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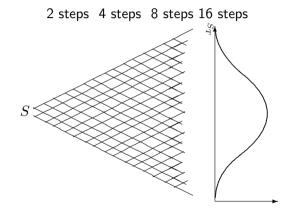
Black-Scholes model

# Using the binomial option pricing model

- In each time step, the price of the asset will either increase to uS or decrease to dS
- The probability that the price increases is p
- > The price will therefore have a binomial distribution
- For a given time period, the number of steps is increased with the size of asset price increases and decreases becoming smaller
- Asset increases and decreases are symmetric:  $u = \frac{1}{d}$
- ⇒ Asset prices become log-normally distributed

Slide 2 of 6

# Increasing the number of steps per time period



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Slide 3 of 6

### The Black-Scholes formula

- For a large number of steps in the binomial option pricing formula then converges to
- $C = SN(d_1) Ke^{-rT}N(d_2)$  $d_1 = \frac{\ln \frac{S}{K} + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$  $d_2 = d_1 \sigma\sqrt{T}$
- This is known as the Black-Scholes formula of option pricing
- ► The variance of the asset replaces the size of the increases and decreases of the asset values:  $u = \frac{1}{d} = e^{\sigma \sqrt{\frac{T}{N}}}$
- ▶ The asset prices themselves are replaced by the cumulative normal distribution

Slide 4 of 6

# Similarity of Black-Scholes formula and binomial option pricing

- The structure of the results from binomial option pricing are retained
- ▶ Black-Scholes formula:  $C = SN(d_1) Ke^{-rT}N(d_2)$
- ▶ Binomial option pricing formula:  $C = \Delta S + B$
- The option price is composed of a number of underlying assets and a loan to finance the holding of these assets

Slide 5 of 6

# Replicating options

- Options are written by some investors to obtain the premium as revenue, often banks, insurance companies, or hedge funds
- This exposes them to risks if the price movement of the underlying asset causes the option to be exercised
- Option writers may want to hedge their risks
- ▶ If creating their own option, they can develop an off-setting position
- ▶ They can do so by holding  $N(d_1)$  of the underlying asset and obtain a loan of  $Ke^{-rT}N(d_2)$
- This is known as option replication

Slide 6 of 6



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