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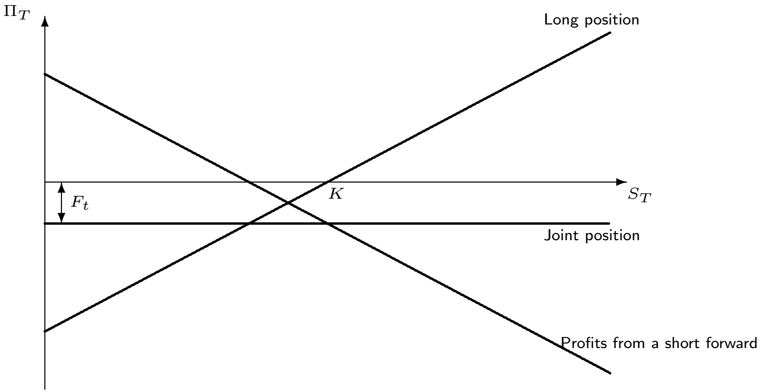
Hedging with futures

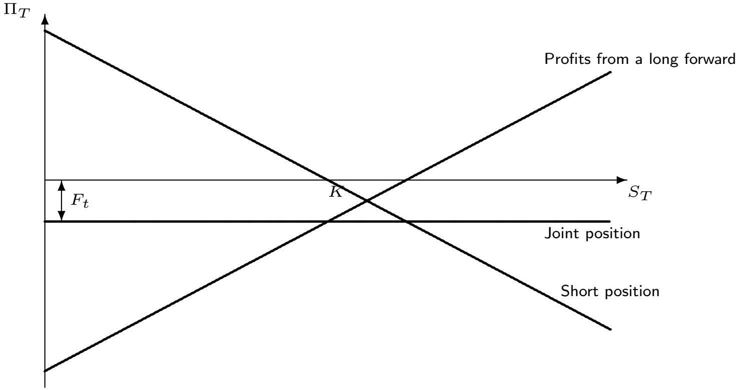
Levered investments and hedging

- Futures and forwards can be used to obtain exposure to the underlying asset without purchasing it, but investing only the premium
- This often referred to as a levered investment
- Futures are also used to eliminate the risk from having exposure to the underlying asset
- Such a use of derivatives if called hedging

Investor holding the underlying asset



Investor having to provide the underlying asset



Unavailability of futures and forwards

- Futures are not always available for the underlying asset desired
- Banks might not be willing to engage in bespoke forwards for the underlying asset desired
- This is a particular problem for stocks, where mostly only stock index futures are available
- Agreeing forwards for specific portfolios might be difficult

$\Rightarrow$ Investors agree a futures or forward contract in a similar underlying asset

Imperfect hedging

- A long position in the asset is hedged with a number of short forward contracts on a different underlying asset
- The value at maturity of the forward is the value of the asset, less the value from the underlying asset of the forward, for each contract
- $\Pi_T = S_T - hF_T$

$\Rightarrow$ $\text{Var}\left[\Pi_T\right] = \text{Var}\left[S\right] - 2h\text{Cov}\left[S, F\right] + h^2\text{Var}\left[F\right]$

- We minimize the variance of the profits: $\frac{\text{Var}[\Pi_T]}{\partial h} = 0$

$\Rightarrow$ $h = \frac{\text{Cov}[S,F]}{\text{Var}[F]}$

- This is known as the hedge ratio, detailing the number of contracts in the underlying asset for each asset sought to hedge

$\Rightarrow$ $\text{Var}\left[\Pi_T\right] = \text{Var}\left[S\right] - \frac{\text{Cov}[S,F]^2}{\text{Var}[F]} > 0$

- The risk is not eliminated, but reduced

β-hedging

- Assume the asset is a stock and the underlying asset of the forward the stock market
- $h = \frac{\text{Cov}[R_i, R_M]}{\text{Var}[R_M]} = \beta_i$
- $\text{Var}\left[\Pi_T\right] = \sigma_i^2 - \beta_i^2 \sigma_M^2$
- Assume the Capital Asset Pricing Model holds: $\mu_i = r + \beta_i \left(\mu_M - r\right)$

$\Rightarrow R_i = r + \beta_i \left(R_M - r\right) + \varepsilon_i$

$\Rightarrow \sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma_{\varepsilon_i}^2$

- The total risk of a stock consists of the systematic risk and the idiosyncratic risk
- $\text{Var}\left[\Pi_T\right] = \sigma_{\varepsilon_i}^2$
- Hedging using stock indices eliminates systematic risk, but not idiosyncratic risk

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