Andreas Krause

Properties of option prices

- We will now investigate the properties of options prices.
- We will see how the option price is affected as we change the parameters, focussing on those of the Black-Scholes model, but the results are equally applicable to the binomial model and other option pricing models.

Influences on option values

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- → Looking at the option pricing formula we will investigate how their changes will affect the option value.
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  - Parameters used in option pricing are not remaining are changing over time, certainly does the time to maturity constantly reduce and the price of the underlying asset will also change. However, the volatility of the underlying asset might also change, as might the risk-free rate.
  - If we know how the option value will change, we can better hedge the exposure of any risks from the underlying asset.
- We can use option pricing theories and take the partial derivative of the option price to determine their respective influences.
- We will see how this information can be used to provide a more perfect hedge against risks.
- → We will look at a number of factors that affect the value of options, focussing here only on call options.

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- Option values are affected by a large number of parameters

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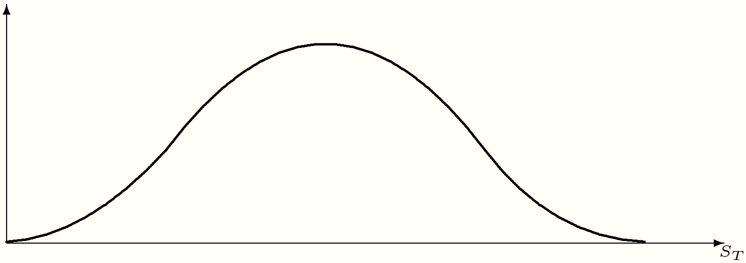

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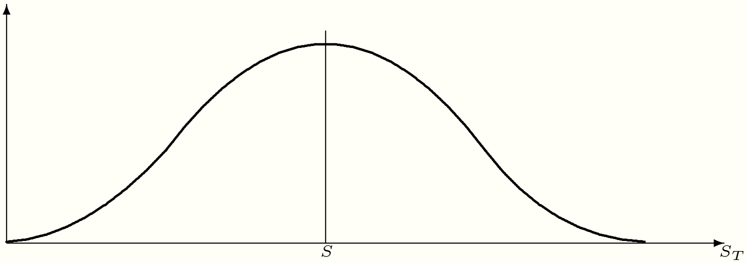

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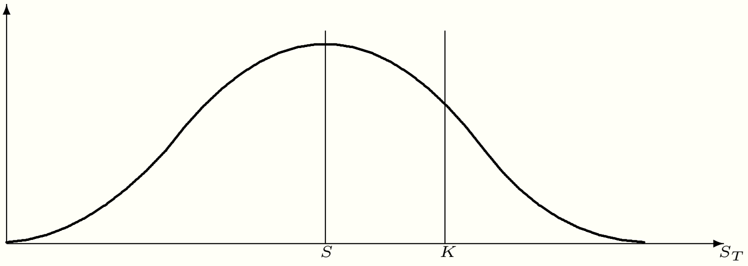

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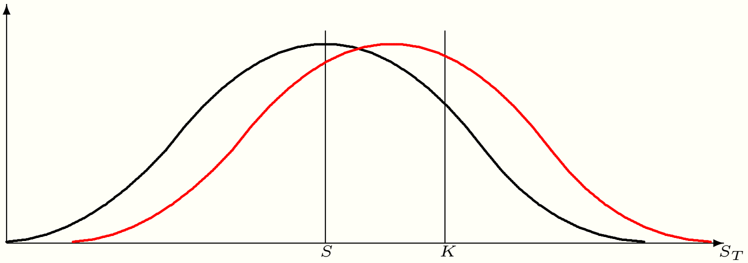

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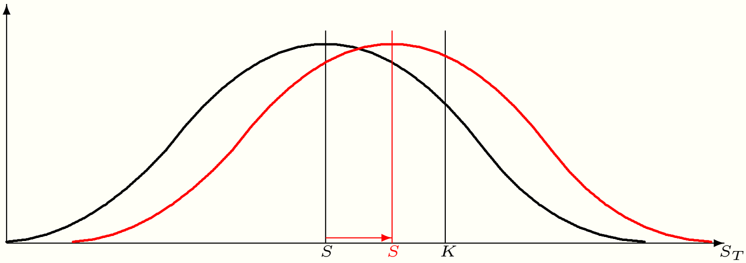

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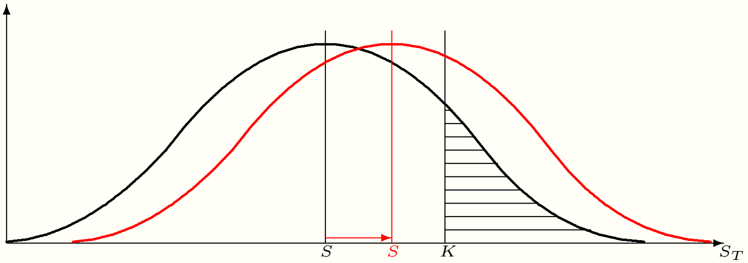

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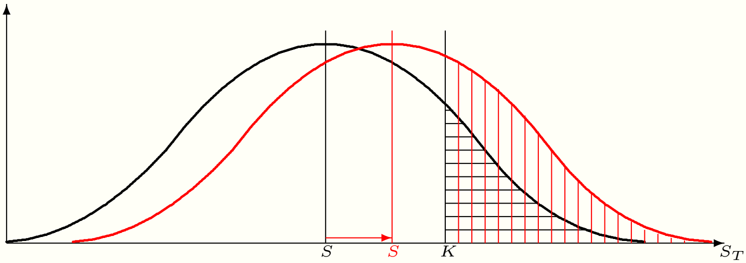

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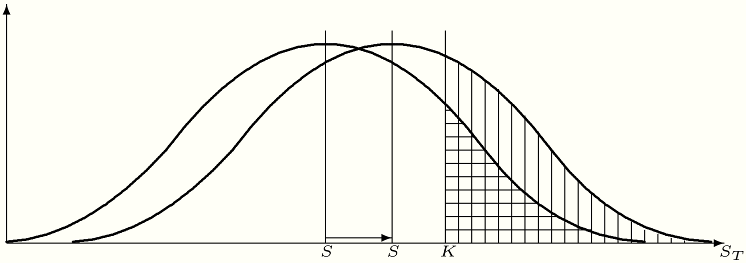

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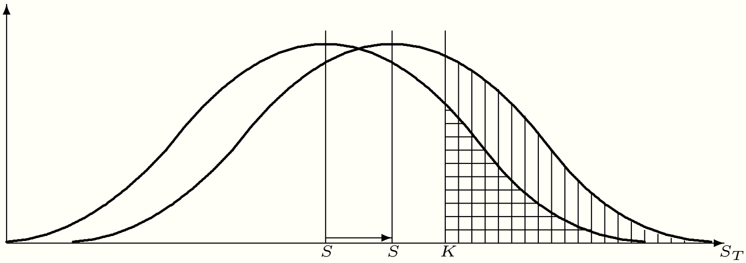

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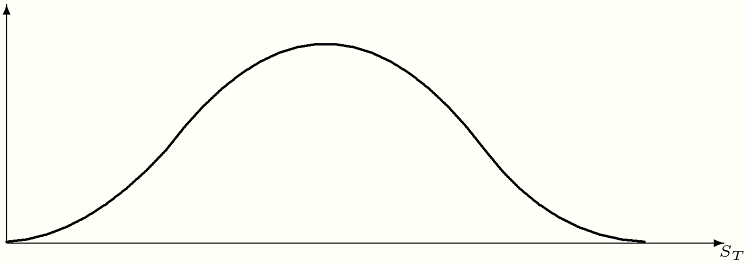

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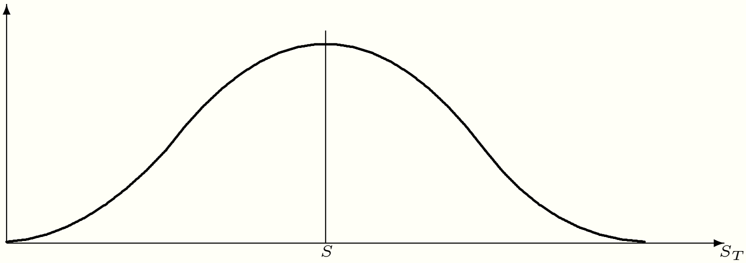

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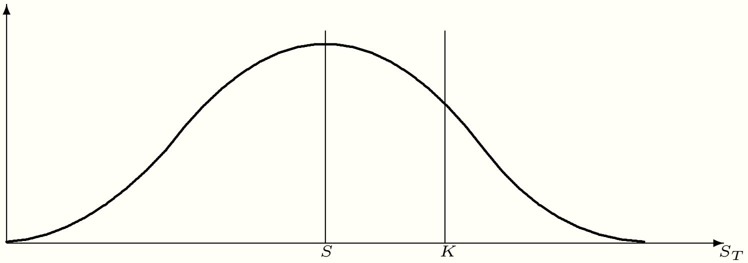

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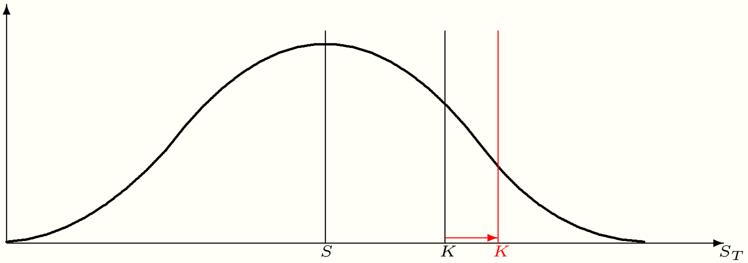

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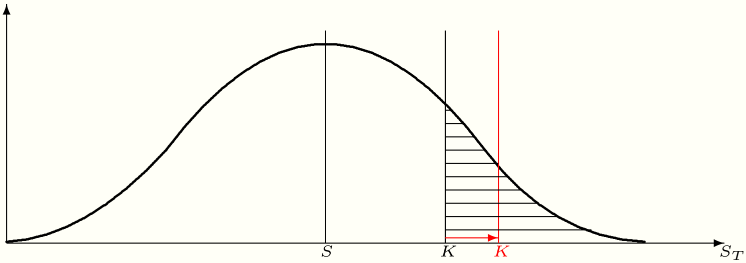

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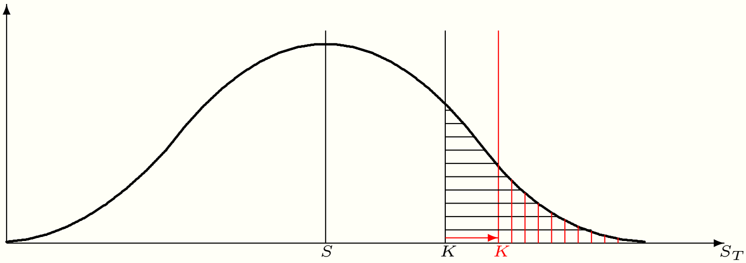

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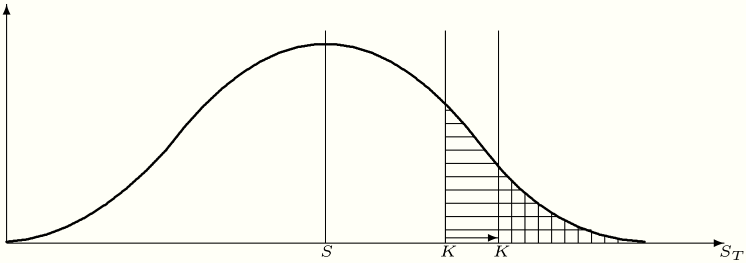

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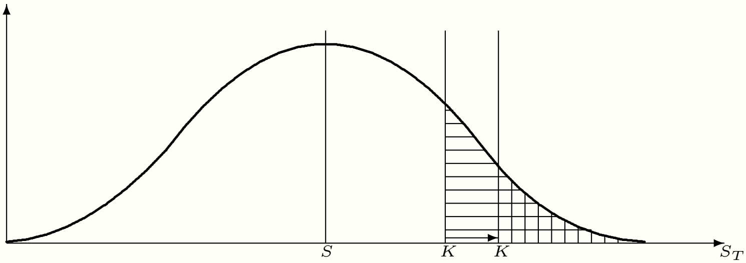

If the strike price increases, call options become less valuable

Strike price

- → We will now look at how an increase in the strike price affects the option value. Although the strike price is not a variable that can change, it is interesting to know how the option price will be affected.
- We will look at the distribution of the price of the underlying asset at maturity of the option.
- We assume a distribution of the asset price at maturity.
- The distribution of the asset price at maturity, given the current asset price is roughly symmetric around the current price.
- We look at a call option with this strike price.
- We can now increase this strike price.
- If the asset price at maturity is above the strike price the call option will be exercised and the buyer will make a profit. The area with horizontal lines shows this region with the original asset price.
- With a higher strike price, the likelihood of the option being exercised is lower and the profits themselves will be lower as the vertical lines show.
- This implies that the option has a lower value to the buyer.
- → Hence the option value decreases as the strike price increases.

Volatility

→ Let us finally look at how an increase in the volatility affects the option value. The volatility of assets might increase if the general economic conditions become more uncertain or companies increase the risks they are taking.

- We will look at the distribution of the price of the underlying asset at maturity of the option.
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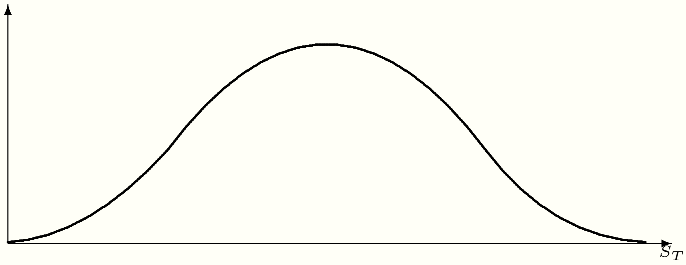

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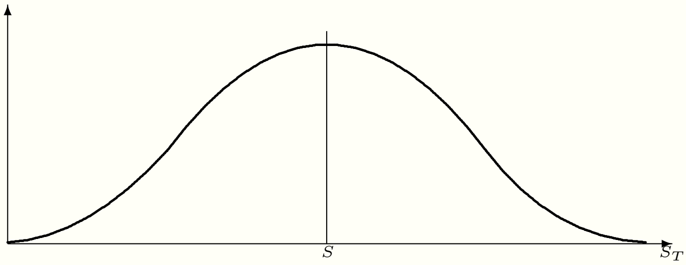

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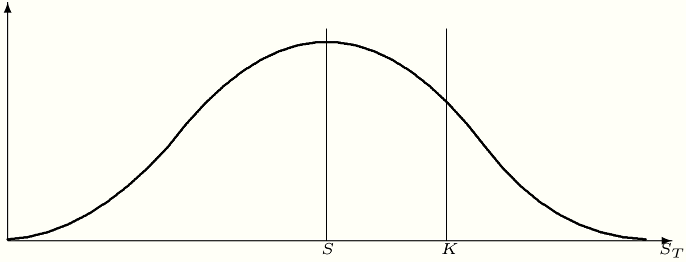

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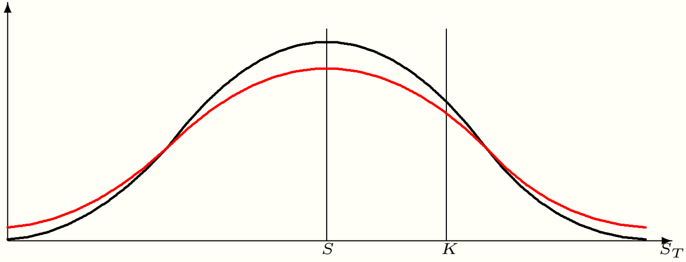

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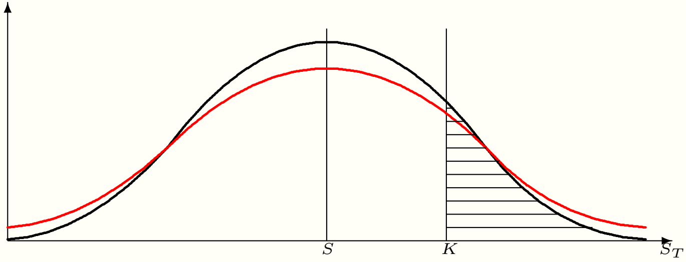

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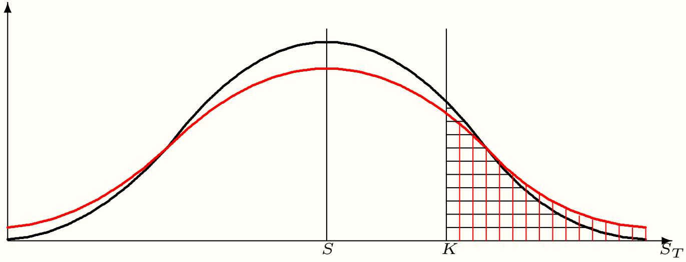

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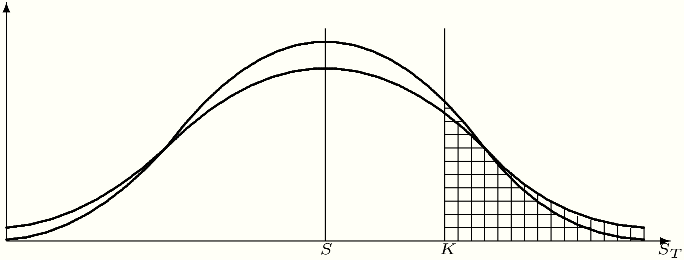

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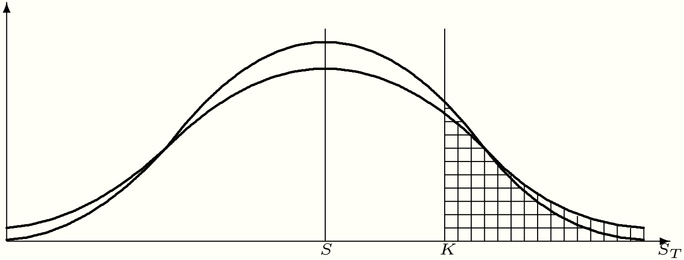

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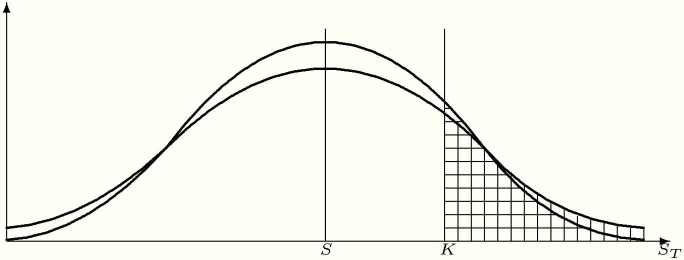

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Option Greeks

Option Greeks

- → We can now explore the the effect of variables on options prices using the Black-Scholes formula.
- The change of the option value if one of the parameters changes is called a 'Greek'.
- The Greeks show the partial derivative of the option value with respect to the parameter and thus show the sensitivity.
- Using the Black-Scholes formula, we can derive these Greeks and we see that some are identical for put and call options, wile other are different. The Greeks are all given Greek symbols from which their names derive and the most common are shown in this table. Of particular importance is the Delta of an option.
- → We will now use the Delta to derive the hedge of a position in the underlying asset that is not only avoiding losses at the maturity of the option, but at any time prior to maturity.

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	Call options	**Put options**
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Vega	$\nu_C = \frac{\partial C}{\partial \sigma} = Sn\left(d_1\right)\sqrt{T} = \frac{\partial P}{\partial \sigma} = \nu_P$	
Theta	$\theta_C = \frac{\partial C}{\partial T} = -\frac{Sn(d_1)}{2\sqrt{T}} - rKe^{-rT}N\left(d_2\right)$	$\theta_P = \frac{\partial P}{\partial T} = -\frac{Sn(d_1)}{2\sqrt{T}} + rKe^{-rT}N\left(d_2\right)$
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Δ-hedging

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→ If we use an option to eliminate all risk from a position at any point until maturity, this referred to as Δ-hedging.

- We will see how how to use options to ensure that a position has no volatility at all, thus there is no risk at any point in time.
- We consider a portfolio of the underlying asset and h put options.
- We now want to ensure that this portfolio is risk-free, that means it does not change its value as the value of the asset changes. We achieve this by setting the first derivative zero. This is not a minimisation, but we seek to avoid a change in the value of the portfolio.
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  - Solving this equation for the optimal number of put options, we get this *formula*.
  - This is 1 divided by the Delta of the put option, hence the name Δ-hedging.
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  - The strategy would be for each asset, the investor hold h put options.
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- Portfolio value hedged with a put option: $V = S + hP$
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$\Rightarrow$ $h = \frac{1}{1 - N(d_1)}$

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