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Hedging with options

- We will now firstly determine the payment option make at maturity.
- This information will then be used to see how option can be used to hedge the position of an investor.

Option types

- Options give the purchaser the right (but not the obligation) to buy or sell the underlying asset in the future at a given price (strike price)
- The seller of the option has to sell or buy the underlying asset on demand of the purchaser
- Call options give the right to buy the underlying asset
- Put options give the right to sell the underlying asset
- European options give the right to exercise the option at maturity only
- American options give the right to exercise the option at any time until maturity

Option types

→ We will first define the different type of basic options that exist.

- If you buy an option you obtain the right to buy or sell the underlying asset,
  - but this is a right and not an obligation, hence the buyer of an option does not have to buy or sell the underlying asset if he does not want to.
  - This right to buy is at a future point of time or during a time period in the future.
  - The price at which the underlying asset can be bought or sold is determined in advance.
  - This price is known as the strike price.
- The seller of an option has the obligation to sell or but the underlying from the purchaser of the option; thus the right is is only on the buyer of the option while the seller has an obligation.
- There are different option type. An option that gives the buyer the right to buy the underlying asset is called a 'Call option'.
- An option that gives the buyer the right to sell the underlying asset is called a 'Put option'.
- We also can distinguish option by the time at which the buyer can exercise his rights. If this is possible only at the maturity of the option, that is the moment this right will extinguish, it is called a 'European option'.
- If exercising the right to buy or sell the underlying asset is possible at any time until the maturity of the option, it is called an 'American option'.

→ In addition, many other option types exist, and they are usually classified as 'exotic options'. The basic types decribed here are the most commonly used options and we will restrict ourselves to such option types here and specifically only consider European options.

European call option payments at maturity

- If at maturity the underlying asset is worth less then the strike price, the option will not be exercised
- Exercising the option would result in buying the asset at a price above its value
- If at maturity the underlying asset is worth more then the strike price, the option will be exercised
- Exercising the option would result in buying the asset below its value
- In this case the profits made by the purchaser are the difference between the asset value and the strike price, provided it is positive, less the option premium paid

$\Rightarrow \Pi_T = \max\{0; S - K\} - C$

European call option payments at maturity

→ We will first see how European call options work.

- The buyer of the option has the right to buy the underlying asset. If the underlying asset is trading at less than the strike price at maturity when the option can be exercised, the buyer would not exercise his right and the option expires without being used.
- Purchasing the underlying asset through the option would be more expensive to buy the asset directly.
- If at maturity the underlying asset is trading at a price above the strike price, the option will be exercised.
- In this case the underlying asset could be purchased at a lower price by exercising the option.
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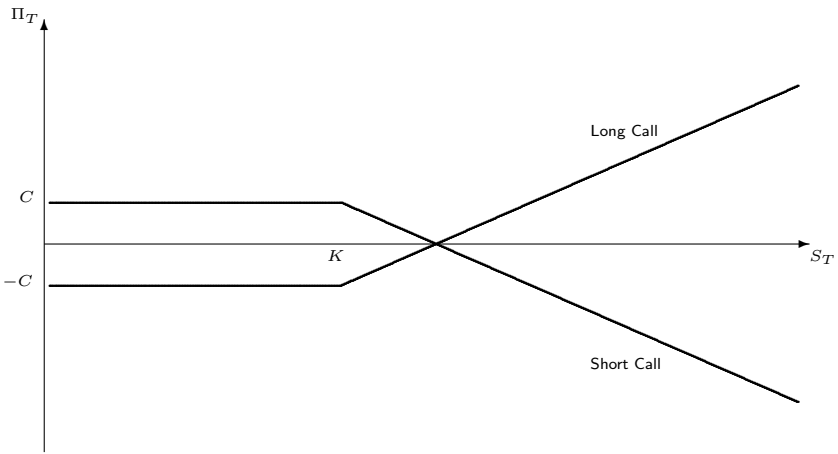
  - The profits from holding the option in this case is the difference between the price of the asset and the strike price.
  - Provided the option is exercised, this will be the profits and the option is exercised if this expression is positive. If this expression is negative, the option is not exercised and the profits from holding the option are zero.
  - Of course, the option had to be bought in the first place and the option premium needed to be paid at that point, reducing the profits by this premium.

⇒ *Formula*

→ We can now illustrate graphically these profits of a call option

Call option payoffs



Call option payoffs

→ We now consider the profits that are generated by a Call option at maturity.
- We will look at the profits the call option generates as a function of the value of the underlying asset.
- We have the strike price of this option.
- The premium paid by the buyer at maturity.
- If the price of the underlying asset is below the strike price the option is not exercised and the buyer makes a loss from the premium he paid.
- Once the price is above the strike price, the option is exercised and the difference between these two is obtained by the buyer.
- As the option is bought, it is referred to as a long call.
- The seller of the option will have the opposite profits. If the option is not exercised, he will retain the premium the buyer has paid.
- If the option is exercised, he will have to sell the underlying asset at a price below the market value at the strike price; his loss would be the difference between these two prices.
- As the call option is sold, this is also known as a short call.

→ We have thus obtained the profits for the buyer and seller of a call option.

European put option payments at maturity

- If at maturity the underlying asset is worth more then the strike price, the option will not be exercised
- Exercising the option would result in selling the asset at a price below its value
- If at maturity the underlying asset is worth less then the strike price, the option will be exercised
- Exercising the option would result in selling the asset above its value
- In this case the profits made by the purchaser are the difference between the asset value and the strike price, provided it is positive, less the option premium paid

$\Rightarrow \Pi_T = \max\{0; K - S\} - P$

European put option payments at maturity

→ We can now repeat the same assessment with a European put option.

- The buyer of the option has the right to sell the underlying asset. If the underlying asset is trading at more than the strike price at maturity when the option can be exercised, the buyer would not exercise his right and the option expires without being used.
- Selling the underlying asset through the option would give him a lower price than selling the asset directly.
- If at maturity the underlying asset is trading at a price below the strike price, the option will be exercised.
- In this case the underlying asset could be sold at a higher price by exercising the option.
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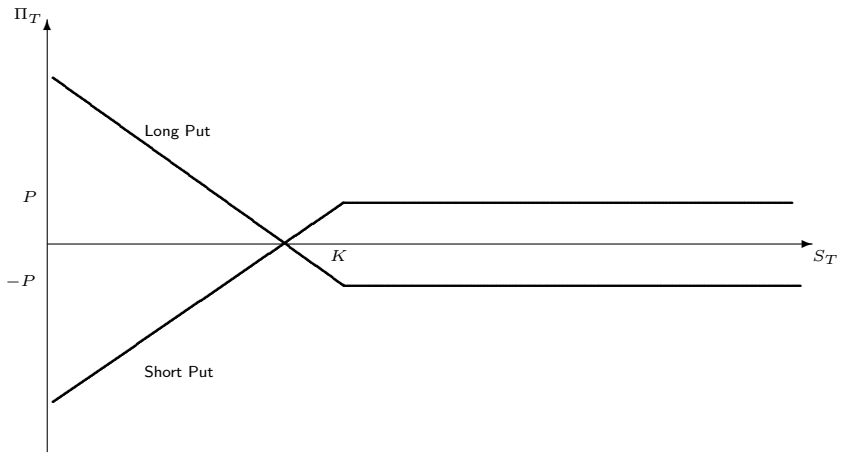
  - The profits from holding the option in this case is the difference between the price of the asset and the strike price.
  - Provided the option is exercised, this will be the profits and the option is exercised if this expression is positive. If this expression is negative, the option is not exercised and the profits from holding the option are zero.
  - Of course, the option had to be bought in the first place and the option premium needed to be paid at that point, reducing the profits by this premium.

⇒ *Formula*

→ We can now illustrate graphically these profits of a put option

Put option payoffs



Put option payoffs

→ We now consider the profits that are generated by a Put option at maturity.
- We will look at the profits the put option generates as a function of the value of the underlying asset.
- We have the strike price of this option.
- The premium paid by the buyer at maturity.
- If the price of the underlying asset is above the strike price the option is not exercised and the buyer makes a loss from the premium he paid.
- Once the price is below the strike price, the option is exercised and the difference between these two is obtained by the buyer.
- As the option is bought, it is referred to as a long put.
- The seller of the option will have the opposite profits. If the option is not exercised, he will retain the premium the buyer has paid.
- If the option is exercised, he will have to buy the underlying asset at a price above the market value at the strike price; his loss would be the difference between these two prices.
- As the put option is sold, this is also known as a short put.

→ We have thus obtained the profits for the buyer and seller of a put option.

Eliminating risks

- Options can be used to eliminate losses from the underlying asset falling below the strike price (long position)
- Options can be used to eliminate losses from the underlying asset rising above the strike price (short position)
- With options, the risk of losses is eliminated, but the possibility of gains is preserved
- Such protection is not free, an option premium has to be paid

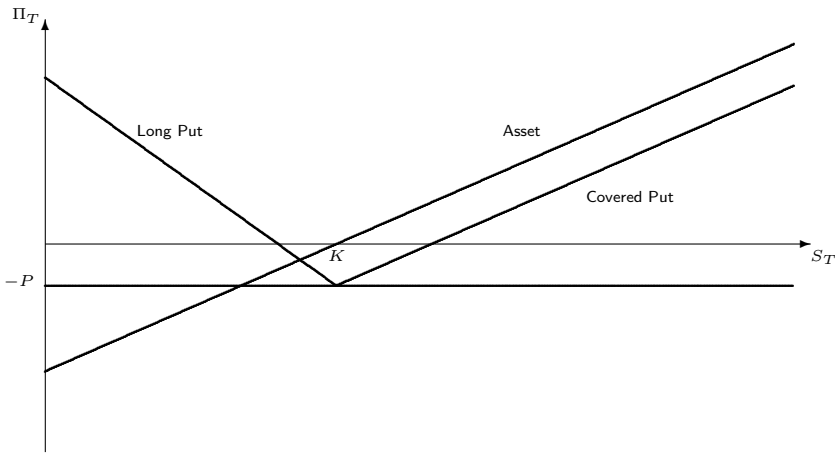
Eliminating risks

→ We can now explore how options can be used by investors to hedge their risks.

- If the asset is held by the investor, options can be used to prevent losses if the price falls below the strike price.
- If the asset is to be bought by the investor in the future, options can be used to prevent losses if the price rises above the strike price.
- We can eliminate any losses for investors, but the use of options allows investors to retain any profits if the market price moves in their favor (increase for long positions, decrease for short positions).
- This type of protection is not free, an option premium has to be paid; therefore the overall profits of the investor are reduced, even if the option is not exercised.

→ We can now illustrate graphically how such hedges with options can be conducted.

Hedging a long position in the underlying asset



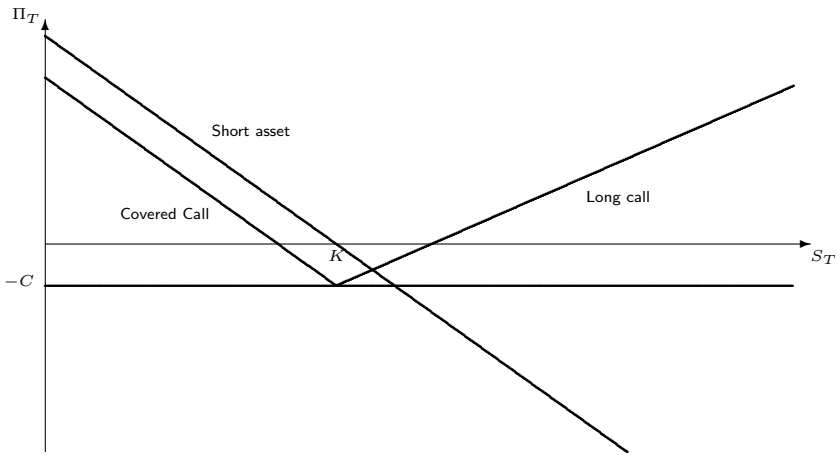
Hedging a long position in the underlying asset

→ We firstly look at the hedging of a long position, thus a situation where an investor holds the asset and wants to prevent losses if its value falls below a threshold. This threshold will be the strike price and the time to maturity will be the investor's time horizon.

- We will look at the profits an investor generates as a function of the value of the underlying asset at the time horizon.
- The investor sets the minimum level of value he wants to achieve by choosing the strike price.
- The profits, relative to the strike price, are increasing in the price at the time horizon.
- Let us combine this long position in the asset with a long put.
- The result, which is often referred to as a covered put, is that investors cannot make a loss, beyond the option premium, but will be able to make a gain.

→ Thus an investor can purchase a put option with a suitable strike price to hedge their long position in an asset and still make profits.

Hedging a short position in the underlying asset



Hedging a short position in the underlying asset

→ We will now at the hedging of a short position, thus a situation where an investor does not own the asset, but will have to purchase it in the future and wants to prevent losses if its value rises above a threshold. This threshold will be the strike price and the time to maturity will be the investor's time horizon.

- We will look at the profits an investor generates as a function of the value of the underlying asset at the time horizon.
- The investor sets the minimum level of value he wants to achieve by choosing the strike price.
- The profits, relative to the strike price, are decreasing in the price at the time horizon. A lower price means that the asset can be bought more cheaply, benefitting the investor and a higher price means that the costs are increasing, reducing profits.
- Let us combine this short position in the asset with a long call.
- The result, which is often referred to as a covered call, is that investors cannot make a loss, beyond the option premium, but will be able to make a gain.

→ Thus an investor can purchase a call option with a suitable strike price to hedge their short position in an asset and still make profits.

Risk elimination only at maturity

- Purchasing an option that allows to sell or buy the position if it is loss-making, eliminates the risk of losses from the underlying asset beyond the strike price
- As an option premium is payable, losses are not fully eliminated but cannot exceed the premium paid
- Risk is only eliminated at maturity of the option, the value of the portfolio can vary prior to maturity

Risk elimination only at maturity

→ We have seen that options can be used to eliminate downside risk, but maintain the ability to benefit from profits in their positions.

- The strategy is to purchase an option that allows the investor to make profits from exercising the option, offsetting the loss of the asset itself. The structure of the option is such that it will not offset any gains the asset generates.
- As the option has to be paid for, the losses cannot be eliminated completely, but they will be limited to the option premium; this premium is usually only a small fraction of the asset value and hence losses will be small.
- The risk is only eliminated at maturity of the option, prior to this time, the value of the asset combined with the option value can vary prior to this point and cause losses, although these are recovered again until maturity of the option. Different hedging strategies exist that can eliminate these risks, but they are dependent on the option pricing model.

→ We have now established that options can be used to eliminate losses for investors, while they retain any profits.

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