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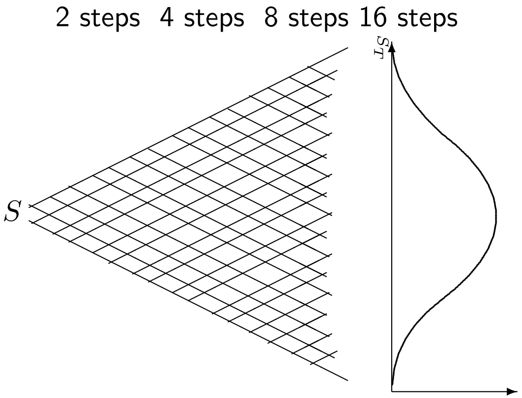
Black-Scholes model

Using the binomial option pricing model

- In each time step, the price of the asset will either increase to uS or decrease to dS
- The probability that the price increases is p
- The price will therefore have a binomial distribution
- For a given time period, the number of steps is increased with the size of asset price increases and decreases becoming smaller
- Asset increases and decreases are symmetric: $u = \frac{1}{d}$

$\Rightarrow$ Asset prices become log-normally distributed

Increasing the number of steps per time period



The Black-Scholes formula

- For a large number of steps in the binomial option pricing formula then converges to
- $C = SN(d_1) - Ke^{-rT}N(d_2)$
  $d_1 = \frac{\ln\frac{S}{K} + \left(r + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$
  $d_2 = d_1 - \sigma\sqrt{T}$
- This is known as the Black-Scholes formula of option pricing
- The variance of the asset replaces the size of the increases and decreases of the asset values: $u = \frac{1}{d} = e^{\sigma\sqrt{\frac{T}{N}}}$
- The asset price changes are replaced by the cumulative normal distribution

Similarity of Black-Scholes formula and binomial option pricing

- The structure of the results from binomial option pricing are retained
- Black-Scholes formula: $C = SN(d_1) - Ke^{-rT}N(d_2)$
- Binomial option pricing formula: $C = \Delta S + B$
- The option price is composed of a number of underlying assets and a loan to finance the holding of these assets

Replicating options

- Options are written by some investors to obtain the premium as revenue, often banks, insurance companies, or hedge funds
- This exposes them to risks if the price movement of the underlying asset causes the option to be exercised
- Option writers may want to hedge their risks
- If creating their own option, they can develop an off-setting position
- They can do so by holding $N\left(d_1\right)$ of the underlying asset and obtain a loan of $Ke^{-rT}N\left(d_2\right)$
- This is known as option replication

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