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Interest rate risk

- Government bonds of most developed countries are seen as risk-free.
- This risk-free assessment is only relevant if the bond is held until maturity as the bond is guaranteed to be repaid in full and any coupon payments will have be made before that.
- With bond yields changing all the time, the value of bonds will vary as well; thus, for any investor seeking to sell a bond before its maturity, they may face risks.
- We will seek to investigate this risk.

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Bond value

- The value of a bond is the present value of all future payments, consisting of the coupon and the repayment of the face value
- $B = \sum_{\tau=1}^{T} \frac{C_\tau}{(1+r)^\tau} + \frac{B_0}{(1+r)^T}$
- Usually we assume that all coupons are identical, the interest the bond pays is fixed
- If interest rates are variable, usually by using to a reference interest rate, these will vary over time
- As bonds are assumed to be risk-free, the discount rate is the risk-free rate

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Bond value

→ We start the investigation by determining the value of a bond.

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  - The bond value is determined by taking the present value of all payments the bond makes to an investor. The discount rate for obtaining the present value is the risk-free rate for this time to maturity.
  - The payments the investor obtains are the coupon payments, which is the interest due on the bond, usually quarterly, semi-annually, or annually. In some cases no coupons are payable, which is known as a zero bond.
  - In addition, at maturity, the bond is repaid, usually at face value.
- *Formula*
-
  - In most cases the coupons paid are identical for the duration of the bond.
  - This is a fixed-rate bond. There are other form of bonds with coupons increasing or decreasing over time.
- We can also have variable interest rates, where the coupon is adjusted regularly, usually quarterly, to match a reference market interest rate, often the Secured Overnight Financing Rate (SOFR) with a small surcharge.
- Bonds are expected to be repaid and all coupons paid on-time, hence the discount rate used, should be the risk-free rate for that maturity, thus the yield on such bonds.

→ We can now use the value of bonds to determine how sensitive their value is to a change in the yield.

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Duration of bonds

- Interest rates will vary over time, while the coupon payment for the bond remain fixed
- If the discount rate changes, the value of the bond will change
- $$\frac{\partial B}{\partial r} = \sum_{\tau=1}^{T} -\tau \frac{C_\tau}{(1+r)^{\tau+1}} - T\frac{B_0}{(1+r)^{T+1}}$$
  $$= -\frac{1}{1+r}\left(\sum_{\tau=1}^{T} \tau \frac{C_\tau}{(1+r)^{\tau}} + T\frac{B_0}{(1+r)^{T}}\right)$$
  $$= -\frac{DB}{1+r}$$
- $D = \frac{\sum_{\tau=1}^{T} \tau \frac{C_\tau}{(1+r)^{\tau}} + T\frac{B_0}{(1+r)^{T}}}{B}$ is the duration of the bond

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- → We can now introduce the key concept in managing interest rate risk, duration.
- ▶
  - Interest rates vary over time, thus the yield of the bond will change.
  - We assume here that the coupon payments are fixed for the life-time of the bond, we thus consider fixed-rate bonds. We can treat variable-rate bonds similarly, but the 'maturity' will be the next time the coupon payment is adjusted to the then prevailing market conditions.
- ▶ As we use the yield of the bond as our discount rate, this change in the yield will affect the bond value.
- ▶ We can now assess the sensitivity of the bond value to a change in the yield by taking the first derivative.
- ▶ [] We can take the term $-\frac{1}{1+r}$ out of the summation
- ▶ [] The term in brackets can now be replaced with DB.
- ▶ We can now define the term D as in the *formula*. This term is called the duration of the bond.
- → This duration is of central importance in the risk assessment of bonds.

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Interpretation of duration

- The duration of a bond measures the time it takes until an investor recovers its initial investment B
- The duration is always less then the maturity of the bond
- The duration of a zero-bond is identical to its time to maturity
- If the bond has variable coupons that track market rates, the time to maturity that is considered is only until the coupon is adjusted to the market rate
- As the coupon is reset to the market rate, the value of the bond will revert to its face value, similar to being repaid at maturity

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Interpretation of duration

→ We can now provide a more intuitive interpretation of the duration of a bond.

- With the investor buying a bond at its value B, the duration gives the time it takes until the investor has recovered this initial investment. Some of the initial investment is recovered through the coupon payments, before the bond is repaid. These coupon payments are discounted to their present value when determining the time it takes to recover the initial investment.
- As coupon payments are received before maturity, the repayment of the initial investment is completed before the bond is repaid, thus the duration is less than the time to maturity.
- Only for zero bonds is the time to maturity identical to the duration. This is because the investor doe snot receive any payments prior to the maturity of the bonds due to the bond having no coupons.
- For variable-rate bonds, the consideration of the bond is only until the coupon is reset.
-
    - At the reset of the coupon the bond has a yield that matches market conditions, hence the value is the face value
    - and this is comparable to the bond being repaid.

→ Thus duration measures the time until the investment into the bond is recovered.

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Bond risks

- We can now approximate the change of the bond value if the interest rate changes
- $\Delta B = \frac{\partial B}{\partial r}\Delta r = -\frac{DB}{1+r}\Delta r$

$\Rightarrow$ $Var\left[\frac{\Delta B}{B}\right] = D^2 Var\left[\frac{\Delta r}{1+r}\right]$

- The variance of the changes to the bond value is dependent on the duration of bonds, and the variance of the interest rate
- The larger the duration of a bond, the more volatile its value will be

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Bond risks

→ We can now return to determine the risk of bonds.

- We had determined the sensitivity of the bond to a change in the yield. WE can now use this sensitivity to make a linear approximation of the change in the bond value.
- If the yield changes by a discrete amount, the bond value will change by a factor equal to this sensitivity. We can insert for the sensitivity from above.
- [⇒] Dividing by the bond value gives us the return of the bond if the yield changes. We can now determine the variance of this change in the bond value as a function of the variance of the bond yield.
-
    - The variance of the bond value increases with the duration of the bond.
    - Of course, the more volatile the yield is, the more volatile the bond value.
- We thus see that bonds with longer durations are more risky.

→ The risk of bonds depends on the duration of bonds, not on their time to maturity. While for bonds with shorter maturities, there is a strong relationship between the two, for bonds with a long time to maturity this relationship doe snot hold anymore as the repayment of the initial investment is achieved by coupon payments on over many years and the timing of final repayment becomes irrelevant. It is thus possible to have a bond with a longer time to maturity, but a shorter duration (this would require the bond to have a higher coupon).

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Bond portfolios

- We consider a portfolio of two bonds: $B = B_1 + B_2$

$\Rightarrow$ $\frac{\partial B}{\partial r} = \frac{\partial B_1}{\partial r} + \frac{\partial B_2}{\partial r} = -\frac{1}{1+r}\left(D_1 B_1 + D_2 B_2\right)$

$\Rightarrow$ $\Delta B = -\left(D_1 B_1 + D_2 B_2\right)\frac{\Delta r}{1+r}$

- We define the weight of a bond in the portfolio as $\omega_i = \frac{B_i}{B}$

$\Rightarrow$ $\frac{\Delta_B}{B} = -\left(\omega_1 D_1 + \omega_2 D_2\right)\frac{\Delta r}{1+r}$

- The duration of the bond portfolio is the weighted average of the durations of the individual bonds

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Bond portfolios

- → So far we assessed the risk of a single bond, but often investors hold portfolios of several bonds with different durations. We will therefore now look at the risk in such bond portfolios.
- ▶ We assume two bonds are held in a portfolio with their respective values.
- ▶ [⇒] We can now determine the sensitivity of this bond portfolio and using the results from above, obtain this *formula*.
- ▶ [⇒] As before, we can make a linear approximation.
- ▶ We now introduce the relative weight of the two bonds in the portfolio.
- ▶ [⇒] Dividing the above expression by the portfolio value B and using the portfolio weights, gives us this *formula*.
- ▶ Comparing this result with that of a single bond, we see that the term $\omega_1 D_1 + \omega_2 D_2$ corresponds to the duration of the portfolio. Thus the duration of a bond portfolio is the weighted average of the durations of the bond, with weights given by the weight of the bonds in the portfolio.
- → We can now established the duration of a bond portfolio.

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Eliminating interest rate risks from bonds

- If we hold a bond and want to eliminate interest rate risk, we can form a portfolio such that $D = \omega_1 D_1 + \omega_2 D_2 = 0$

$\Rightarrow$ $\omega_2 = \frac{D_1}{D_1 - D_2}$

- By adding another bond with a different duration to our portfolio. we can eliminate interest rate risk
- If the other bond used to hedge our interest rate exposure has a longer duration than the bond we seek to hedge, $D_2 > D_1$, we will have to hold a short position
- If $D_2 > D_1$, then we easily see that the new bond needs a value of $\omega_2 > 1$ and hence a short position of the original bond, $\omega_1 < 0$ which is not feasible
- This hedge is only effective against equal changes in the interest rates for all maturities

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Eliminating interest rate risks from bonds

→ Using the duration of a bond portfolio, we can now see how we can eliminate the interest rate risk of holding a bond.

- In order to eliminate the risk from holding a bond, we form a portfolio of bonds whose duration is zero. Such a portfolio would not change in value as the yield of bonds changes and therefore be free of interest rate risk.
- [⇒] Assuming we hold bond 1, we can solve for how much another bond we need to hold such that any risk is eliminated. We note that the weights must sum up to 1, hence $\omega_1 + \omega_2 = 1$, allowing us to solve for ω_2.
- We can add another bond with a different duration to our existing bond (or a portfolio of bonds with that duration) and eliminate all interest rate risk.
- We have thus obtained a hedge against interest rate risk by using another bond. If this other has a longer duration, the weight will be negative, meaning we will hold a short position in this bond, otherwise it will be along position.
- If the duration of the bond used for hedging is shorter than the bond we hold, we would need to hold a short position in this original bond; this is not possible and hence we need to hedge with a bond of a longer duration using a short position.
- Our analysis suggested that the change in the yield was Δr, regardless of the time to maturity. This implies that we only considered parallel shifts in the yield curve.

→ We thus can hedge our exposure to interest rate risk by using bonds of different durations.

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Duration gap in banks

- Banks use the differences in duration between their assets (loans) and their liabilities (deposits) as a measure for their interest rate risk
- This is referred to as the duration gap
- Banks then manage this duration gap by granting more short-term loans or attracting more long-term deposits

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Duration gap in banks

→ Banks by the nature of their business are very exposed to interest rate risk, in addition to credit risk in the loans they provide. While the do not hold bonds, providing a loan is in principle the same as holding a bond, it has the same characteristics; similar do deposits have the characteristics of bonds. Both require fixed payments and a repayment of the initial loan or deposit.

- Banks provide long-term loans (long duration), but finance these loans with short-term deposits (short duration). The total exposure to interest rate risk is the difference between the durations of the loans and deposits.
- This difference is known as the 'duration gap'.
- Banks manage this duration gap, and hence their interest rate risk, by granting more short-term loans (lower duration for loans) and seeking more long-term deposits (longer duration for deposits). The can do so by making short-term loans more attractive by offering lower loan rates for these loans and attract more long-term deposits by offering attractive deposot rates for such deposits.

→ Banks are actively managing their exposure to interest rate risk through adjusting the duration of the loans and deposits they obtain.

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