

Put-Call parity



- It is common in option theory to focus most attention on call options only, rather than put options.
- We will here use the ideas to develop an option strategy that shows how call and option values are related.

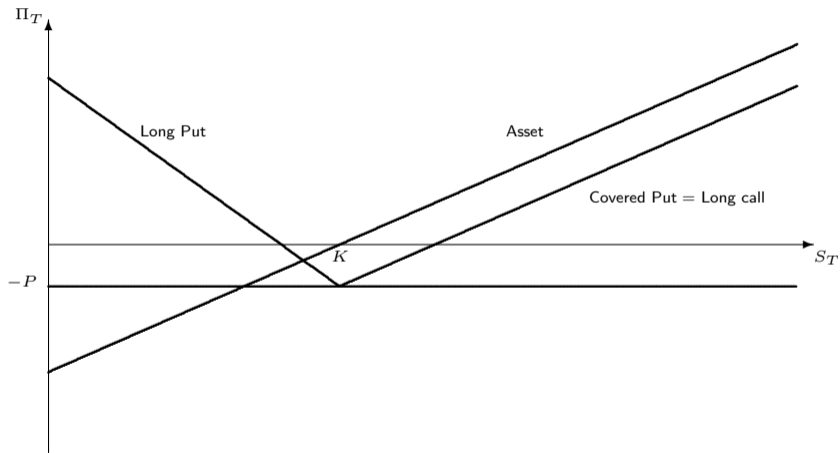
Option payoffs at maturity

- ▶ If we combine options and the underlying asset, we are able to derive different payoff profiles
- ▶ Hedging a long position in the underlying asset with a long put option gives us the payoff of a call option at maturity
- ▶ We can therefore use the asset and the put option to create a call option

Option payoffs at maturity

- We compare the payoff of different combinations of options and the underlying asset at the maturity of the options.
- ▶ We can use combination of options and the underlying asset to generate different pay-off profiles.
- ▶ We know that hedging a long position in the underlying asset by buying a put option is equivalent to a call option.
- ▶ We can use this strategy to of underlying asset and put option to obtain a call option.
- We thus have created a connection between the put and the call option.

Hedging with a put option



- We graphically can show how this call can be created.
- ▶ We look at the profits that an investor obtains for different values of the underlying asset.
- ▶ Firstly, we select a strike price.
- ▶ Relative to the strike price, we see that the underlying asset generates profits the higher its value is.
- ▶ The investor now buys a put option with this specified strike price.
- ▶ Combining these two positions gives us a covered put.
- ▶ This covered put has the same characteristics as buying a call option.
- We have thus shown that there is a relationship between put and call options that we are going to explore further.

Investment strategies

- ▶ We can invest into the underlying asset and will always obtain the value at maturity of the option, S_T
- ▶ Value of call option: $C_T = \max\{0; S_T - K\}$
- ▶ Value of put option: $P_T = \max\{0; K - S_T\}$
- ▶ We can also invest into a risk-free asset and accumulate interest

- We now compare the investment strategies with options in more detail and look at the values at maturity of the option.
- ▶ The underlying asset will give its value at the time of maturity.
- ▶ The call option will give the difference to the strike price, provided it is positive and zero otherwise as then the option is not exercised.
- ▶ The put option will give the difference to the strike price, provided it is positive and zero otherwise as then the option is not exercised. The sign will be different to that of a call option as a put option gives the right to sell the underlying asset, while a call option gives the right to buy.
- ▶ In addition we have available a risk free asset that pays some interest.
- We can now look at the details of the investment strategies required to establish the relationship between the value of call and put options.

Comparing portfolio payoffs

Portfolio A		
Initial investment	Payoff at maturity	
	$S_T < K$	$S_T \geq K$
S	S_T	S_T
P	$K - S_T$	0
	K	S_T

Portfolio B		
Initial investment	Payoff at maturity	
	$S_T < K$	$S_T \geq K$
C	0	$S_T - K$
Ke^{-rT}	K	K
	K	S_T

- We can now compare the payoffs at maturity of two investment strategies.
- ▶ We compare two portfolios and investigate the payoffs they generate at maturity of the options. With the payment from the options changing at the strike price, we will distinguish cases where the underlying asset has a value at maturity which is below and above the strike price.
- ▶ The first portfolio consists of the underlying asset and the put option; the second portfolio of a call option and a risk-free asset.
- ▶ The underlying asset will be worth S_T whether the value is below the strike price
- ▶ or above the strike price.
- ▶ The put option is exercised if the value of the underlying asset is below the strike price, paying the difference to the strike price.
- ▶ If the underlying asset is worth more than the strike price the put option is not exercised and the investor makes no profits.
- ▶ We can now see the value of this portfolio in the case of the asset being worth less than the strike price
- ▶ and if it is worth more than the strike price.
- ▶ The call option would not be exercised if the asset value is below the strike price, giving the investor no profits.
- ▶ If the asset value is above the strike price, the option is exercised and the investor obtains the difference between the value of the asset and the strike price.
- ▶ The risk-free asset will accumulate interest. The initial investment can be seen as the discounted value, discounted at the risk-free rate. At maturity, there is no discounting anymore.
- ▶ The value of the risk-free asset is independent of the value of the underlying asset.
- ▶ We can now see the value of this portfolio in the case of the asset being worth less than the strike price
- ▶ and if it is worth more than the strike price.
- ▶ We now see that the value of the two portfolios is identical in the case the asset value is below the strike price
- ▶ and also the same if it is above the strike price.
- Hence we have developed two portfolios that have the same payoff profile at maturity of the option.

Relationship between put and call prices

- ▶ The portfolios have the same value at maturity of the option, then they must have the same value prior to maturity

$$\Rightarrow P = C - S + Ke^{-rT}$$

- ▶ This relationship is called the Put-Call parity for European options

Relationship between put and call prices

- We can now use these two portfolios to establish the relationship between call and put values.
- ▶
 - Regardless of the value of the underlying asset, the two portfolios have the same value at maturity of the options.
 - If they have the same value at maturity, they must have the same value before maturity to prevent arbitrage.
- ▶ [⇒] Using the initial values of the underlying asset, put option, call option and the investment into the risk-free asset, we set these two portfolio values equal and solve for the value of the put option.
- ▶ This formula is known as the Put-Call parity for European options. It applies to put and call options on the same underlying asset and the same strike price.
- We have therefore established how put and call prices are related and knowing the value of one, allows us to determine the value of the other.

Implications of the Put-Call parity

- ▶ Assume that the strike price is equal to the current price: $K = S$
- ⇒ $P = C - (1 - e^{-rT})K < C$
- ▶ If the strike price is equal to the current price of the underlying (at-the-money), put options are worth less than call options
- ▶ Profits from put options are limited to the value of the underlying asset, but profits from call options are unlimited
- ⇒ Profits of call options are potentially higher, requiring a higher premium

- We can now explore some implications of the Put-Call parity.
- ▶ Let us now assume that the strike price is equal to the current value of the underlying asset.
- ▶ [⇒] • Inserting this relationship into the Put-Call parity, we get this *formula*.
 - It is immediately clear that this expression is smaller than the value of the call option.
- ▶ • If the strike price and the current value of the underlying asset are the same, a put option is worth less than a call option.
- ▶ • If the strike price is equal to the current value of the underlying asset, this is also known as at-the money; if the current value of a call option is above the strike price this is in-the money and if it is below the strike price it is out-of the money, for put options this is reversed.
- ▶ • This is because profits from put options are limited, given the lowest possible price of underlying asset is zero.
- ▶ • The possible profits from a call are unlimited as the price increase of the asset is potentially unlimited.
- ▶ [⇒] Therefore, the higher potential profits from buying call options makes them more valuable.
- We have thus established a relationship between the value of put and call options, allowing any valuation of standard European options to focus on call options only.



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