Andreas Krause

Hedging with futures

- Forwards and futures can be used to speculate and obtain potentially large profits from small initial investments, although this might also turn into large losses.
- Forwards and futures can also be used to hedge existing positions in the underlying asset.
- We will explore this hedging process in more detail here.

Levered investments and hedging

Levered investments and hedging

→ Forwards and futures can be used for speculation, but also to hedge exposures to the underlying asset.

- - The value of a forward or future at its maturity is given by the difference between the price of the underlying asset and the strike price, thus the profits rise (and fall) with the price of the underlying asset, giving an investor the same movements as when holding the underlying asset directly.
  - These profits (and losses) are obtained with no initial investment if the strike price was the forward rate or a small investment in form of the premium.
- This is referred to as a levered investment and the returns are a multiple of the return on the underlying asset due to the small initial investment.
- Apart from their use for speculators, the reason forwards and futures were introduced was to allow investors to transfer risk between them as they have exposure to the underlying asset.
- Using derivatives to re-distribute risks is called hedging.

→ Risk is not eliminated but only transferred to another party. The counterparty selling the forward or future will take on the risk that has been eliminated by the investor.

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Investor holding the underlying asset

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- → Let us first consider the case where an investor holds the underlying asset and seeks to eliminate the risk of this position.
- We will look at the profits an investor would make, depending on the value of the underlying asset at maturity of the forward or future.
- We choose a strike price at which we want to insure the current position and this will define our benchmark to determine the profits and losses to the investor. This strike price might be the current price of the underlying asset.
- Holding an asset is also known as a long position. The profits relative to the strike price are then given as a straight line; if the price at maturity is above the strike price, the investor makes a profits, otherwise a loss.
- The investor now uses a forward that allows him to sell the underlying asset at the strike price. As it involves selling the underlying asset this is known as a short forward or short future. The profits from this forward at maturity ($T = 0$) can be obtained in the same way as when determining the forward value, but taking into account that we look at the position of the seller. A higher price of the underlying asset at maturity implies that he has to sell the asset at the lower strike price, making a loss. A lower price at maturity allows him to sell the underlying asset at a higher price, making a profit.
- If we add these two positions together, we see that both have the same slope of opposite signs, thus the joint position is horizontal as indicated here. Regardless of the value of the underlying asset at maturity, the value of the joint position is fixed. Thus there is no risk to the investor and he is perfectly hedged.
- Overall he may make a loss as indicated here as the two positions do not cancel each other out perfectly.
- This difference is the value of the forward contract. This premium can be negative or positive; it will be zero if the strike price is the forward rate.
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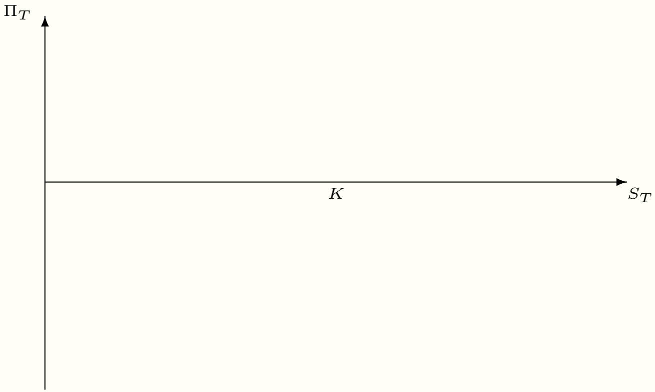

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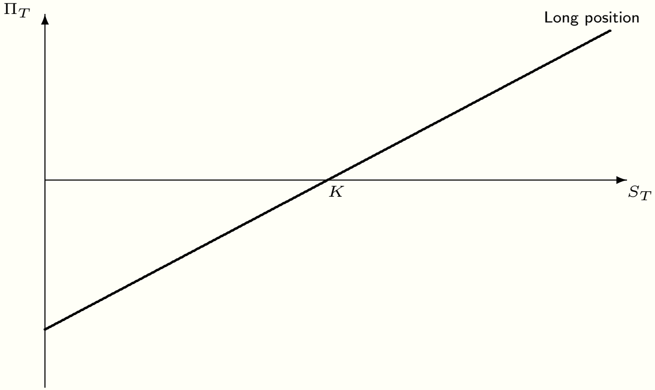

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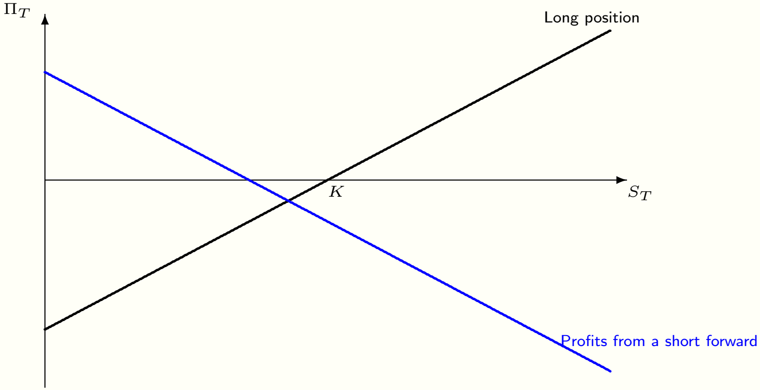

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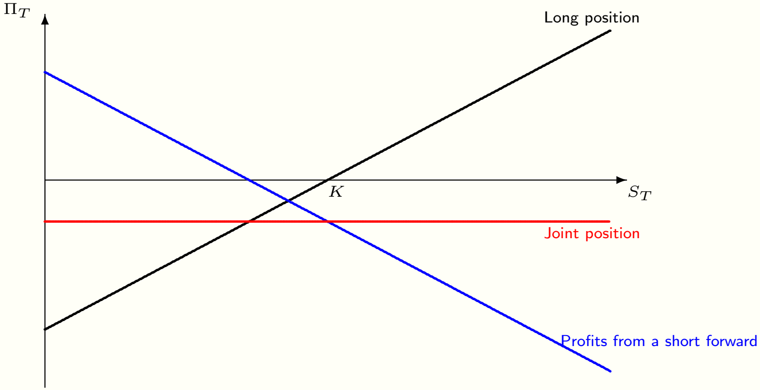

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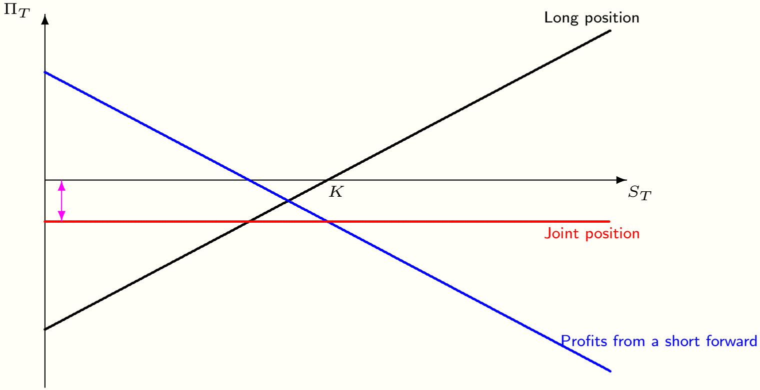

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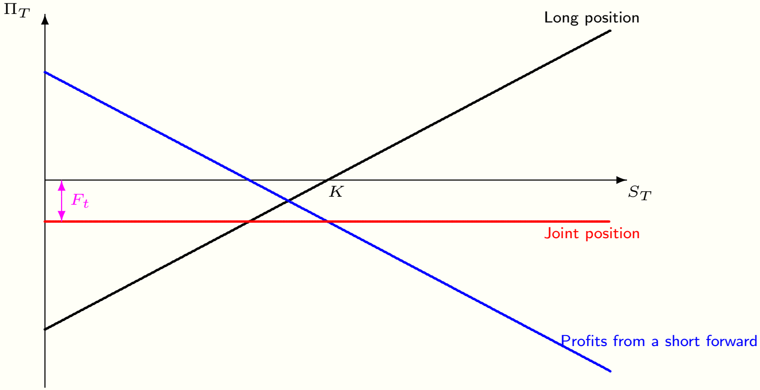

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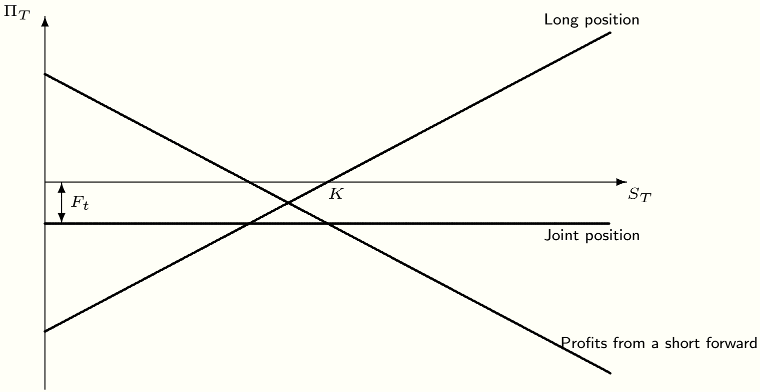

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Investor having to provide the underlying asset

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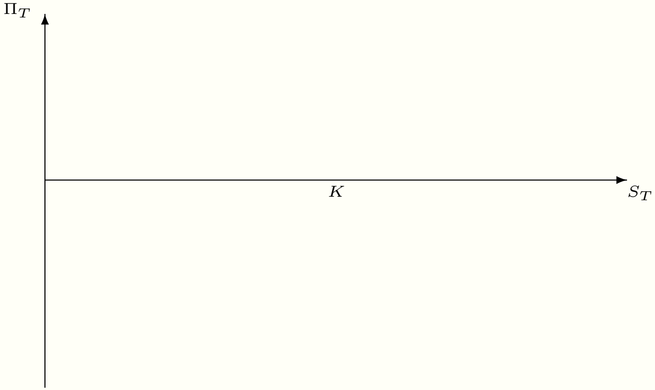

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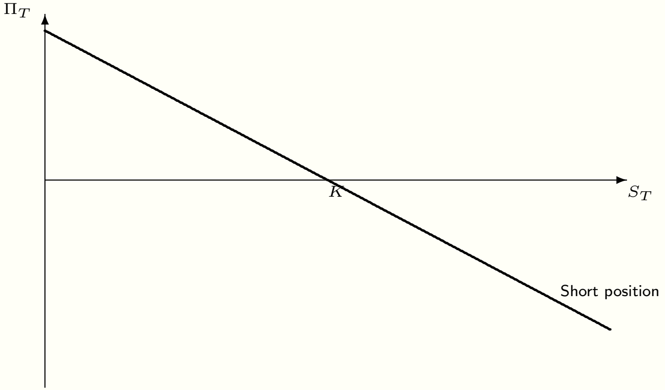
→ We can use forwards and futures to hedge a position when having to purchase the underlying asset in the future, eliminating all risk to the investor.

Investor having to provide the underlying asset



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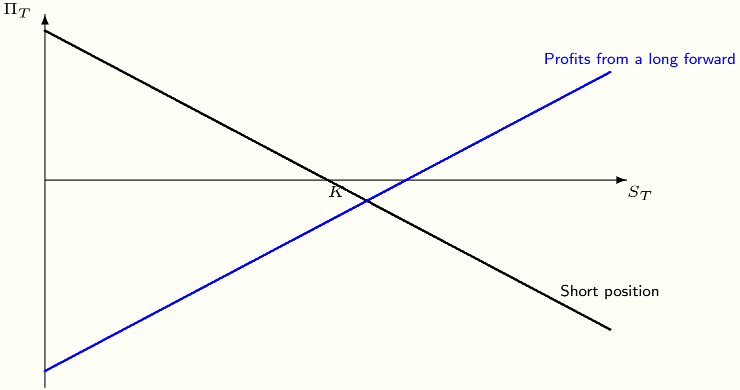

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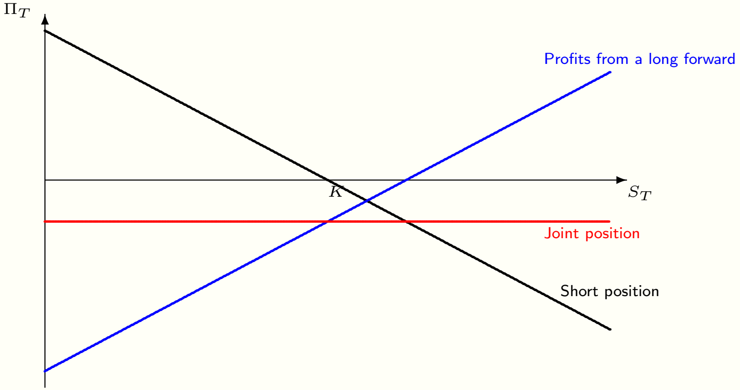

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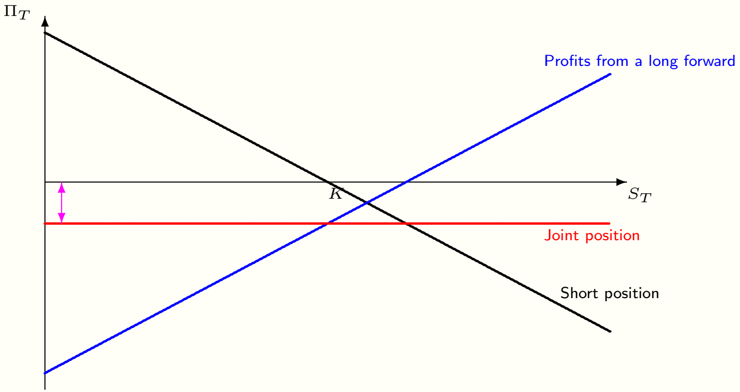

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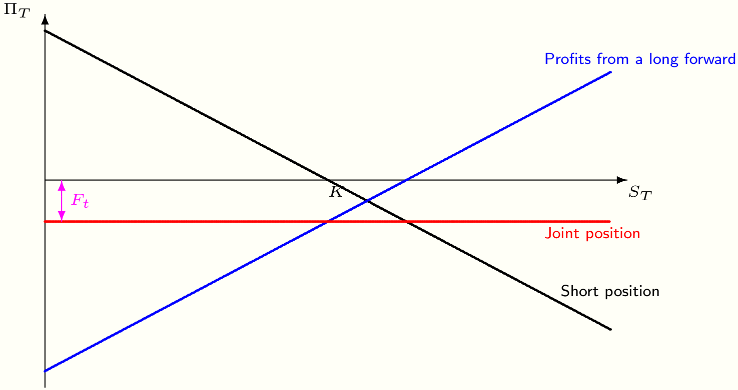

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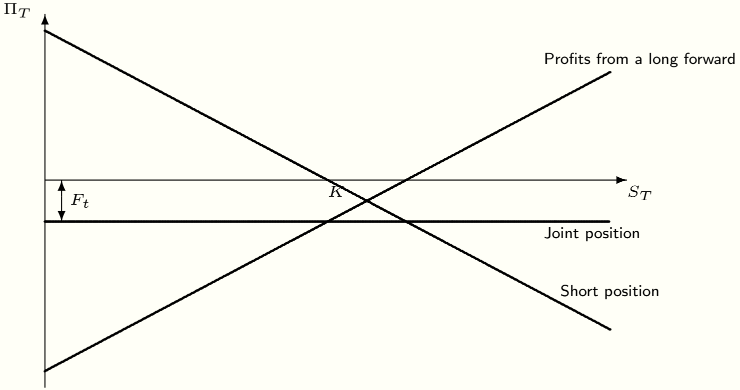

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- While forwards can be agreed bilaterally between partners, futures are only available for selected underlying assets.
- In addition, forwards might not be easily agreed on some underlying assets with counterparties if there is little demand.
- While stock index futures are available, futures are only available on a small number of individual stocks, often only the most widely held stocks. Even where futures on individual stocks are available, these are often not frequently traded and prices might not be efficient.
- Investors holding portfolios of stocks would have to either agree a larger number of forwards on individual stocks or a forward on their specific portfolio, both of which would be time consuming and often not feasible.
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Imperfect hedging

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→ We consider now how such a hedge with a different underlying asset can be conducted.

- We know that a long position in an asset can be hedged with a short forward contract. We now generalise this idea by stating that we will use a forward on a different underlying asset and allow for any number of forward contracts to be used.
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  - At maturity of the forward, the profits are given by the value of the underlying asset which the investor still holds.
  - Less the value of the forward on the different underlying asset, which at maturity is the difference between its value and the strike price.
  - The sign is negative due to it being a short forward and we use h such contracts for each of the assets we want to hedge.
- *Formula*
- [⇒] We can determine the variance of these profits at maturity.
- As we want to hedge the risk, we want to minimize this risk.
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  - The number of forward contracts per asset is known as the hedge ratio.
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- [⇒] Inserting the hedge ratio, we can determine the variance of the joint position. We can see that this expression will be positive; dividing by Var $[S]$, we see that the last term is the squared correlation and hence, unless the expression will be positive unless the correlation is perfect.
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β-hedging

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→ We will now look at the properties of this hedge in a special case of using stock index futures.

- We look at the case of the asset whose position is to be hedged being a stock and the underlying asset of the future is a stock market index.
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  - Rather than looking at the variance of the value of the stock and the index, we can look at their respective returns. The return of the asset we denote by R_i and that of the stock market index by R_M.
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→ We this see that forwards and futures can be used to eliminate risk completely if the underlying asset is identical to the asset that is to be hedged. If this is not the case, only the systematic risk can be hedged; thus hedging eliminates only systematic risk in this case, but idiosyncratic risk could be eliminated through diversification.

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