Andreas Krause

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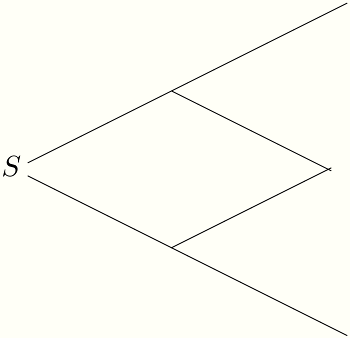
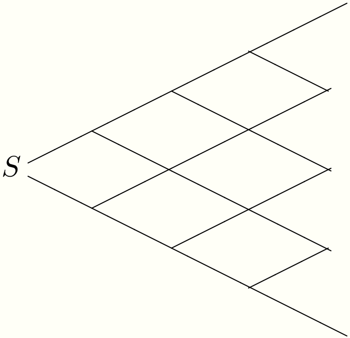
2 steps



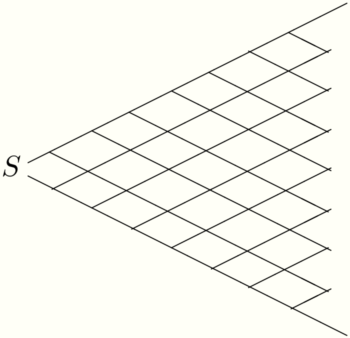
Increasing the number of steps per time period

4 steps



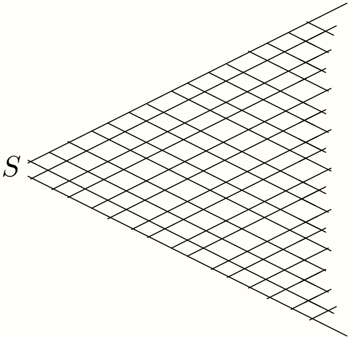
Increasing the number of steps per time period

8 steps



Increasing the number of steps per time period

16 steps



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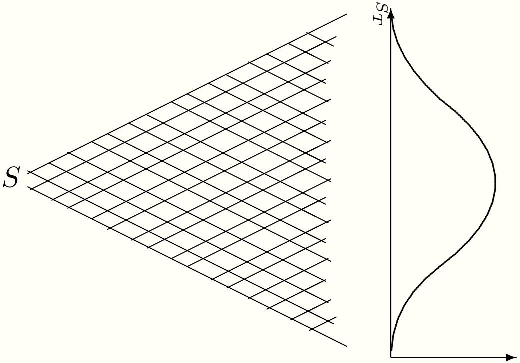

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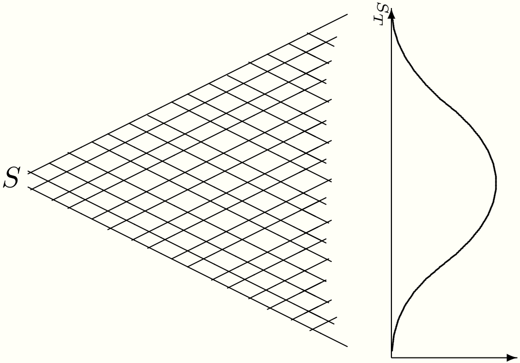

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