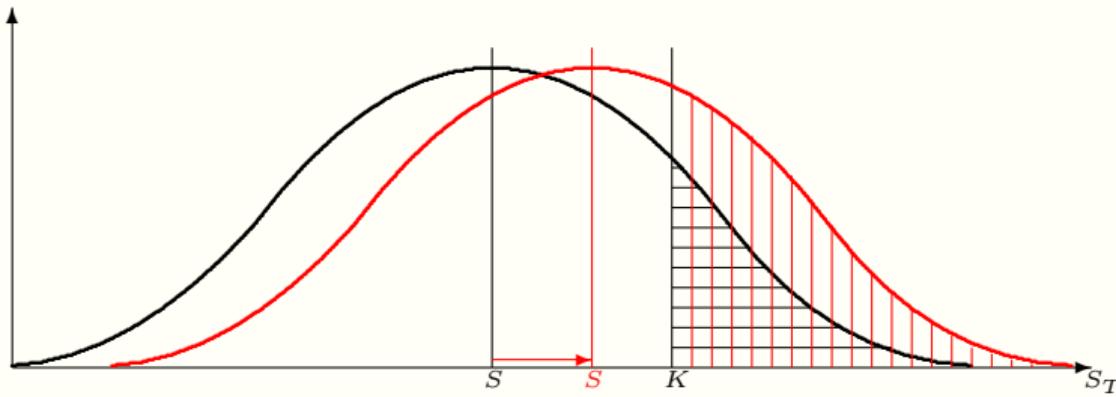
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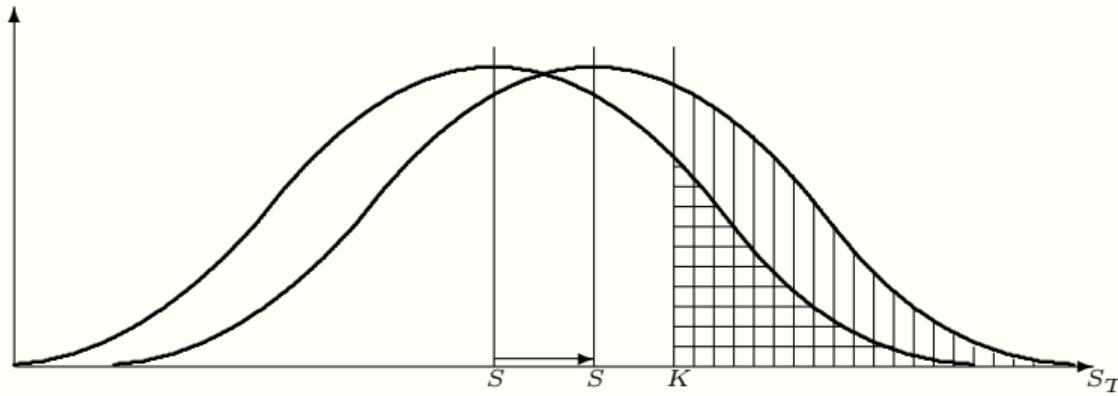
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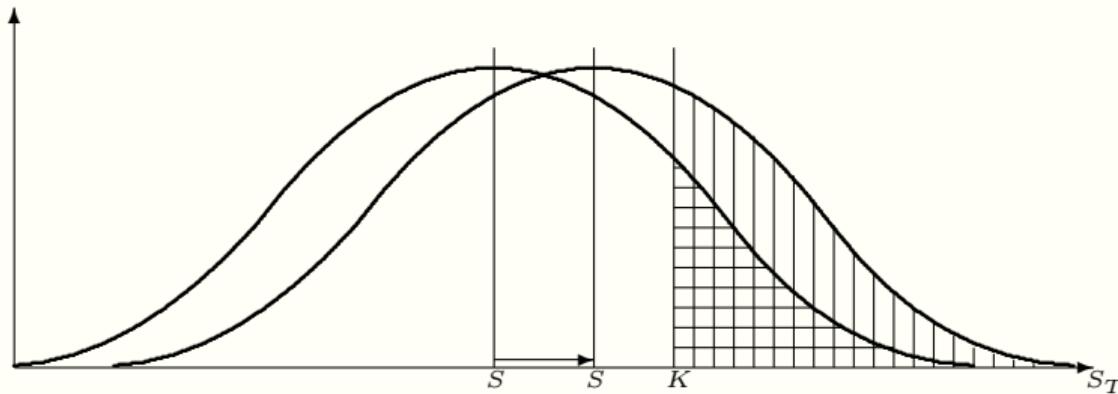
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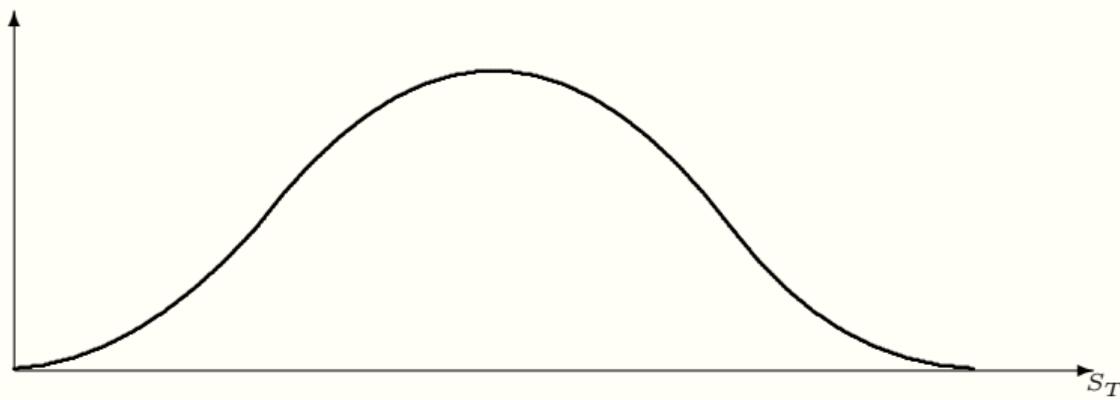
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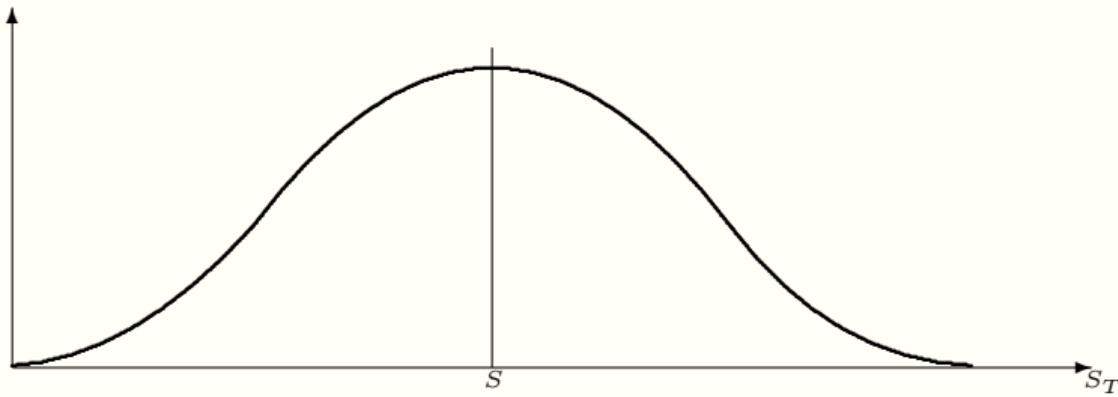
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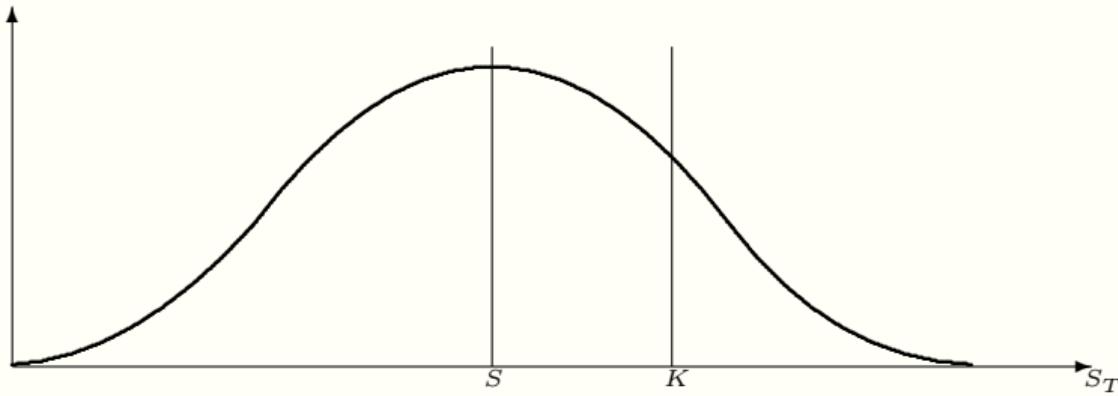
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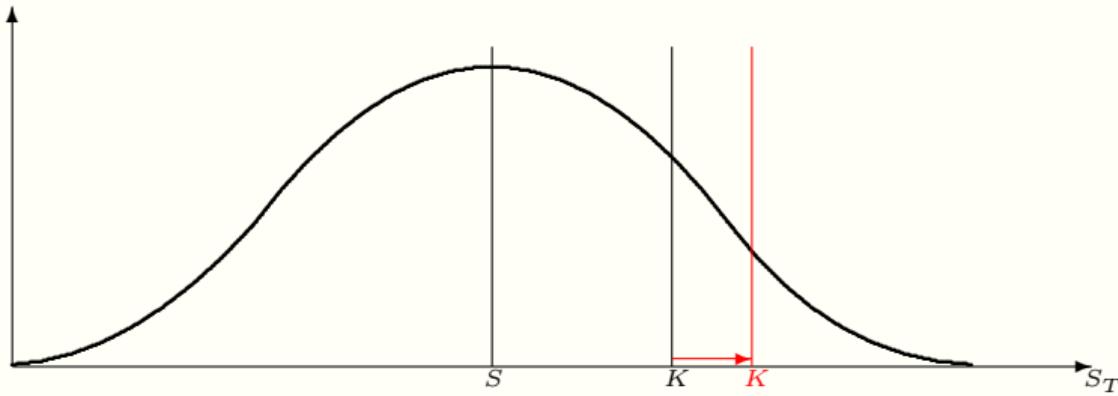
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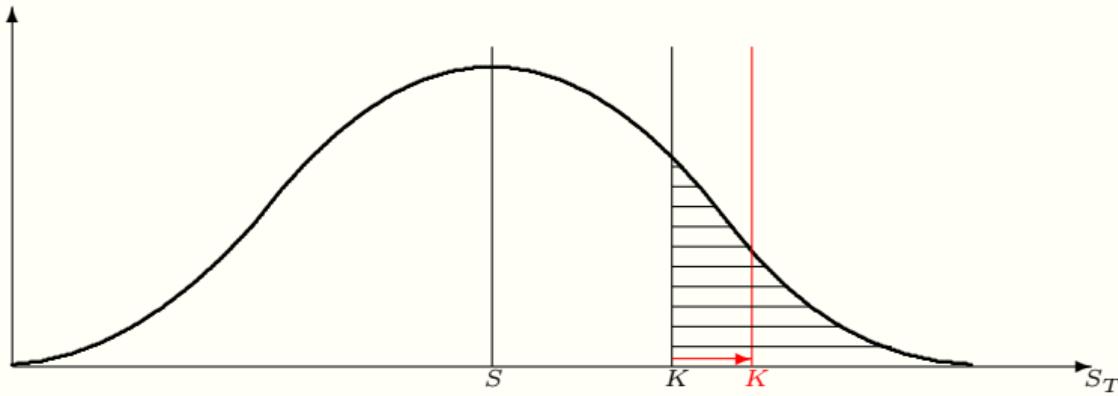
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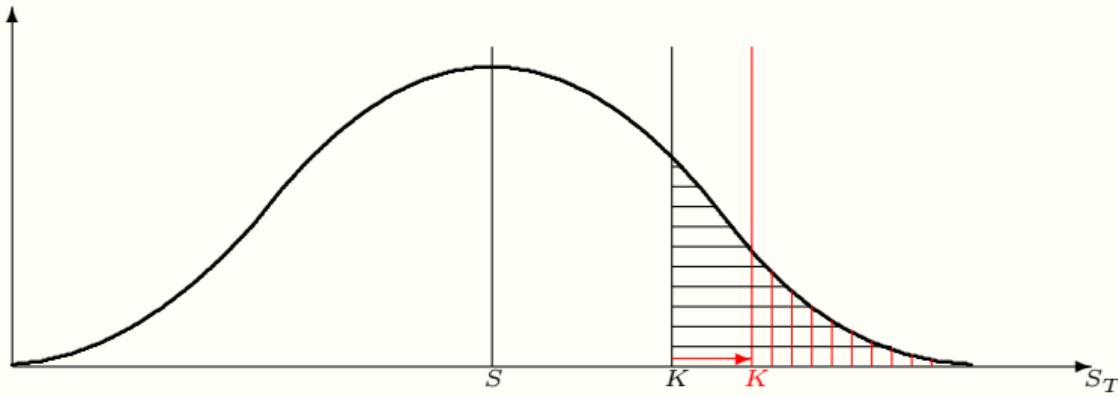
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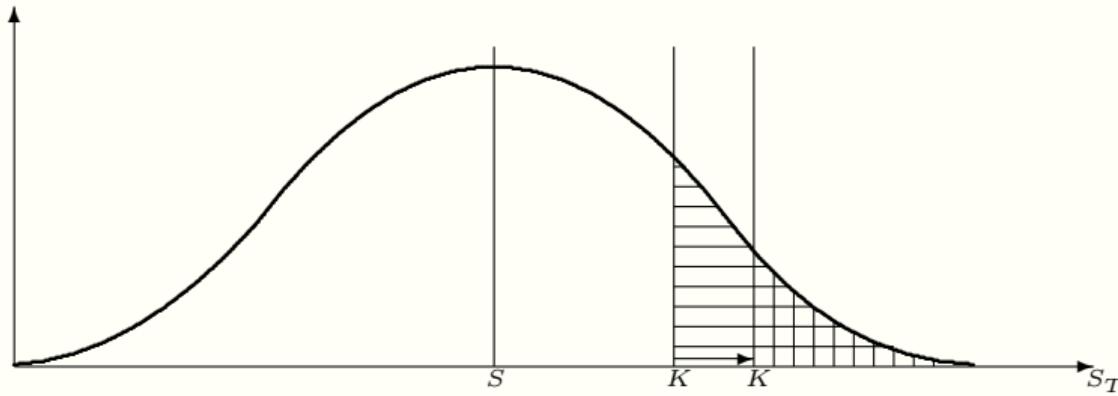
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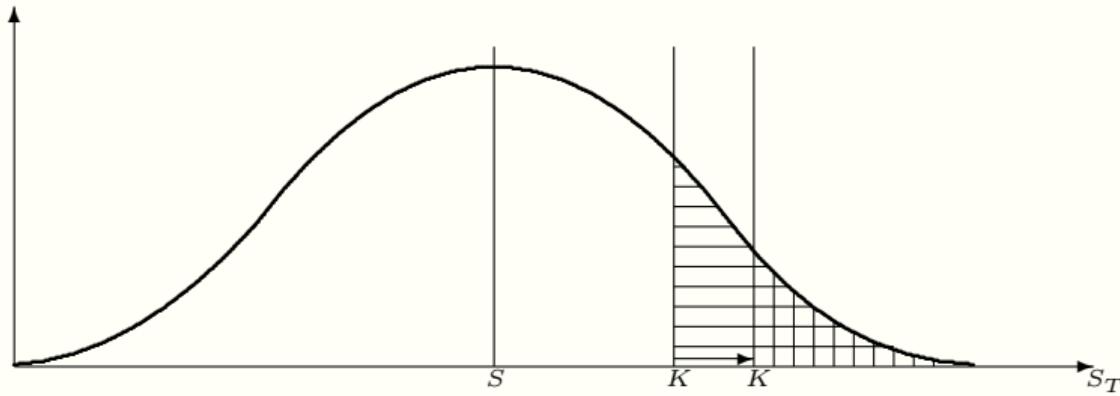
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# Volatility

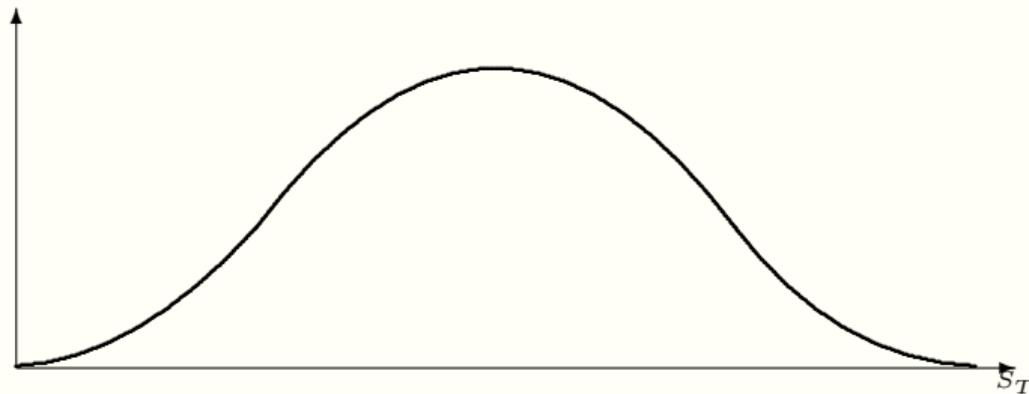
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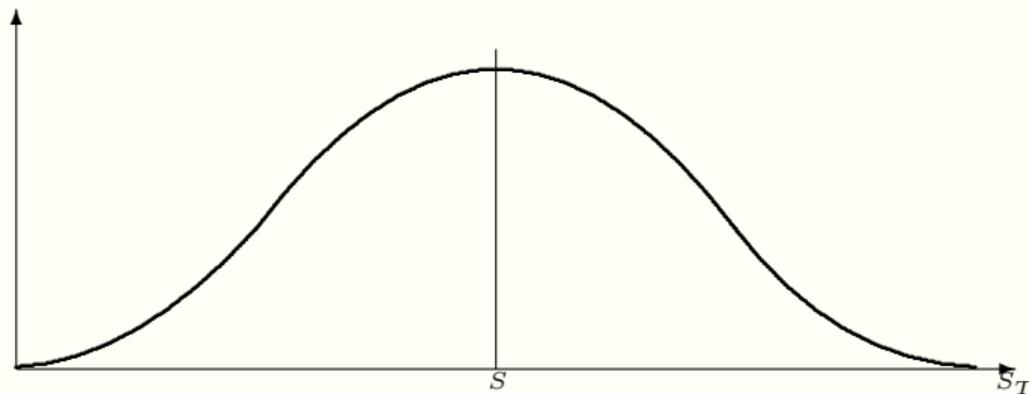
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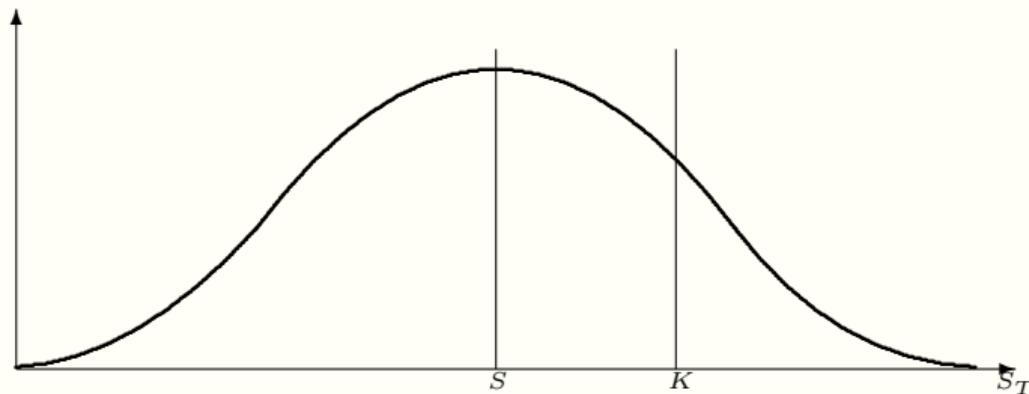
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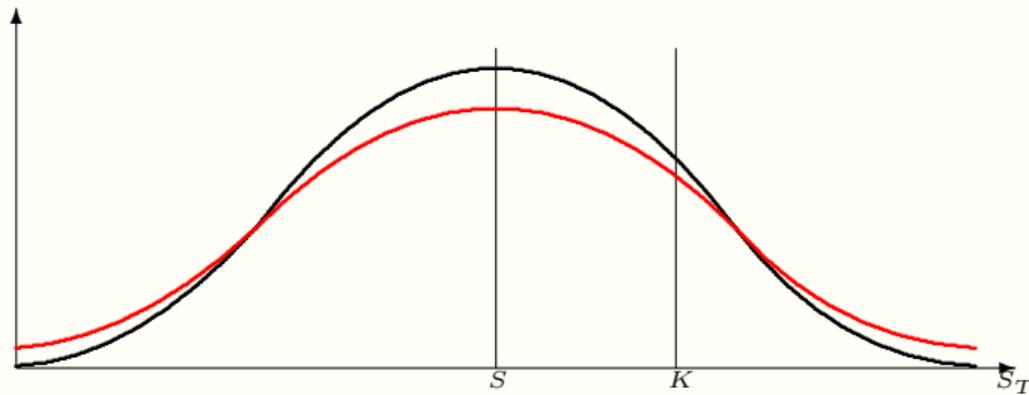
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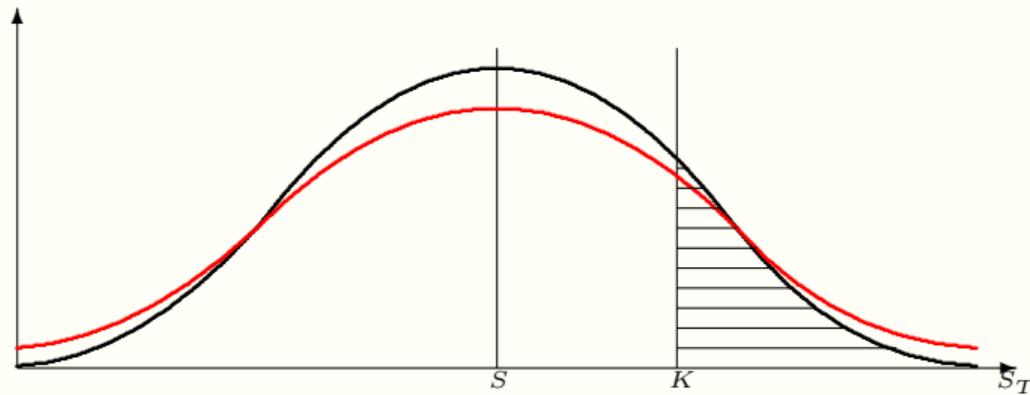
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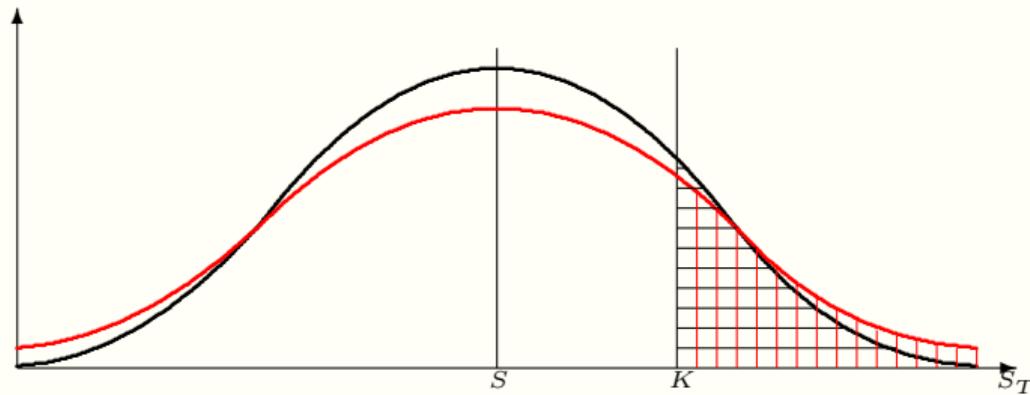
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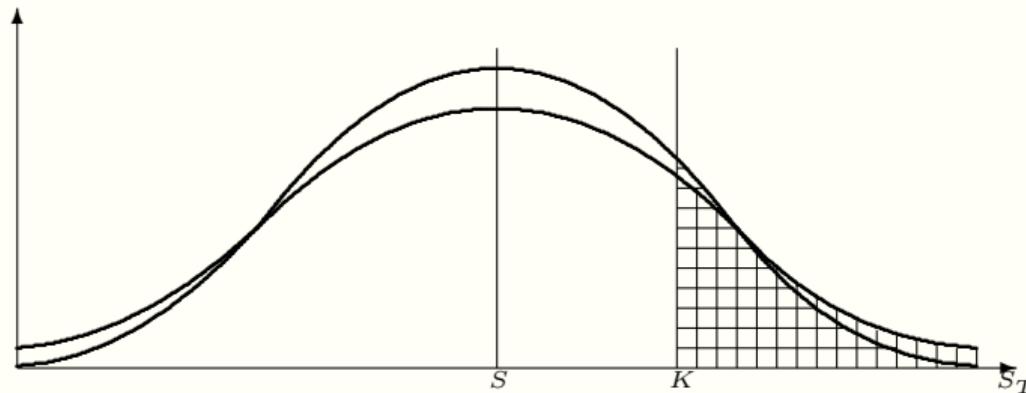
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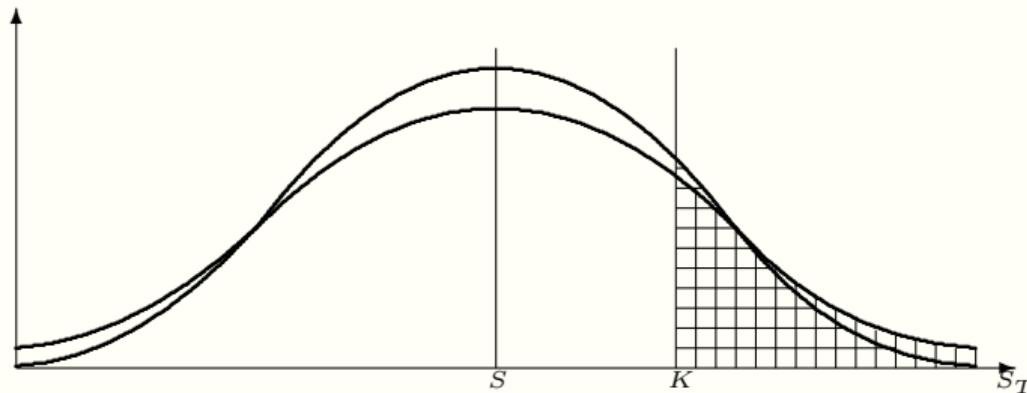
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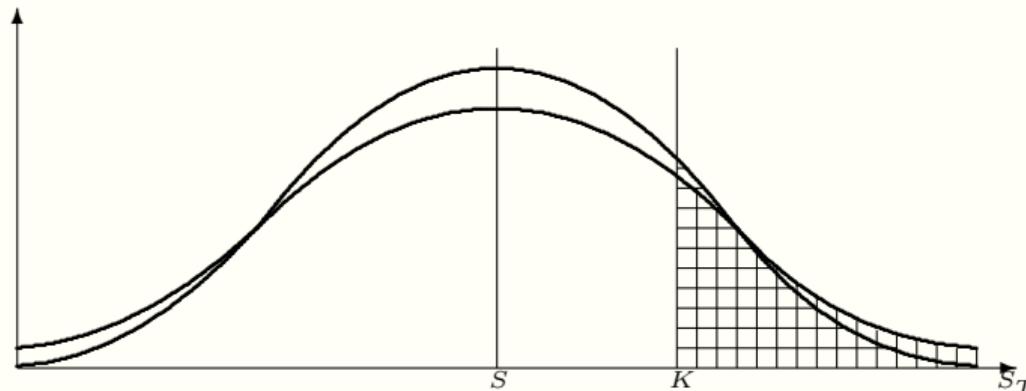


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# Option Greeks

- We can now explore the the effect of variables on options prices using the Black-Scholes formula.
- The change of the option value if one of the parameters changes is called a 'Greek'.
- The Greeks show the partial derivative of the option value with respect to the parameter and thus show the sensitivity.
- Using the Black-Scholes formula, we can derive these Greeks and we see that some are identical for put and call options, while other are different. The Greeks are all given Greek symbols from which their names derive and the most common are shown in this table. Of particular importance is the Delta of an option.
- We will now use the Delta to derive the hedge of a position in the underlying asset that is not only avoiding losses at the maturity of the option, but at any time prior to maturity.

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Vega	$\nu_C = \frac{\partial C}{\partial \sigma} = S n(d_1) \sqrt{T}$	$\nu_P = \frac{\partial P}{\partial \sigma} = \nu_C$
Theta	$\theta_C = \frac{\partial C}{\partial T} = -\frac{S n(d_1)}{2\sqrt{T}} - r K e^{-rT} N(d_2)$	$\theta_P = \frac{\partial P}{\partial T} = -\frac{S n(d_1)}{2\sqrt{T}} + r K e^{-rT} N(d_2)$
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- We will see how to use options to ensure that a position has no volatility at all, thus there is no risk at any point in time.
- We consider a portfolio of the underlying asset and  $h$  put options.
- We now want to ensure that this portfolio is risk-free, that means it does not change its value as the value of the asset changes. We achieve this by setting the first derivative zero. This is not a minimisation, but we seek to avoid a change in the value of the portfolio.
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$\Rightarrow h = \frac{1}{1 - N(d_1)} = -\frac{1}{\Delta_P}$

- If we use an option to eliminate all risk from a position at any point until maturity, this referred to as  $\Delta$ -hedging.
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