

Forwards and futures

- Forwards are the most basic financial derivatives and have been used since the 17th century, they have been common to use for agricultural products throughout the 20th century in US.
- They have seen much more widespread use since the 1970s for interest rates and currencies after the breakdown of the Brtton Woods agreement on fixed exchange rate and the volatility of exchange rates and interest rates that followed.
- Since the 1980s they have also been used on stock indices and occasionally individual stocks.

Definitions

- ▶ A Forward is a contract in which the buyer agrees to purchase the underlying asset at a future date (time to maturity) for a given price (strike price)
- ▶ In return the seller agrees to sell the underlying asset at this future date
- ▶ A Futures contract is similar to a forward, but has standardised times to maturity and strike prices
- ▶ Futures have usually short times to maturity, while forwards may have longer times to maturity
- ▶ In addition, purchasers and sellers have to provide collateral for their commitments, the margin

- Forwards are contracts that settle terms today for transactions at a future point of time/
- ▶
 - A forward is a contract that is based on another asset, such as a security, currency, commodity. It is a contract to purchase (or sell) this so-called underlying asset. This purchase (or sale) will be completed in the future. Purchases (sales) that are completed immediately are called spot transactions.
 - The time period until the purchase is completed is also called the time to maturity.
 - The price at which the purchase (sale) is to be conducted, will already be agreed at the time the forward is agreed.
 - This price at which the asset is to be purchased (sold) is called the strike price.
- ▶ The purchaser agrees to buy the asset at these conditions and the setter agrees to sell it to him. Thus the contract is binding on both parties. This is different to options, where the contract is binding only on the seller of the option.
- ▶ A futures contract is the same as a forward contract, but it has standardised maturities and strike prices that allow these contracts to be traded on an organised exchange.
- ▶
 - Typically futures have a short time to maturity, often within 3 months, and rarely over 1 year.
 - Forwards are agreed bilaterally between the two parties, only of which is usually a bank, and can be bespoke in all aspects, the strike price but also the time to maturity, which can be substantial, often many years.
- ▶ In futures both parties have to provide collateral as this is a commitment to engage in a transaction. This collateral is called a margin.
- Thus forwards and futures are in essence the same and we will here neglect the impact the margins will have on any decisions.

Forward value

- ▶ At maturity of the forward, the profits are the value of the underlying asset, less the strike price, and the price paid for the forward
- ▶ $\Pi_T = S_T - K - F_t$
- ▶ If buying the underlying asset directly, the profits are the value of the underlying asset, plus any returns on that asset, less the costs of financing the asset
- ▶ $\hat{\Pi}_T = S_T + r_S T S_t - (1 + r_L T) S_t$
- ▶ To prevent arbitrage, these profits have to be equal, $\Pi_T = \hat{\Pi}_T$
- ⇒ $F_t = S_t - K + (r_L - r_S) T S_t$
- ▶ The value of a future consists of the profits to be made and the cost of carry (basis)
- ▶ The value of forwards can be positive or negative

- We now seek to determine the value of a forward (or a future) and we focus on the value of a future from the perspective of the buyer.
- - We consider the value of the future at its maturity, that is at the time the purchase or sale is completed. At that time we can determine that the value will depend on the value of the underlying asset.
 - The asset is bought at the strike price, hence the profits of the buyer is the difference between these two values.
 - From these profits we have to deduct the price the buyer paid to obtain the forward.
- *Formula*
- - Rather than using a forward, the buyer could purchase the asset directly. After the transaction the buyer holds the asset and obtains its value at maturity of the forward.
 - They also obtain any payments the asset makes during these T time periods until maturity of the forward. These returns might be dividends for stocks, interest paid on bonds, and similar.
 - From this the purchase price, the current price of the asset, needs to be conducted. As with a forward we only pay the purchase price at maturity, we have additional financing costs for buying the asset already now.
- *Formula*
- At maturity of the forward, both strategies yield the same outcome, namely the purchase holds the underlying asset. Thus the profits these two strategies yield must be the same. If this were not the case, then either forwards would not be demanded or everyone would demand forwards and not purchase the asset directly. It would thus enable arbitrage by engaging in one strategy and offsetting it with the other (short sale), giving a profit as that would be the difference between the two strategies.
- ⇒ *Formula*
- - The value of the futures is then consisting of the profits to be made from purchasing the asset through the forward at the strike price compared to purchasing it outright now.
 - To this we add the financing costs of the direct purchase as a benefit for delaying the purchase, but also take into account the benefits the asset provides and which do not accrue to the purchaser if delaying the purchase through a forward. This difference is known as 'cost of carry' or the 'basis'.
 - The cost of carry is also called the 'basis'.
- We can easily see that the value of a forward can be positive or negative. If the strike price of the asset is high compared to the current price or the yield on the asset is high (r_S), the forward value can be negative. The value of a forward to the seller of the asset (and hence the seller of the forward) would be the same, just with the opposite sign.
- Forwards are often quoted using a the forward or futures rate. We will determine this value now.

Forward rate

- ▶ Forwards are commonly set such that no initial premium is to be paid: $F_0 = 0$
- ⇒ $K = S_0 + (r_L - r_S) TS_0$
- ▶ This is known as the forward rate
- ▶ For futures the strike prices are given and they will have an initial premium to pay

- In the context of forwards it is common to hear of the forward rate, which we will explore now.
- When a forward is agreed, the strike price agreed is often set such that the value of the forward is zero and no payments are necessary until maturity.
- ⇒ We solve the value of a forward for the strike price and obtain the *formula*.
- This strike price is known as the forward rate.
- - If we are using futures, the strike prices are set by the rules of the exchange on which they are traded.
 - For this reason we cannot set the strike price such that no premium is payable when entering a futures contract.
- Once a forward has agreed, its value will become positive or negative; this is because the current value of the underlying asset will change (S_t), but also the cost of carry (basis) changes as the time to maturity becomes shorter.

Types of forwards

	Financing rate r_L	Yield on underlying asset r_S
Stock index	risk-free rate	dividend yield
Currency	interest rate domestic currency	interest rate foreign currency
Interest rate	risk-free rate	bond yield
Commodities	risk-free rate	negative of storage costs

- We will now look at the different types of forwards and futures and determine what the basis for the cost of carry is. The strike price, the price of the underlying asset, and the time to maturity are straightforward.
- We will look at what the financing rate is and what constitutes the yield on the underlying asset.
- Stock index futures had become popular in the 1980s as an alternative to investing into a broad portfolio of stocks in an index. These were mostly used as futures rather than forwards. The financing rate would be the risk-free rate, the interest rate at which the buyer could borrow funds. The yield of the underlying asset arises from any dividend payments of the stocks in the index that are due until time to maturity of the futures. This would only apply if the index value is not adjusted for dividend payments.
- Currency forwards and futures are both available, where the majority of transactions are forward agreements. The financing rate would be the rate of the domestic currency as that is the currency the loan would have to be taken up in. The yield of the underlying asset would then be the interest that could be earned in the foreign currency bought.
- Interest rate forwards and futures are agreeing to purchase a bond in the future; while both futures and forwards exist, forwards are the more common form. The financing rate would again be the risk-free rate and the yield of the underlying asset the yield on the bond, whose interest does not accrue to the purchaser of a forward or future.
- Commodity futures are available on a wide range of commodities and the use of futures is quite widespread, especially for agricultural products, but also oil. The financing rate is the risk-free rate again, but commodities do not yield any return, instead holding commodities may require to pay storage costs; such storage costs can be seen as a negative yield on the commodity. Storage costs are only relevant for forwards and futures for which a physical delivery is agreed; many such forwards and futures are agreed to be settled by cash, paying the difference between the market price at maturity and the strike price, without taking delivery of the commodity. In this case storage costs would be zero.
- While there are small differences arising from the various underlying assets, the principle idea of forwards and futures applies equally across all underlying assets.

Properties of forwards and futures

- ▶ Forwards and futures are valued based on the current value of the underlying asset, adjusted for the benefits of purchasing the asset later (cost of carry)
- ▶ Forward rates can be lower than the current price if the yield of the underlying asset is higher than the financing costs
- ▶ Forwards and futures can have a negative value

Properties of forwards and futures

- We have seen how the value of forwards and futures can be determined and what the forward rate is.
- ▶
 - The value of a forward is based on the difference between the current price and the strike price, thus the profits or losses that would be made when purchasing the underlying asset using the forward or future.
 - These profits or losses are adjusted by net financing costs if purchasing the underlying asset directly now, the so-called cost of carry.
- ▶ If the yield on the underlying asset is higher than the financing costs, this cost of carry is negative and the forward rate will be below the current value of the underlying asset.
- ▶ We have also seen that forwards and futures can have either a positive or negative value.
- Forwards and futures are used for speculators as they require no or a very small initial investment, but as the profits (or losses) depend on the value of the underlying asset, they fully participate in the movements of the underlying asset. This give the potential for substantial profits with very small initial investment, but also the potential for very large losses. As profits and losses often are a multiple of the forward or future value, the investment is often referred to as 'levered'.



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