



Andreas Krause

Black-Scholes model

Using the binomial option pricing model

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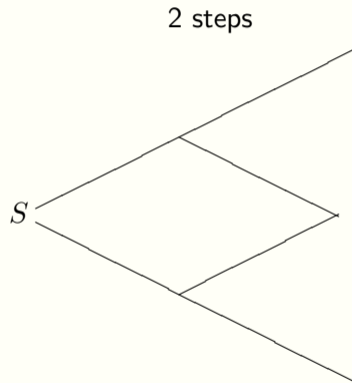
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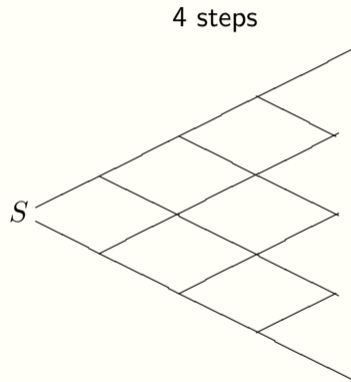
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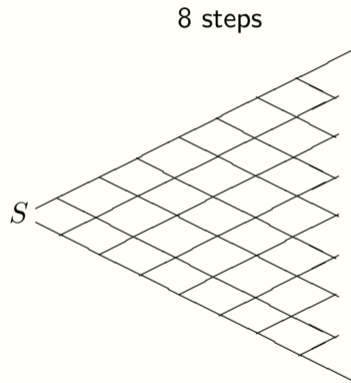
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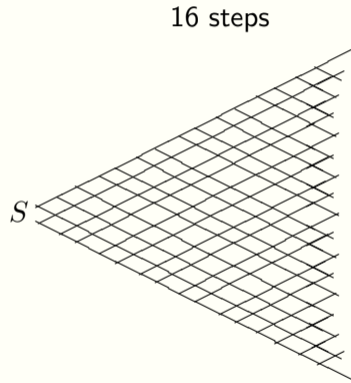
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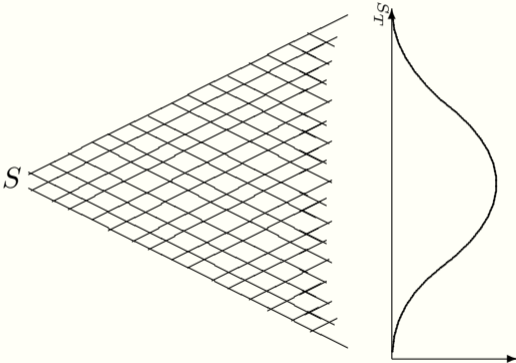
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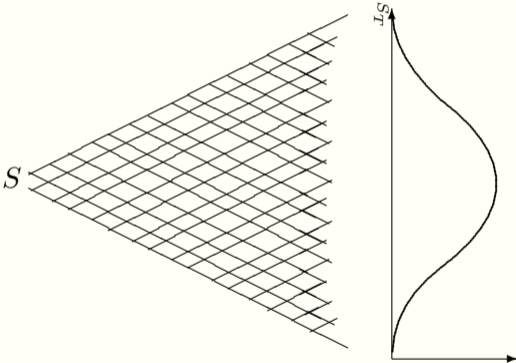
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Andreas Krause
Department of Economics
University of Bath
Claverton Down
Bath BA2 7AY
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E-mail: mnsak@bath.ac.uk